Astron. & Astrophys. 31, 185–188 (1974)

Pulsational Instability towards Non-radial Oscillations in Homogeneous Stars of Small Mass

A. Noels, A. Boury, R. Scuflaire and M. Gabriel
Institut d'Astrophysique de l'Université de Liège

Received January 3, 1974

Summary. The pulsational stability of a 0.5 $M_\odot$ star composed of a radiative core and an extended convective envelope is studied. The star is found strongly unstable because of the high temperature sensitivity of the He^3 reaction. This instability should affect only a very small range of masses.

Key words: non-radial oscillations – pulsational stability

I. Introduction

Stars are vibrationally stable or unstable, depending upon which of two effects is dominant. The first one, destabilizing, comes from nuclear reactions and has its seat in the central layers. Oppositely, the external layers usually exert a stabilizing influence. An oscillation mode with a large amplitude in the core and a vanishing one in the external layers will of course favour the presence of a vibrational instability towards that particular mode. In the case of non-radial adiabatic oscillations, such an amplitude distribution can occur, under certain conditions, for the gravity modes g^+ (see for example Ledoux and Smeyers, 1966). As a matter of fact, in a homogeneous star of small mass, the amplitude of those modes is large and oscillating in the radiative core and decreases exponentially as one penetrates in the extended convective envelope. In such a star, favourable conditions for vibrational instability exist. An investigation of this effect has already been done (Robe *et al.*, 1972) in the case of a homogeneous 0.5 $M_\odot$ star. In fact, despite the large amplitudes reached by the g^+ modes in the radiative core, the model considered was found vibrationally stable. The temperature sensitivity, ν, of the nuclear energy generation rate was however taken incorrectly equal to its value in the static model. As the stability was already rather marginal, this question deserved more attention. In the present work, we have tested the vibrational stability of a 0.5 $M_\odot$ population II star, described in Section II, towards non radial g^+ modes, using the effective sensitivity to temperature, ν_{eff}, in the expression of the perturbation of the energy rate. The model is found unstable, with an e-folding time of amplification much shorter than the evolutionary time.

II. Models

The evolution of a 0.5 $M_\odot$ star from the gravitational contraction to the post main sequence phases has been computed with a Henyey method. The adopted chemical composition was characteristic of an old population II star (X = 0.900, Y = 0.099, Z = 0.001). All details concerning the physics used and the description of the evolutionary models can be found in Noels (1972). We only give here, in Table 1, the properties of the model (1) tested for vibrational instability. That model was chosen so that the two most important conditions for instability be fulfilled: (1) the nuclear reactions have to take place in a *radiative* core and (2) the chemical composition must be as *homogeneous* as possible (see Section III). The model finally adopted corresponds

Table 1. Properties of the models of 0.5 $M_\odot$ (Noels, 1972) and of 0.6 $M_\odot$ (Toma, 1972)

Mass	$\frac{X_{\text{center}}}{X_{\text{surface}}}$	$T_c(\cdot 10^{-6})$	ϱ_c	$\varrho_c/\bar{\varrho}$	$L(\cdot 10^{-33})$	$\log T_e$	$\frac{m_{\text{env}}}{M_{\text{tot}}}$
0.5 $M_\odot$ (1)	0.9008	8.384	82.99	7.396	0.1811	3.6304	0.476
(2)	0.0003	18.42	3772.	3105.	0.8971	3.6428	0.091
0.6 $M_\odot$	0.9925	10.69	96.85	21.67	0.3883	3.6334	0.106

exactly to the disappearance of the convective core which had developed in the early hydrogen burning phase. The chemical composition is still very nearly homogeneous.

In Table 1, we have also listed the properties of a more evolved model (2) of the same star and those of a similar model of a 0.6 $M_\odot$ star, obtained by Toma (1972), which has been tested here for comparison purpose.

III. Non-radial Adiabatic Oscillations

The theory of non-radial pulsations can be found in Ledoux (1969). We have integrated the fourth order system, taking into account the perturbation of the gravitational potential. Two independent solutions satisfying the central boundary conditions are computed using Heun's method. The eigenvalues are determined by demanding that a linear combination of those solutions satisfy the surface boundary conditions.

