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29. — STABILITY OF G+ MODES IN A 30 $M_\odot$ STAR

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ABSTRACT

The evolution of a population I 30 $M_\odot$ star during the main-sequence phases has been computed without semi-convective zone. Three models are tested for stability towards g^+ modes of non-radial oscillation. Whereas the Z.A.M.S. model is found stable, evolved models are unstable.

In our interpretation, the cause of the instability resides in the existence of super-adiabatic temperature gradient in the region of varying chemical composition.

I. INTRODUCTION

Kato (1966) suggested that in a region of varying molecular weight, some mixing takes place, due to vibrational instability (overstability) appearing as soon as the temperature gradient $\nabla_T = \dfrac{\partial\, ln\, T}{\partial\, ln\, p}$ exceeds the adiabatic value $\nabla_{a,T} = \dfrac{\Gamma_2 - 1}{\Gamma_2}$.

Such a situation is encountered in stars of mass larger than $\sim 10\, M_\odot$ during the mains sequence phase when their evolution is computed ignoring semiconvection (see Taylor 1969, Gabriel 1970). In these stars, according to Kato, the overstability will produce a partial mixing of the material in the region of varying molecular weight and it will reduce the gradient in such a way that $\nabla_T = \nabla_{a,T}$.

However Kato's discussion is a strictly local one and before concluding to any instability the influence of the other regions of the star must be taken into account. This has been attempted by Gabriel (1969) and by Aure (1971) in the case of asymptotic modes. For these modes no instability was found. The aim of the present study is to check whether Kato's mechanism is able to destabilize low order g^+ modes. We find a positive answer for evolved M.S. models. However if this mechanism were the only one to operate in order to modify the molecular weight profile, it would achieve a value of ∇_T intermediate between Ledoux's and Schwarzschild's value. Nevertheless, the mechanism proposed by Gabriel (1970) is more efficient in forming semiconvective zones and could prevent this instability to appear.

II. THE MODELS

The evolution of a 30 $M_\odot$ star with a chemical composition given by $X = .602$, $Z = .044$ was started during the gravitaional contraction just before the onset of the CNO cycle and carried on until the end of central hydrogen burning. No semi-convective region was included in the models. Therefore during the main sequence phases models may derived into 4 zones shown schematically in fig. 1 : a convective core, a region of variable molecular weight left by the withdrawing of the core, a small convective shell and the radiative envelope. The mass at bottom of the convective shell 0,635 $M_{\odot}/M$ is equal to the mass of the convective core on the Z.A.M.S. This point is not convectively neutral ($\nabla_T > \nabla_{a,T}$) but in the region just

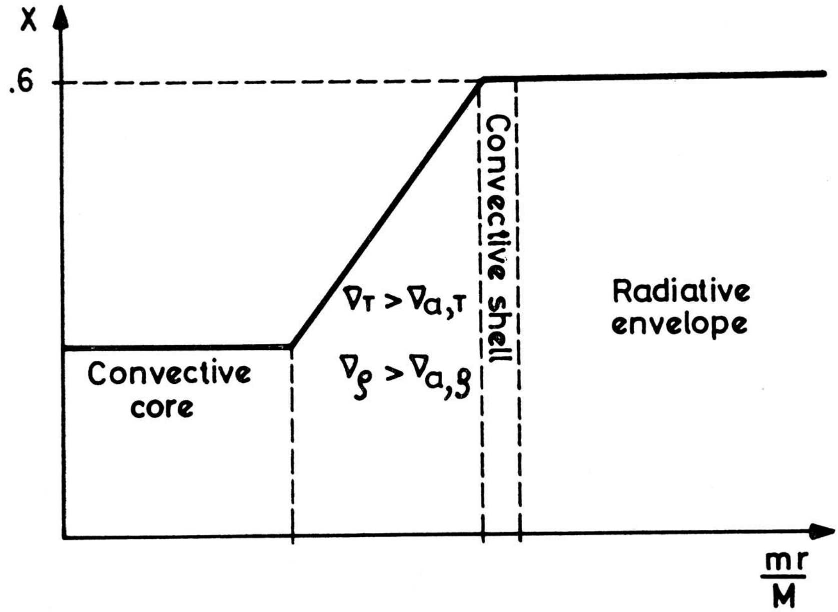

Fig. 1. — Schematic hydrogen profile in massive M.S. stars and the four zones of the corresponding models.

beneath, the material is stabilized against convection by the μ-gradient. The convective shell grows steadily during the course of evolution and its maximum mass is 3.7 %. In the region of varying molecular weight the temperature gradient is intermediate between Schwarzschild's and Ledoux's values i.e.

$$\nabla_{a,T} < \nabla_T < \nabla_{a,T} + \left(\frac{\delta \ln T}{\partial \ln \mu}\right)_{\rho,p} \frac{d \ln \mu}{d \ln p} \quad (1)$$

and the influence of the Kato's mechanism may be tested in these models.

