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Oscillatory Periods in the Sun and Theoretical Models with or without Mixing

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Summary. Theoretical eigenvalues corresponding to periods less than one hour are presented for a standard solar model and for models of about the solar age which are undergoing a thermal pulse following a fast mixing. The two sets of eigenvalues differ very little, the differences being probably less than the accuracy of Hill's observations, except that the standard solar model presents a quadrupolar oscillation with a 43 min period while the mixed models do not. This period does not seem to be present in Hill's observations. Attempts to observe that particular period are important since the presence or absence of this 43 min period and its possible identification with a quadrupolar oscillation would be a strong argument in favour of one of the two types of solar models studied here.

Key words: sun — solar neutrinos — stellar pulsation

I. Introduction

Recently Hill and his collaborators have observed long-period oscillations at the solar surface which they identify as normal modes. Ten periods are given by Hill (1975) and shorter ones can be seen in the observed power spectrum.

Previously, other long period oscillations of the sun had been reported (Kaufmann, 1972; Kobrin and Korshunov, 1972; Fossat and Ricort, 1973) but this is the first time that such an extended set of periods is given.

These observations furnish indirect information about the deep layers of the sun and allow a new test of stellar evolution theory and especially of solar models to be made.

We may hope that the comparison between observed and computed periods will allow one to discriminate between various evolutionary schemes proposed recently (Fowler, 1972; Prentice, 1973, 1975; Faulkner *et al.*, 1975; Wheeler and Cameron, 1975) to solve the solar neutrino problem.

We have performed this comparison for two kinds of solar model: firstly, for a model obtained with the usual evolutionary scheme; secondly, for models of about the solar age, during a thermal pulse supposedly initiated by a fast mixing of the inner 83% of the total mass (Fowler, 1972; Gabriel *et al.*, 1975). In both cases, the comparison is made with radial quadrupolar and hexadecupolar oscillations. For both types of model, quadrupolar oscillations fit the observations best, and the fitting may even be better for the mixed models than for the standard one.

In the next section, the observed periods are identified with theoretical normal modes. In the last section, the vibrational stability of these modes is discussed.

II. Identification of the Oscillatory Periods

The observed periods are given in the first column of Table 1. The corresponding observational dimensionless eigenvalues ω^2 are given in the second column. ω^2 is defined by

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 \left(\frac{GM}{R^3}\right)^{-1}.$$

The theoretical dimensionless eigenvalues ω^2 are given in the other columns for radial ($\ell=0$) quadrupolar ($\ell=2$) and hexadecupolar ($\ell=4$) oscillations.

Only even ℓ values are considered here because since Hill measures variations in the solar diametre he will not detect oscillations with odd ℓ values.

In columns under the heading S we give the values for the standard solar model. Under the heading M_i we give values for models in the thermal pulse. We also give in parentheses the identification of the mode in the usual nomenclature of Cowling (1941).

Model M_1 is the model which is going to be mixed and model M_4 is that with the minimum surface luminosity. Table 2 gives for models in the thermal pulse the time counted from the beginning of the mixing, the variation in effective temperature, bolometric magnitude and

Table 1. Hill's periods and the dimensionless eigenvalues ω^2 for radial ($\ell=0$) quadrupolar ($\ell=2$) and hexadecupolar ($\ell=4$) oscillations of the standard sun (S) and of solar models during a thermal pulse (M_i). The identification of the modes is given in parentheses

Hill's values		$\ell=0$		$\ell=2$							$\ell=4$	
$P_{(min)}$	ω^2	S	M_4	S	M_1	M_2	M_3	M_4	M_5	M_6	S	M_4
		7.24 (p_0)	6.89 (p_0)	8.874 (g_1)	7.616 (f)	7.504 (f)	7.099 (f)	6.924 (f)	7.062 (f)	7.468	10.71 (g_2)	8.807 (f)
47.8	12.46			12.52 (f)	13.29 (p_1)	13.23 (p_1)	12.87 (p_1)	12.75 (p_1)	12.87 (p_1)	13.23	13.64 (g_1)	
		16.85 (p_1)	15.19 (p_1)	15.14 (p_1)							17.22 (f)	
30.3	30.99	29.14 (p_2)	27.29 (p_2)	26.76 (p_2)	26.00 (p_2)	25.84 (p_2)	25.64 (p_2)	25.58 (p_2)	25.66 (p_2)	25.91 (p_2)	19.76 (p_1)	19.37 (p_1)
		46.48 (p_3)	44.86 (p_3)	44.14 (p_3)	43.22 (p_3)	42.94 (p_3)	42.88 (p_3)	42.92 (p_3)	43.02 (p_3)	43.15 (p_3)	36.40 (p_2)	35.85 (p_2)
21	62.22	68.28 (p_4)	67.13 (p_4)	65.95 (p_4)	64.83 (p_4)	64.47 (p_4)	64.77 (p_4)	64.77 (p_4)	64.93 (p_4)	64.79 (p_4)	57.82 (p_3)	56.96 (p_3)
17.1	97.31	94.51 (p_5)	93.70 (p_5)	91.83 (p_5)	90.49 (p_5)			90.54 (p_5)			82.84 (p_4)	81.64 (p_4)
14.6	133.5	124.6 (p_6)		121.6 (p_6)	120.1 (p_6)			120.1 (p_6)			112.1 (p_5)	110.7 (p_5)
		159.1 (p_7)		155.6 (p_7)				153.9 (p_7)				143.6 (p_6)
11.8	204.4	196.9 (p_8)		193.2 (p_8)				191.6 (p_8)				
		238.0 (p_9)		234.2 (p_9)				233.0 (p_9)				
10.5	258.1	282.0 (p_{10})		278.0 (p_{10})				277.1 (p_{10})				
		329.5 (p_{11})		325.4 (p_{11})				324.5 (p_{11})				
8.8	367.4	380.7 (p_{12})		376.6 (p_{12})				375.9 (p_{12})				
7.9	456.2	435.6 (p_{13})		431.6 (p_{13})				430.9 (p_{13})				
7.2	548.5	493.9 (p_{14})		490.1 (p_{14})				489.7 (p_{14})				