The adiabatic frequencies of oscillation, $\sigma_{k,l}$ corresponding to the first g^+ modes, computed for 3 values of the degree l of the spherical harmonic ($l=1, 2, 4$) are listed in Table 2. To illustrate the behaviour of the corresponding eigenfunctions, we have plotted in Fig. 1, for $l=2$ (g_1, g_2, g_3, g_4), the radial displacement, $\delta r/R$, as a function of r/R. It should be noticed here (but more emphasis will be put on that point in a forthcoming paper) that such an amplitude distribution in a g^+ mode remains similar, as long as the model is close enough to the zero age main sequence. For example, in the case of a more evolved model (2) of the same star, the amplitude becomes important again in the external convective layers (see Fig. 2).

Table 2. Adiabatic frequencies of oscillation, $\sigma_{k,l}$, and pulsational stability results (see text) in the case of the 0.5 $M_\odot$ star

	σ^2	E_n	E_F	σ'	$\lvert\sigma'\rvert^{-1}$
$l=1$ g_1	5.989(−7)	1.567(33)	9.054(32)	−2.244(−15)	1.413(7)
g_2	2.112(−7)	4.703(32)	3.075(32)	−2.359(−15)	1.345(7)
g_3	1.008(−7)	2.544(32)	1.891(32)	−2.068(−15)	1.533(7)
g_4	5.609(−8)	1.858(32)	1.552(32)	−1.589(−15)	1.996(7)
$l=2$ g_1	1.011(−6)	6.660(31)	4.334(31)	−1.682(−15)	1.886(7)
g_2	4.435(−7)	1.157(31)	8.378(30)	−1.695(−15)	1.871(7)
g_3	2.374(−7)	4.225(30)	3.319(30)	−1.576(−15)	2.013(7)
g_4	1.390(−7)	2.033(30)	1.844(30)	−8.082(−16)	3.925(7)
$l=4$ g_1	1.416(−6)	1.233(29)	1.133(29)	−3.393(−16)	9.348(7)
g_2	8.120(−7)	1.205(28)	1.163(28)	−1.822(−16)	1.741(8)
g_3	5.079(−7)	2.463(27)	2.269(27)	−5.257(−16)	6.033(7)
g_4	3.275(−7)	7.419(26)	7.552(26)	+1.295(−16)	2.449(8)

IV. Vibrational Stability

With a time dependence of the perturbations of the form $e^{i\sigma_{k,l}t}\, e^{-\sigma'_{k,l}t}$, the damping coefficient $\sigma'_{k,l}$ of the