III. THE VIBRATIONAL STABILITY

The fourth order system of equations for non radial adiabatic oscillations was integrated taking into account the perturbation of the gravitational potential. The damping coefficient σ' is evaluated in terms of the solution of the adiabatic problem and is given as (see for instance P. Ledoux 1973)

$$\sigma' = -\frac{2\sigma_a}{1} \frac{\int_0^{M_a} \frac{\delta T}{T} \delta \left[\varepsilon - \frac{1}{\rho} \vec{\nabla} . \vec{F} \right] dm}{\int_0^M \delta \vec{r} . \delta \vec{r} \, dm}$$

where δ is the lagrangian perturbation symbol and σ_a is the eigenvalue of the adiabatic problem. The upper limit, M_a, of the integral of the numerator is determined as the mass point where

$$\frac{\Gamma_2 - 1}{\Gamma_2} \left| \frac{\delta p}{p} \right| = \frac{1}{\sigma_a C_p T} \left| \delta \left[\varepsilon - \frac{1}{\rho} \vec{\nabla} . \vec{F} \right] \right| \tag{2}$$

(both members being calculated with the adiabatic solution)

The term $\delta \left(\frac{1}{\rho} \vec{\nabla} . \vec{F} \right)$ is (Gabriel et al. (1974))

$$\delta \left(\frac{1}{\rho} \vec{\nabla} . \vec{F} \right) = \frac{d \delta L^r}{dm} - \frac{l(l+1)}{r^2} \left[\frac{\delta F^h}{\rho} + \frac{\chi}{\sigma_a^2} \frac{dL}{dm} - \frac{F}{\sigma_a^2 \rho} \frac{\chi}{r} \right] \tag{3}$$

where

$$\frac{\delta L^r}{L} = 2 \frac{\delta r}{r} + \frac{\delta F^r}{F^r}$$

$$\chi = \frac{p'}{\rho} + \varphi'$$

$$\delta F^i = \delta F^i_R + \delta F^i_c$$

$\vec{F}$, $\vec{F}_R$, $\vec{F}_c$ designate respectively the total, the radiative and the convective fluxes, δF^h is the component of the perturbation of the flux perpendicular to the radius, p' and φ' are the eulerian perturbations of the pressure and the gravitational potential.

This expression is valid both in a convective and in a radiative region. The perturbation of the convective flux $\vec{F}_c$ is given by a generalization of Unno's work (1967) to non-radial oscillations (for details see to Gabriel et al. 1974).

The results are given in tables I and II for a degree l of the spherical harmonic equal to 1 and 2 respectively. The tables give for 3 models and for several g modes

TABLE I

Periods P *and* e-*folding times* σ'^{-1} *for the 4 first* g^+ *mode* ($l = 1$) *of 3 main sequence models (a negative value of* σ'^{-1} *means instability).*

	Nº	1	2	3
g_1	P(x)	6.1081(4)	3.3220(4)	3.2588(4)
	σ'^{-1}(yr)	3.771(3)	2.023(3)	2.884(0)
g_2	P	9.6652(4)	7.6119(4)	3.7840(4)
	σ'^{-1}	6.706(3)	9.642(3)	1.103(2)
g_3	P	1.3162(5)	1.0486(5)	6.0035(4)
	σ'^{-1}	1.196(2)	2.838(3)	8.146(3)
g_4	P	1.6702(5)	1.3802(5)	8.443(4)
	σ'^{-1}	2.827(1)	— 8.921(4)	— 1.227(3)

TABLE II

Same as Table I. Besides, the 3 first lines give the mass concentration $\rho_c/\bar{\rho}$, the central hydrogen abundance X_c and the age of the models

N°		1	2	3
$\rho_c/\bar{\rho}$		30.4	167	667
	X_c	0.595	0.182	0.001
Age		1.620(4)	2.874(6)	3.570(6)
g_1	P(x)	3.802(4)	2.852(4)	2.508(4)
	σ'^{-1}(yr)	2.547(3)	4.965	7.884(— 2)
g_2	P	5.815(4)	4.592(4)	3.351(4)
	σ'^{-1}	4.733(2)	7.129(3)	1.651
g_3	P	7.802(4)	6.439(4)	3.556(4)
	σ'^{-1}	9.790(1)	1.700(3)	30.28
g_4	P	9.830(4)	8.059(4)	4.865(4)
	σ'^{-1}	2.480(1)	4.620(3)	— 1.909(3)
g_5	P		9.844(4)	6.122(4)
	σ'^{-1}		1.434(4)	5.063(3)
g_6	P		1.090(5)	6.604(4)
	σ'^{-1}		— 2.542(3)	5.744(3)
g_7	P		1.331(5)	
	σ'^{-1}		6.722(2)	
g_8	P		1.448(5)	
	σ'^{-1}		8.344(2)	
g_9	P		1.597(5)	
	σ'^{-1}		1.820(2)	

the period P and the e-folding time σ'^{-1} (A negative sign for the e-folding time means instability). Table II also gives the central condensation $\rho_c/\bar{\rho}$, the central hydrogen abundance X_c and the age of the models. They show that the Z.A.M.S. model is stable while the other two are unstable towards one overtone for each l .The characteristic time of the instabilities is always much shorter than the time scale. Figure 2 shows $\frac{\delta p}{p}$ for the 4 first modes of $l = 2$ in model 1. Figures 3 through 9 show the same eigenfunctions for model 3.

