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Space Oscillations of Stellar Non Radial Eigen-functions

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Summary. It is generally believed that the g^- modes of non radial oscillations of a star may oscillate only in convective zones, and that the g^+ modes may oscillate only in radiative zones. Whereas the first statement may be proved quite generally, the second one is true only in the inner part of the star. With this restriction it is valid also for p modes.

Key words: non radial oscillations — convective zones — evolved stars

1. Introduction

Noels *et al.* (1974) have computed the first g^+ modes of non radial oscillations for two models of a star of $0.5\,M_\odot$. For the second model (the more evolved one) these modes oscillate in the exterior convective zone. Now it is generally admitted that the g^+ modes may oscillate only in radiative zones (Ledoux and Smeyers, 1966; Smeyers, 1966; see also Ledoux, 1969). In fact, the conclusions in these papers were based upon a hypothesis which is satisfied in the whole star for the g^+ modes only in the asymptotic case (σ^2 small). It seems worthwhile therefore to reexamine this question in a more general context.

2. Oscillation Theorems

When the perturbations of gravity are neglected, the non radial oscillations of the spherical harmonic of degree l satisfy the Eqs. (1) and (2) (Ledoux and Walraven, 1958)

$$\frac{dv}{dr} = \left[\frac{l(l+1)}{\sigma^2} - \frac{\varrho r^2}{\Gamma_1 P}\right]\frac{P^{2/\Gamma_1}}{\varrho} w \tag{1}$$

$$\frac{dw}{dr} = (\sigma^2 + Ag)\frac{\varrho}{r^2 P^{2/\Gamma_1}} v\,. \tag{2}$$

Most of the notations are fairly conventional.

$$v = r^2 \delta r P^{1/\Gamma_1}$$

$$w = P'/P^{1/\Gamma_1}$$

where P' is the local perturbation of the pressure.

$$A = \frac{d\ln\varrho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr}.$$

The sign of A is related to the convective instability: $A<0$ means stability and $A>0$ means instability.

Let a and b be defined as follows

$$a = \left[\frac{l(l+1)}{\sigma^2} - \frac{\varrho r^2}{\Gamma_1 P}\right]\frac{P^{2/\Gamma_1}}{\varrho}$$

$$b = (\sigma^2 + Ag)\frac{\varrho}{r^2 P^{2/\Gamma_1}}\,.$$

The system (1)–(2) may be rewritten

$$\frac{dv}{dr} = aw \tag{3}$$

$$\frac{dw}{dr} = bv\,. \tag{4}$$

Let us consider an interval $[r_1, r_2]$ where a and b keep a constant sign. The solutions of (3)–(4) may oscillate in this interval only if a and b have opposite signs. This is a consequence of the Sturm's comparison theorem and may be verified directly as follows. Let r' and r'' be two zeros of v, then dv/dr vanishes at an intermediate point. Thus a zero of w always seperates two zeros of v. In the same manner a zero of v always seperates two zeros of w. Let now r' and r'' be respectively a zero of v and a zero of w and let us consider the product vw. It vanishes at r' and r'', thus its derivative

$$\frac{d}{dr}(vw) = aw^2 + bv^2$$

must vanish at an intermediate point. This is possible only if a and b have opposite signs. These two propositions allow us to conclude that v and w cannot oscillate in $[r_1, r_2]$ if a and b have the same sign.

(a) g^- Modes

If $\sigma^2<0$, $a<0$ in the whole star. The solutions of (1)–(2) may oscillate only if $b>0$ that is if

$$Ag>-\sigma^2>0\,. \tag{5}$$

This condition may be satisfied only in convective zones. Thus Theorem 1 follows: the g^- modes may oscillate only in convective zones ($A>0$).

(b) g^+ and p Modes

Let us write

$$a=a_1\frac{P^{2/\Gamma_1}}{\varrho}$$

with

$$a_1=\frac{l(l+1)}{\sigma^2}-\frac{\varrho r^2}{\Gamma_1 P}\,.$$

If $\sigma^2>0$, a_1 is a decreasing function of r (except possibly if Γ_1 increases rapidly in a short interval or at a discontinuity of the mean molecular weight, we shall neglect these particular cases) positive near the center and negative near the surface. For a given mode, i.e. given l and σ^2, let us denote by r_0 the value of r where a_1 vanishes. Let us remark that r_0 increases with l and decreases when σ^2 increases. For a given mode let us consider separately the regions of the star $r<r_0$ and $r>r_0$.