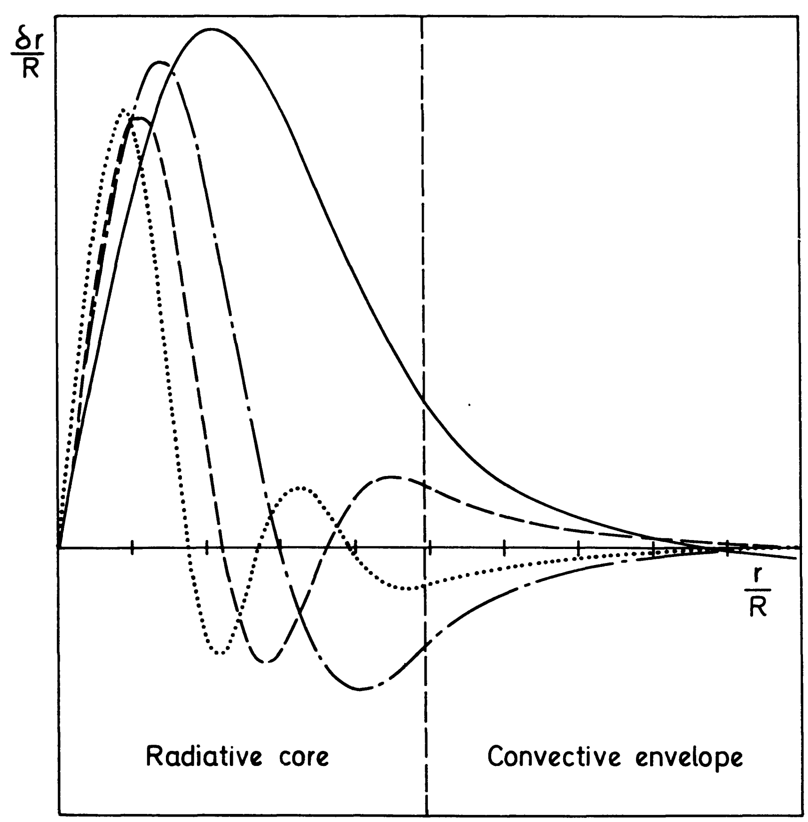

Fig. 1. Radial component, $\delta r/R$, of the displacement for $l=2$: mode g_1(———), g_2(–·–·–), g_3(– – – –), g_4(······), as a function of r/R, in the case of a 0.5 $M_\odot$ homogeneous star (1) ($\delta r/r=1$ at the center; maximum scale limit $=1.6\cdot10^{-1}$ for g_1 and $1.3\cdot10^{-1}$ for g_2, g_3, g_4)

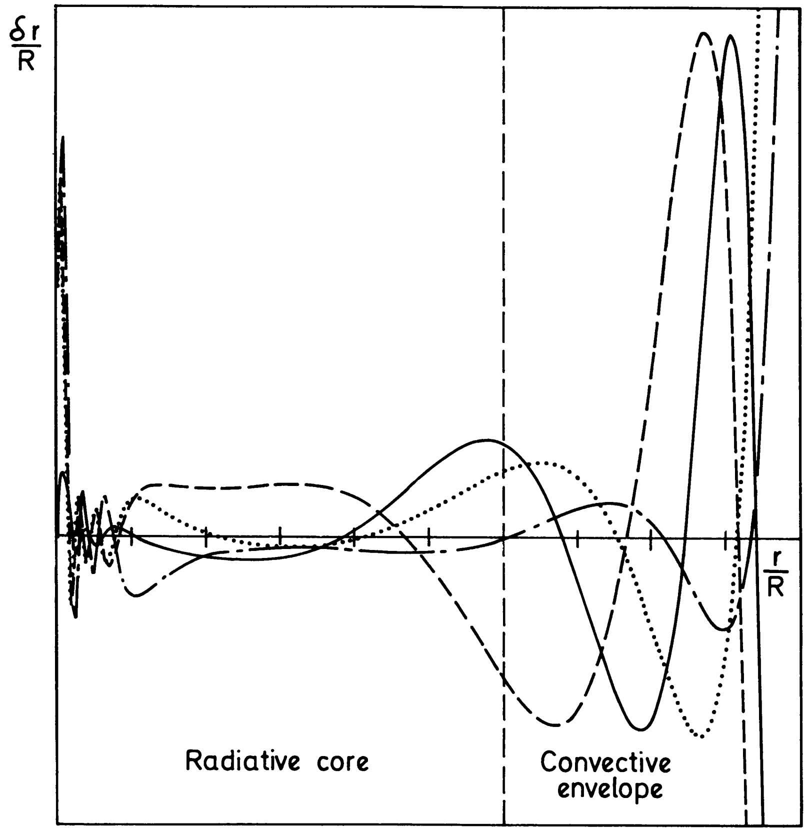

Fig. 2. Same as Fig. 1, in the case of a 0.5 $M_\odot$ evolved star (2) ($\delta r/r = 1$ at the center; maximum scale limit $= 4 \cdot 10^{-2}$ for g_1 and $6 \cdot 10^{-3}$ for g_2, g_3, g_4)

mode k associated with the l^{th} harmonic writes (see Ledoux, 1969):

$$\sigma'_{k,l} = -\frac{1}{2\sigma_{k,l}^2} \frac{\int_0^M \left(\frac{\delta T}{T}\right)_{k,l} \delta\left[\varepsilon - \frac{1}{\varrho} \boldsymbol{\nabla} \cdot \boldsymbol{F}\right]_{k,l} dm}{\int_0^M (\delta \boldsymbol{r} \cdot \delta \boldsymbol{r}^*)_{k,l}\, dm} \tag{1}$$