We notice two important differences. First, in model 3 all the nodes but one are concentrated in a small region just above the convective core, whereas in model 1 the distance between nodes is much larger. As a result for the g_4 and higher modes of model 3 the derivatives of the eigenfunctions are large in the region mentioned and the leading term in the perturbation of the flux is the perturbation of the temperature gradient $\delta\vec{\nabla}T$. Second, if $(\delta p/p)_N$ stands for the maximum value of $\delta p/p$ in the region where the nodes are located, the ratio $R \equiv \left(\frac{\delta p}{p}\right)_{surf} \Big/ \left(\frac{\delta p}{p}\right)_N$ in model 1 steadily increases with the degree of the harmonic, but in model 3 the same ratio decreases at first, reaches a minimum for the g_4 mode. Compare now the general expression of $\delta(\vec{\nabla}.\vec{F})$ in a radiative region with its asymptotic approximation.

For high order modes the spatial dependance of the eigenfunction has the form $\exp\,[i\vec{k}\,.\,\vec{r}]$ and k_j^{-1} is supposed much smaller than any scale height. Therefore one has

$$\frac{\delta F_j}{F} \simeq \frac{\delta \nabla_j T}{\nabla_r T} \simeq i\,k_j \left[\frac{\delta T}{T} \Big/ \frac{d\,ln\,T}{dr} - \delta r\right] \tag{4}$$

$$\delta\,(\vec{\nabla}.\vec{F}) \simeq \vec{\nabla}.\delta\vec{F} \simeq -k^2 F \left[\frac{\delta T}{T} \Big/ \frac{d\,ln\,T}{dr} - \delta r\right] \tag{5}$$

with

$$k^2 = k_H^2 + k_r^2 \tag{6}$$

If one now supposes $p' = 0$, the equation of energy conservation writes, in the adiabatic approximation

$$\frac{\delta T}{T} = \nabla_{a,T} \frac{d\,ln\,p}{dr}\,\delta r \tag{7}$$

Introducing the expression in eq (5) one gets

$$\delta\,(\vec{\nabla}.\vec{F}) \simeq k^2 F \frac{H_T}{\nabla_{a,T}} (\nabla_{a,T} - \nabla_T) \frac{\delta T}{T} \tag{8}$$

where H_T is the temperature scale height. If one considers eq. (1) one sees that $\delta\,(\vec{\nabla}.\vec{F})$ has a stabilizing (destabilizing) influence if $\nabla_T < \nabla_{a,T}$ ($\nabla_T > \nabla_{a,T}$). This reproduces Kato's result.

The general expression for $\delta\,(\vec{\nabla}.\vec{F})$ is in a radiative zone

$$\delta\left(\frac{\vec{\nabla}.\vec{F}}{\rho}\right) = \frac{d\delta L^r}{dm} - \frac{l\,(l+1)}{r^2}\frac{F}{\rho}\left[\frac{\delta T}{T} \Big/ \frac{d\,ln\,T}{dr} - \delta r\right] - \frac{l\,(l+1)}{r^2}\frac{\chi}{\sigma_a^2 \rho}\frac{dL}{dm} \tag{9}$$

with

$$\frac{\delta L^r}{L} = 2\frac{\delta r}{r} + \left(4 - \frac{\delta\,ln\,\varkappa}{\delta\,ln\,T}\right)\frac{\delta T}{T} - \left(1 + \frac{\delta\,ln\,\varkappa}{\delta\,ln\,\rho}\right)\frac{\delta\rho}{\rho} + \frac{\frac{dr}{d}\left(\frac{\delta T}{T}\right)}{\frac{d\,ln\,T}{dr}} - \frac{d\,\delta r}{dr} \tag{10}$$

If one compares this formula with eq. (5), he recognizes the term given by the perturbed horizonta lflux since $l\,(l+1)\,r^2$ corresponds to $k^2{}_H$. The term in k_r^2 is associated to δF^r and comes from the last two terms of eq. (10). In the asymptotic approximation the other terms of (9) and (10) are negligible.