If $r<r_0$, $a>0$ and the solutions of (1)–(2) may oscillate only if $b<0$, that is if

$$Ag<-\sigma^2<0\,. \tag{6}$$

There follows Theorem 2: a given g^+ or p mode may oscillate in the region $r<r_0$ only in radiative zones ($A<0$).

Let us now consider the case $r>r_0$. Then $a<0$ and oscillations are possible only if $b>0$, that is

$$Ag>-\sigma^2\,. \tag{7}$$

This condition cannot exclude radiative or convective zones. g^+ and p modes, when $r>r_0$ may oscillate in both radiative or convective zones. However oscillations are not allowed in a radiative zone if it is too stable, that is if the Väisälä frequency is greater than the frequency of the oscillation

$$n^2=-Ag>\sigma^2\,. \tag{8}$$

As g^+ modes have lower frequencies than p modes and r_0 is a decreasing function of σ^2, the domain of application of Theorem 2 is larger for g^+ modes than for p modes. As $\sigma^2\to 0$, $r_0\to R$ and Theorem 2 becomes valid throughout the star. This confirms the conclusions of Ledoux and Smeyers (1966) for asymptotic g^+ modes. On the contrary as $\sigma^2\to\infty$, that is for asymptotic p modes, the domain of validity of Theorem 2 shrinks to zero.

For the lower g^+ and p modes there is always an exterior layer in which Theorem 2 does not apply. However it has to be sufficiently thick in order that oscillations may effectively be observed in that layer. This condition seems to be realized in evolved models (see next section).

3. g^+ Modes of Evolved Models

Figures 1 and 2 give respectively for the first and the second models of 0.5 $M_\odot$ (Noels *et al.*, 1974) the function $r_0(\sigma^2)$ in dimensionless variables

$$x_0=r_0/R\,,$$

$$\omega^2=\frac{R^3\sigma^2}{GM}\,.$$

These curves are very similar for both models. We have also represented the square of the frequencies of the first four $l=2$ g^+ modes. For the first model the frequencies of these modes are such that $x_0\simeq 0.95$. Theorem 2 applies in about 95% of the star (in radius). Thus in the larger part of the convective zone oscillations are not allowed, as it is shown in Fig. 1 of Noels *et al.* (1974). For the second model, the dimensionless frequencies of the same modes are much higher than in the first one and Theorem 2 is valid only up to about

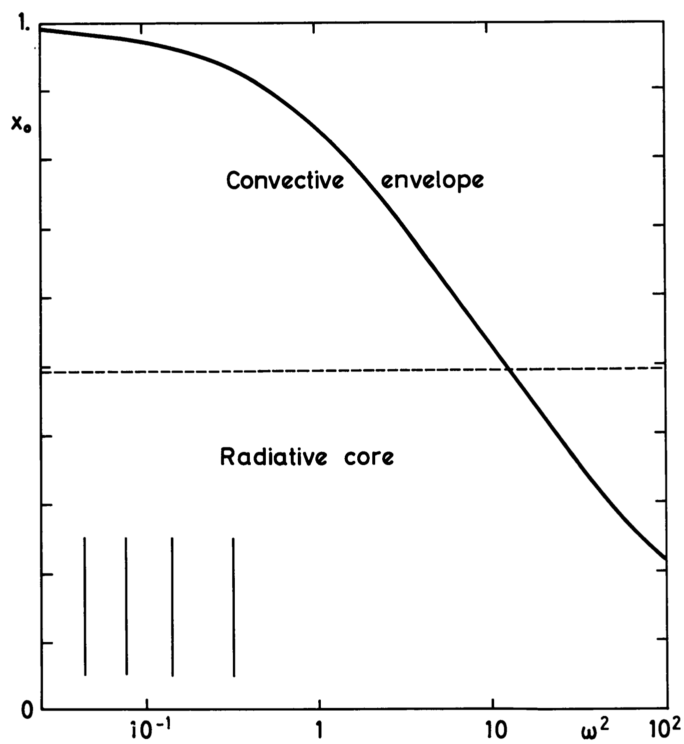

Fig. 1. x_0 as a function of ω^2 ($l=2$) for Model 1. The vertical lines represent the values of ω^2 for modes $g_1^+,\dots,g_4^+$ from right to left