where δ is the lagrangian perturbation symbol. As usual, the non adiabatic terms are calculated by means of the adiabatic solution. A positive (negative) value of $\sigma'_{k,l}$ means stability (instability) towards the mode $[k, l]$ The method used to obtain $\delta\varepsilon$ is described in Ledoux and Sauvenier-Goffin (1950) and in Schatzman (1951). In the case considered here, ε can be written as the sum of 3 contributions: $\varepsilon_{(\mathrm{H}^1+\mathrm{H}^1)}$, $\varepsilon_{(\mathrm{H}^2+\mathrm{H}^1)}$ and $\varepsilon_{(\mathrm{He}^3+\mathrm{He}^3)}$. For each contribution, the corresponding perturbation writes:

$$\left(\frac{\delta\varepsilon_{ij}}{\varepsilon_{ij}}\right)_{k,l} = \left(\frac{\delta\varrho}{\varrho}\right)_{k,l} + \nu_{ij}\left(\frac{\delta T}{T}\right)_{k,l} + \left(\frac{\delta \mathrm{X}_i}{\mathrm{X}_i}\right)_{k,l} + \left(\frac{\delta \mathrm{X}_j}{\mathrm{X}_j}\right)_{k,l} \tag{2}$$

$$= \left(\frac{\delta\varrho}{\varrho}\right)_{k,l} + \nu_{\text{eff}}\left(\frac{\delta T}{T}\right)_{k,l} \tag{3}$$

which defines the effective sensitivity to temperature.

$\delta \mathrm{X}_3/\mathrm{X}_3$ is found to be negligible and $\delta \mathrm{X}_2/\mathrm{X}_2$ is given in Boury and Noels (1973).

The central value of ν_{eff} was found to be 11.8. In the work of Robe *et al.* (1972), relation (3) was used with the static ν value of the entire proton-proton chain (that is to say ~ 5) instead of ν_{eff}. That error led those authors to underestimate appreciably the destabilizing factor in the expression of $\sigma'_{k,l}$. The second term in expression (1), $\delta\left[\frac{1}{\varrho}\boldsymbol{\nabla}\cdot\boldsymbol{F}\right]$, is written in the following way:

$$\delta\left[\frac{1}{\varrho}\boldsymbol{\nabla}\cdot\boldsymbol{F}_T\right] = \frac{d(\delta L_C)_{\text{radial}}}{dm} + \frac{d(\delta L_R)_{\text{radial}}}{dm} - l(l+1)\frac{\chi}{\sigma^2 r^2}\frac{dL_T}{dm} + l(l+1)\frac{\chi}{\sigma^2 r^2}\frac{F_T}{\varrho r} - \frac{F_R}{\varrho r^2} l(l+1)\left[\frac{\delta T}{T}\left(\frac{d\ln T}{dr}\right)^{-1} - \delta r + \frac{\chi}{\sigma^2 r}\right] \tag{4}$$

where subscripts (k, l) have been omitted for brevity. Subscripts C, R, T mean respectively convective, radiative and total. $(\delta L_C)_{\text{radial}}$ has been taken from Boury *et al.* (1964). The term corresponding to the non-radial part of $\left[\frac{1}{\varrho}\boldsymbol{\nabla}\cdot\delta\boldsymbol{F}_C\right]$ has been taken equal to zero, which is of no consequence in this case, as the amplitudes are small in the envelope.

In the nuclear burning core, the predominant contribution to σ' comes from the nuclear term while, in the outer layers, term (4) remains the only contribution.