TABLE III

Contribution of the region of varying molecular weight to numerator of eq. (1) for σ'

	$\int \frac{\delta T}{T} \frac{d\delta L^r}{dm} dm$	$\int \frac{\delta T}{T} \frac{l(l+1)}{r^2} L \left[\frac{\delta T}{T} \Big/ \frac{d \ln T}{dr} - \delta r \right] dr$
g_1	7.315(39)	4.894(38)
g_2	2.002(40)	— 5,802(38)
g_3	2.844(40)	— 1.323(39)
g_4	— 3,686(41)	— 3,328(39)
g_5	— 1,341(39)	— 1,579(37)
g_6	— 2,817(38)	— 5,916(35)

Since for evolved models the derivatives of the eigenfunctions are the most important terms in $\delta(\vec{\nabla}.\vec{F})$ in the region of variable (where $\nabla_T > \nabla_{a,T}$), we may expect this region to have a destibilizing influence. Table III shows the contribution of the first two terms in eq. (9) to the integrals of the numerator of eq. (1). Negative values mean a destabilizing influence. For the g_2 mode and higher overtones the horizontal term has a destabilizing influence. For the radial term, this happens only for and above the g_4 mode.

In the other radiative region, as is often the case, the term $\delta(\vec{\nabla}.\vec{F})$ has a stabilizing influence. Therefore if the amplitude is large enough in the outer layers (i.e. if R is large enough), the mode will be stable. For the g_4 models R is small enough for the destabilizing influence of the region of variable μ to overcome the stabilizing effect of the outer layers.

CONCLUSION

Kato's mechanism is able to destabilize at least one low order mode for several values of l in evolved massive M.S. stars. If this mechanism were the only one to produce a semiconvective zone, one might expect that some mixing would occur until the μ gradient is reduced sufficiently in order to achieve vibrational neutrality($\sigma' = 0$). This could lead in the semi-convective region to a temperature gradient intermediate between Ledoux's and Schwarzschild's value.

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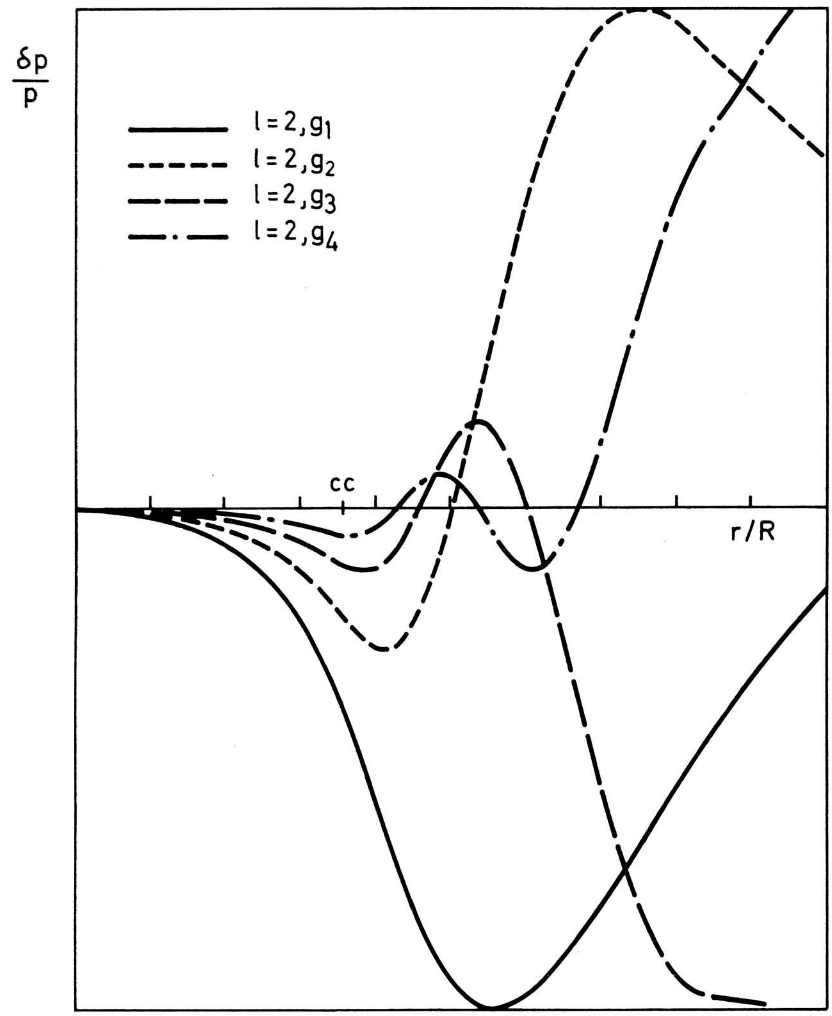

Fig. 2. — $\frac{\delta p}{p}$ for the 4 first g^+ modes ($l = 2$) of the zero age main sequence model vs distance to the centre. $\frac{\delta p}{p}$ is in arbitrary units. Label c. c. indicates the surface of the convective core.