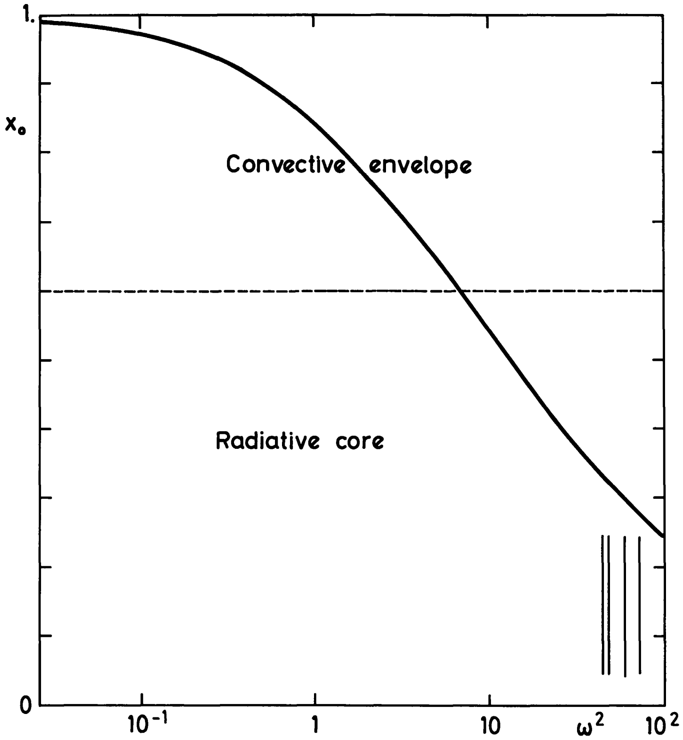

Fig. 2. Same as Fig. 1 but for Model 2

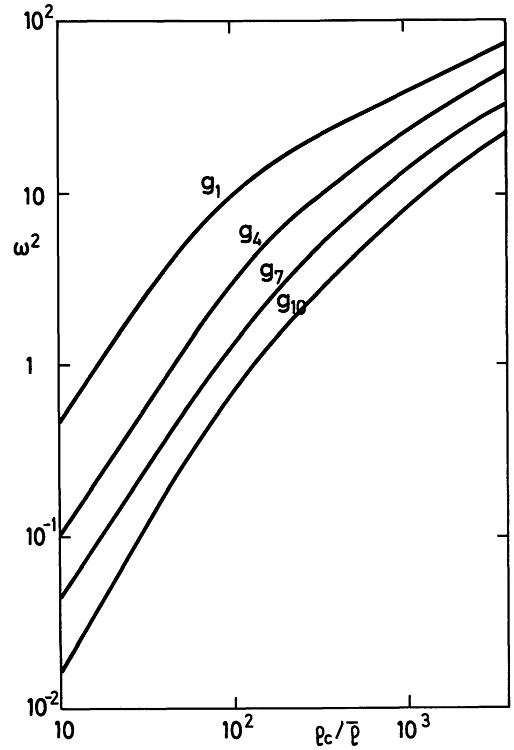

Fig. 3. ω^2 of a few $l=2$ g modes as a function of the central condensation $\varrho_c/\bar{\varrho}$ for polytropic models ($\Gamma_1 = 5/3$)

30% of the radius. Thus oscillations are allowed in the convective zone. They are effectively present (Fig. 2 of Noels *et al.*).

Figure 3 gives for a few $l=2$ modes ω^2 as a function of the central condensation $\varrho_c/\bar{\varrho}$ for a sequence of polytropic models ($\Gamma_1 = 5/3$). The increase of ω^2 with the central condensation seems to be a general fact, confirmed also by numerous computations of non radial oscillations of physical models. We can say with great confidence that the oscillations of g^+ modes in exterior convective zones is not a peculiar characteristic of the model considered here but a general feature of all evolved models.

4. Conclusions

In evolved models, g^+ modes do not decrease exponentially in exterior convective zones. They present oscillations and may have large amplitudes. In vibrational stability computations this behaviour gives more weight to external convective layers than was thought before numerical computations brought out this phenomenon.

This work originated in discussions with Professor Ledoux about the behaviour of g^+ modes in the model discussed above.

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