Table 3. Same as Table 2 in the case of the 0.6 $M_\odot$ star

	σ^2	E_n	E_F	σ'	$\lvert\sigma'\rvert^{-1}$
$l=1\ g_1$	1.173(−6)	1.008(34)	9.802(33)	−5.802(−17)	5.467(8)
g_2	5.723(−7)	9.770(33)	8.351(33)	−7.967(−16)	3.981(7)
g_3	3.181(−7)	4.609(33)	4.553(33)	−9.872(−17)	3.212(8)
g_4	1.950(−7)	2.508(33)	3.319(33)	+2.924(−15)	1.084(7)
$l=2\ g_1$	2.568(−6)	5.379(32)	1.212(33)	+7.942(−16)	3.994(7)
g_2	1.318(−6)	2.688(32)	3.831(32)	+1.467(−16)	2.162(8)
g_3	7.638(−7)	1.071(32)	1.490(32)	+2.447(−15)	1.296(7)
g_4	4.897(−7)	4.414(31)	7.583(31)	+5.442(−15)	5.828(6)
$l=4\ g_1$	4.751(−6)	1.353(30)	1.513(31)	+3.522(−15)	9.005(6)
g_2	2.685(−6)	3.994(29)	1.893(30)	+5.529(−15)	5.737(6)
g_3	1.692(−6)	1.135(29)	3.787(29)	+7.984(−15)	3.973(6)
g_4	1.162(−6)	3.271(28)	1.043(29)	+9.021(−15)	3.516(6)

Table 2 gives, for each computed mode, the nuclear term, $E_n\left(=\int_0^M \frac{\delta T}{T}\,\delta\varepsilon\, dm\right)$, the flux term

$$E_F\left(=\int_0^M \frac{\delta T}{T}\,\delta\left(\frac{1}{\varrho}\,\nabla\cdot \boldsymbol{F}\right) dm\right),$$

the damping coefficient σ' and the e-folding time of amplification $|\sigma'|^{-1}$. For all, but one, modes tested here, the model is unstable towards the corresponding mode. For $l=1$, 2, the instability is maximum for the first two g^+ mode and then decreases. For $l=4$, the stability is in fact restored for the g_4^+ mode.

In order to display the importance of the presence of an extended envelope, we have applied a similar treatment to a homogeneous 0.6 $M_\odot$ star, whose physical properties are given in Table 1. The corresponding stability results are listed in Table 3. In a case like this, where the radiative core is much larger than the nuclear burning region, the star remains stable towards the g^+ modes of oscillation, except in the case $l=1$, where the instability towards the g_1, g_2 and g_3 modes is nevertheless somewhat marginal.

V. Conclusions

In the case of a homogeneous star, composed of a radiative core and a convective envelope, non-radial g^+ modes have a large, oscillating amplitude in the core and a vanishing one in the convective envelope. If the convective region is sufficiently extended, the destabilizing effect of the nuclear reactions overwhelms the stabilizing one. This type of instability is susceptible to prevent the building up of a gradient of molecular weight in the nuclear burning region and the subsequent evolution will be correspondingly modified. The range of stellar masses affected by this effect is however very narrow, on both sides of 0.5 $M_\odot$. Higher masses (already $\gtrsim 0.6\,M_\odot$) do not have a convective envelope extended enough to be unstable. On the other hand, for smaller masses, the nuclear burning region does not remain in radiative equilibrium, so the stability could be quickly restored The extension of the convective envelope and consequently, the exact range of stellar masses unstable towards the g^+ modes of oscillations naturally depend on the convective mean free path in the external layers.

References

Boury, A., Gabriel, M., Ledoux, P. 1964, *Ann. Astrophys.* **27**, 92
Boury, A., Noels, A. 1973, *Astron. & Astrophys.* **24**, 255
Ledoux, P. 1969, La Structure Interne des Etoiles, XI cours de perfectionnement de l'association vaudoise des chercheurs en Physique, Saas-Fee, Suisse, p. 44
Ledoux, P., Sauvenier-Goffin, E. 1950, *Astrophys. J.* **111**, 611
Ledoux, P., Smeyers, P. 1966, *Compt. Rend. Acad. Sci. Paris* **262**, 841
Noels, A. 1972, *Bull. Soc. Roy. Sci. Liège* **41**, 50
Robe, H., Ledoux, P., Noels, A. 1972, *Astron. & Astrophys.* **18**, 424
Schatzman, E. 1951, *Ann. Astrophys.* **14**, 305
Toma, E. 1972, *Astron. & Astrophys.* **19**, 76

A. Noels, A. Boury, R. Scuflaire, M. Gabriel
Institut d'Astrophysique
5, avenue de Cointe
B-4200 Cointe-Ougrée, Belgique