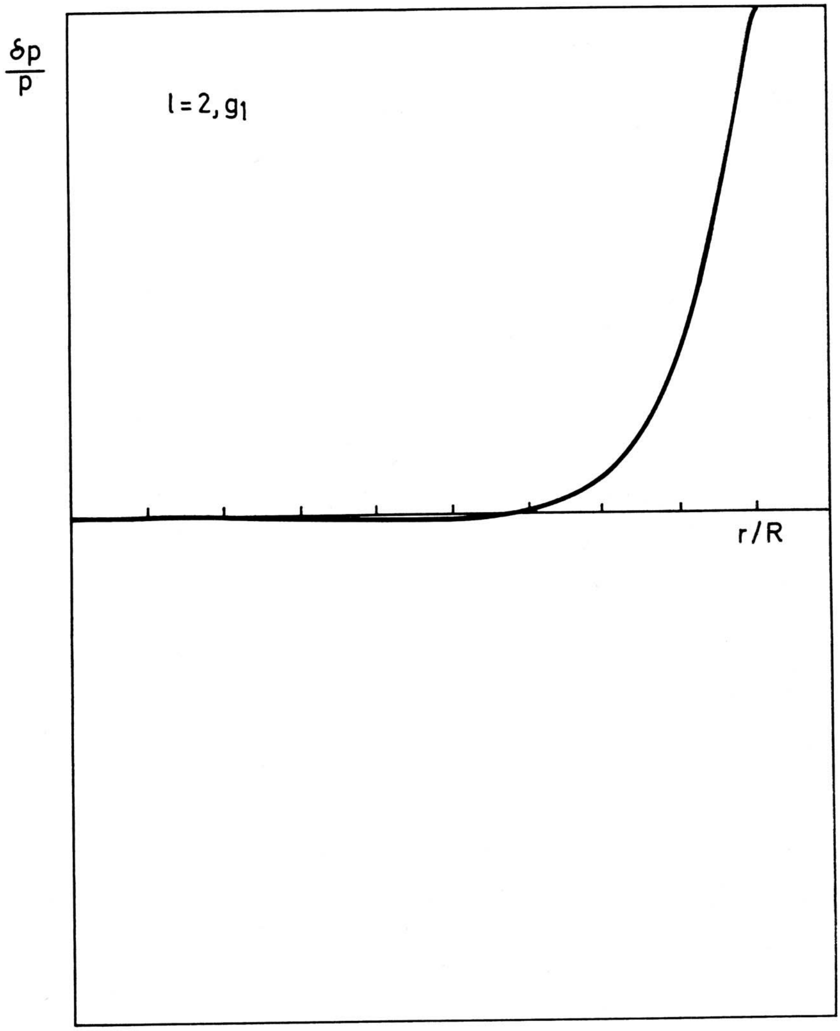

Fig. 3. — $\frac{\delta p}{p}$ for the g^{+} mode ($l = 2$) for model 3.

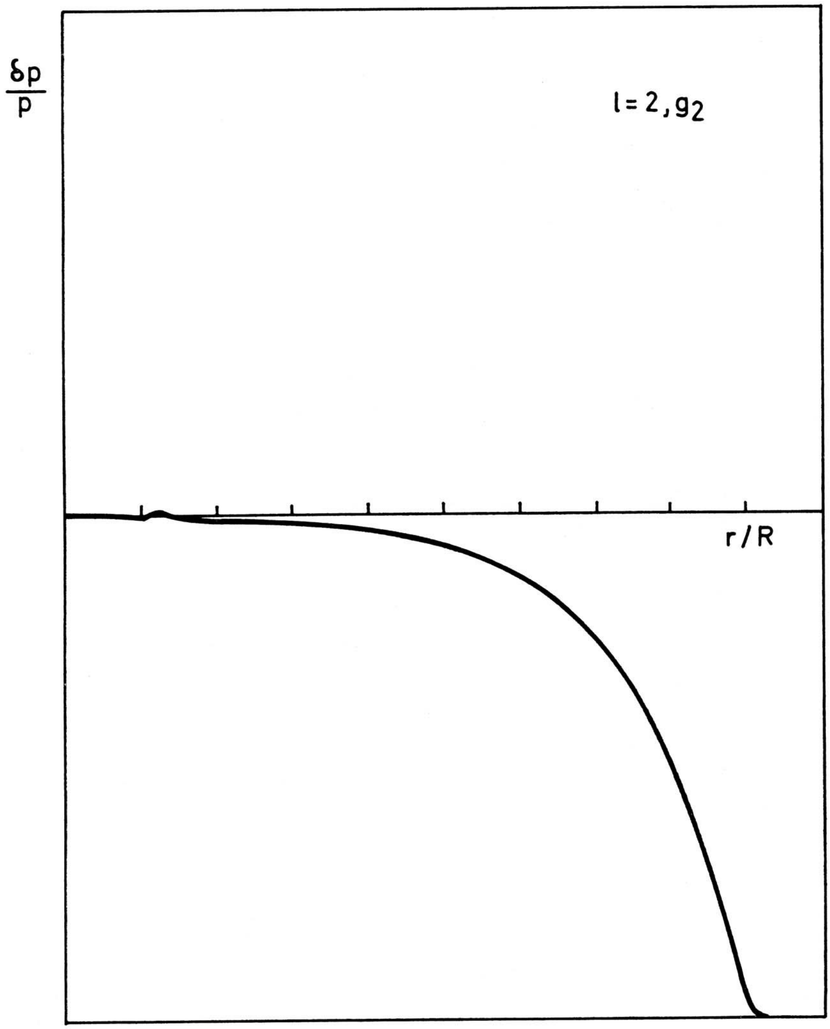

Fig. 4. — Same as fig. 3 for the g_2^+ mode.

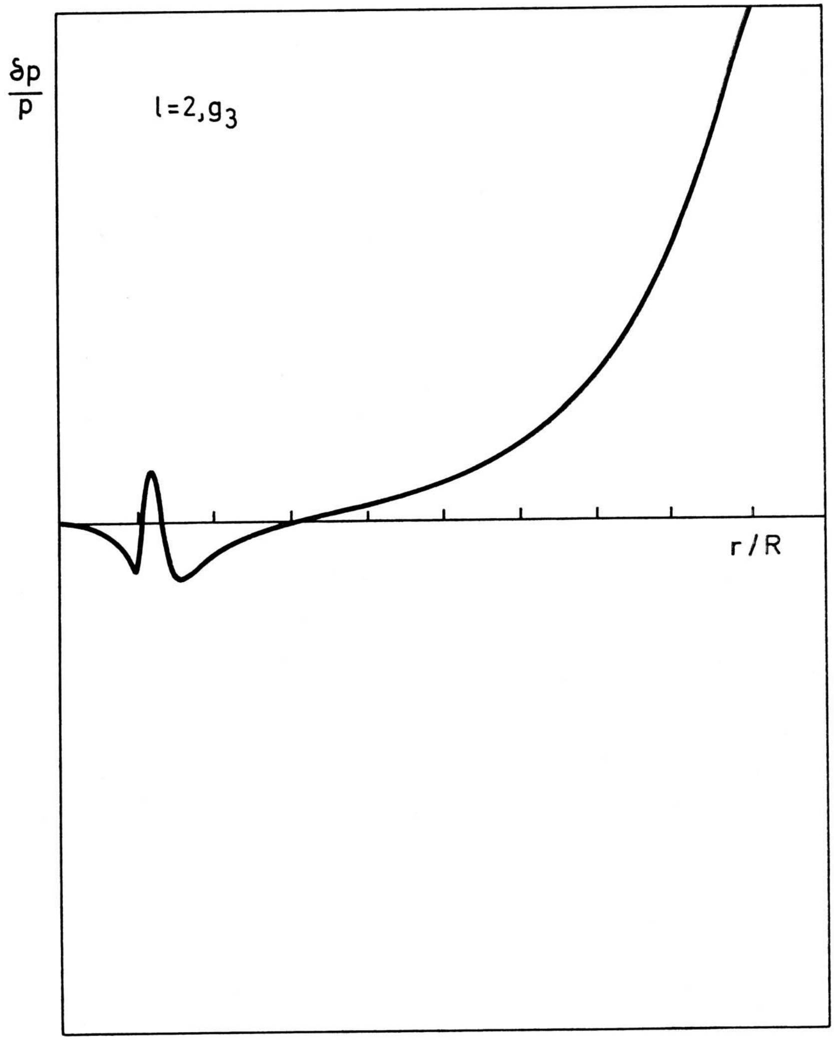

Fig. 5. — Same as fig. 3 for the g_3^+ mode.

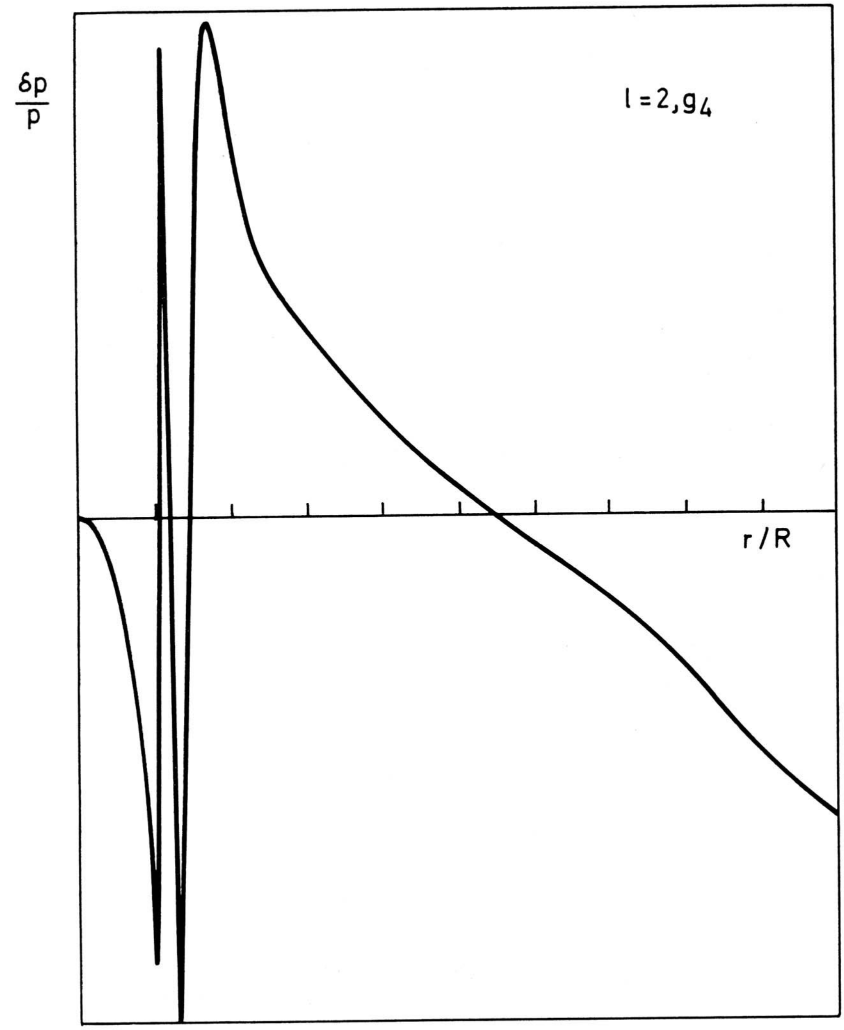

Fig. 6. — Same as fig. 3 for the g_4^+ mode.

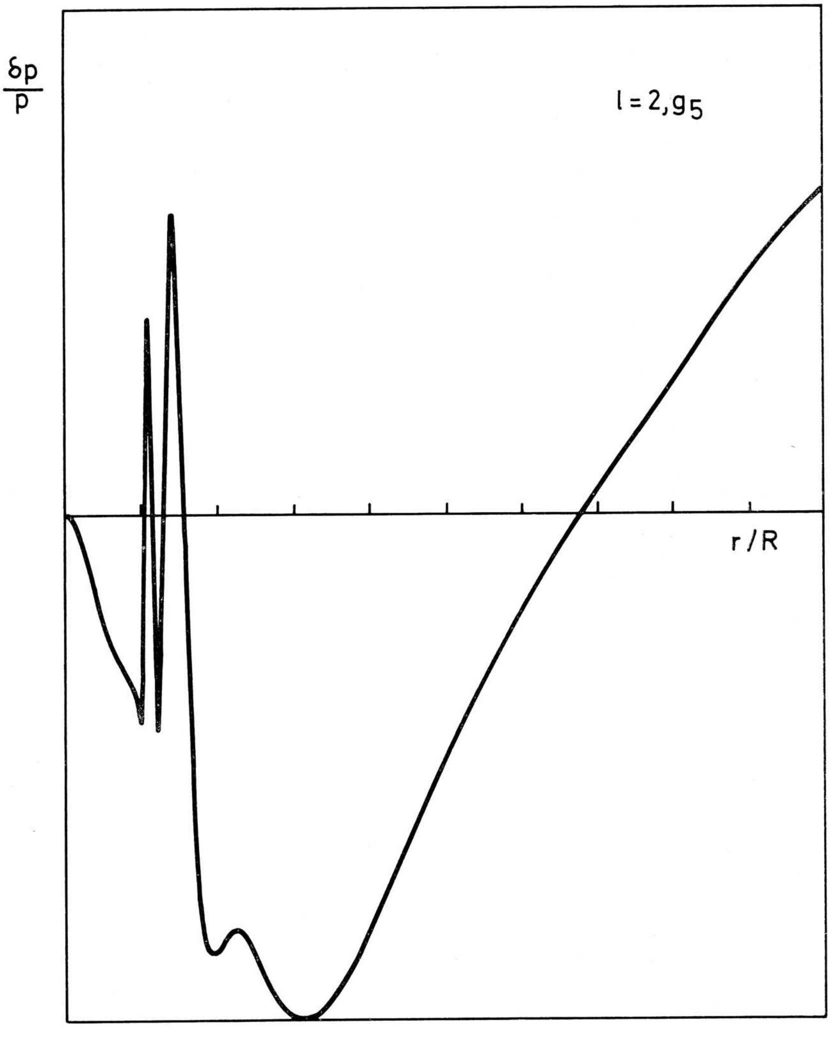

Fig. 7. — Same as fig. 3 for the g_5^+ mode.

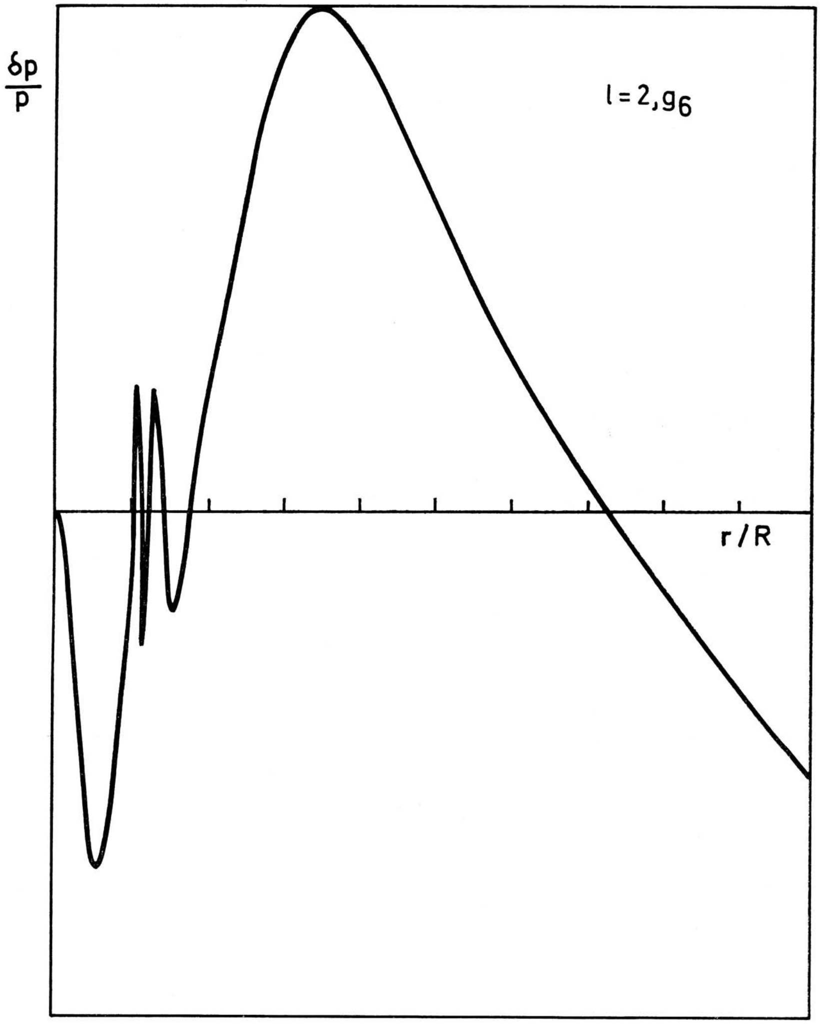

Fig. 8. — Same as fig. 3 for the g_6^+ mode.

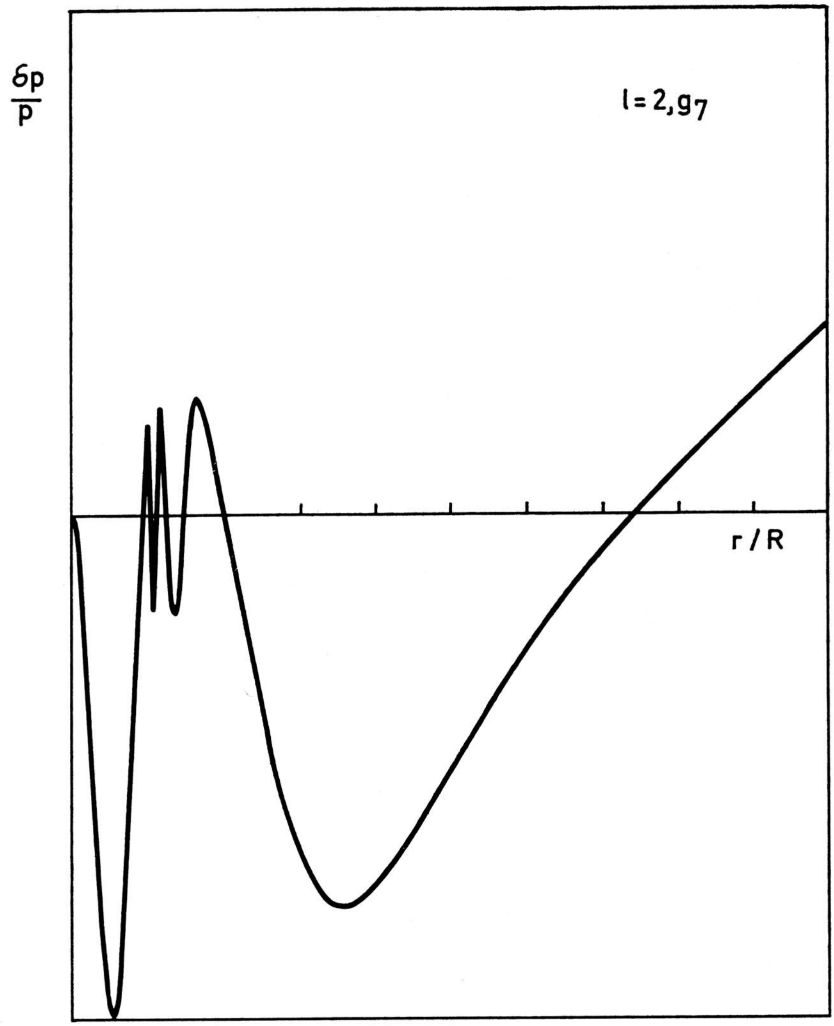

Fig. 9. — Same as fig. 3 for the g_7^+ mode.