Table 2. Data for mixed models, variation in time, effective temperature, bolometric magnitude, radius with respect to model M_1 where the pulse stars and central condensation

	$\Delta t_{(years)}$	$\Delta \log T_e$	ΔM_{Bol}	$\Delta R/R$	$\varrho_c/\bar{\varrho}$
M_1	0	0	0	0	61.27
M_2	5.257 (5)	−0.0053	0.090	−0.0161	45.51
M_3	1.085 (6)	−0.0085	0.159	−0.0338	41.82
M_4	2.051 (6)	−0.0103	0.199	−0.0451	42.21
M_5	2.851 (6)	−0.0101	0.185	−0.402	45.14
M_6	3.982 (6)	−0.073	0.124	−0.0226	50.44

radius relative to model M_1 and the central condensation. These models are about the solar age; they have been obtained through an evolution in which each time a vibrationally unstable model is encountered, it is partially mixed, as suggested by Dilke and Gough (1972). We choose a mass fraction of the mixed core of .83 (Gabriel *et al.*, 1975).

In the hypothesis of evolution with periodic mixing, the luminosity of the sun outside the pulse and the present position of the sun inside the pulse are not known. Thus, there is no justification for matching exactly the present sun with one of the mixed models. That is why we make here our comparison between observation and theory in terms of the dimensionless parameter ω^2. Supposing that the position of the sun in the pulse were known, it would then be sufficient to make a homologous transformation from a model of our sequence to get the proper solar model (Gabriel *et al.*, 1975).

Hill *et al.* do not give the error on the observed periods. By comparing periods yielded by the 1973 and 1975 observations we can however estimate its order to be about 10%.

From Table 1 we first notice that for $\ell=2$ the eigenvalues for a given mode change by less than 10% from a mixed model to any other one in the thermal pulse. This variation is about twice smaller than our estimate of the observational error. Therefore for $\ell=0$ and 4 we limited our computations to model M_4.

Comparing observed and theoretical values of ω^2, we notice that the fitting is bad for $\ell=4$ and therefore the oscillations are most likely not hexadecupolar modes.

The fitting for $\ell=0$ and 2 is much better. Clearly, the 48 min period may be identified only with a quadrupolar oscillation. For the other periods, however, it is difficult to distinguish between radial and quadrupolar oscillations, since the difference between the two sets of eigenvalues is less than the accuracy of the observations, except for eigenvalues smaller than 30.

It is also impossible to use the periods shorter than about 30 min to distinguish between the standard solar model and the mixed ones. The reason for this can be understood from the data of Table 3 where we give for the standard sun and for model M_4, the mass fraction

Table 3. Mass fraction where the energy of pulsation reaches half ($q^{1/2}$) and a quater ($q^{1/4}$) of its total value; fraction of the energy of pulsations in the convective envelope (f_e). Datas concern a few quadrupolar modes (f to p_4) of the standard sun and of the mixed model M_4

	S			M_4		
Mode	$q_{1/2}$	$q_{1/4}$	f_e	$q_{1/2}$	$q_{1/4}$	f_e
f	0.180	0.045	0.168	0.947	0.63	0.283
p_1	0.790	0.30	0.262	0.955	0.75	.425
p_2	0.988	0.94	0.507	.988	0.94	.464
p_3	0.993	0.94	0.549	.993	0.93	.544
n_4	0.993	0.93	0.543	.991	0.93	.500

$q_{1/2}$ and $q_{1/4}$ at which the energy of a few quadrupolar modes reaches half and a quarter of its total value and the fraction, f_e, of the total energy of pulsation in the convective envelope. Thus the period of these modes will not be sensitive to the structure of the inner region of the sun, We can consider that modes p_1 to p_5 fit the observations by Hill *et al.* (1975 b). They do not find any period at about 25 min ($\omega^2 \simeq 45$) but there is an inflection in their power spectrum which suggests that a normal mode might be present there. For higher modes the fitting is less good showing that the models inevitably suffer small imperfections in the description of the outer layers of the sun.
The most interesting difference between the standard sun and the mixed models is the presence of a quadrupolar mode at $\omega^2 \simeq 15$ for the standard sun which does not exist in the mixed models. In the observations we see no indication of an oscillation of period 43 min. If we admit that all the observed modes are of quadrupolar type, it seems hard to understand why the 43 min mode is not excited while the observed 48 and 30 min modes have the largest amplitudes and that the other theoretical periods fit very well the observations at least up to the p_5 mode ($P=17.1$ min). If observation shows with certainty a 43 min – period in the oscillatory spectrum of the sun and if it can be identified with a quadrupolar mode, we will have a good argument in favour of the standard solar model. On the contrary, if a 43 min – period is not observed, serious doubts would be cast upon that model; other types of solar models, like the mixed ones considered here, would become better candidates for representing the sun. In that event, however, we would not be allowed to push further the distinction among mixed models, that is between models which, like ours, are periodically rapidly mixed and others continuously slowly mixed, since the eigenvalues vary little from a mixed model to another. Moreover, we cannot exclude that other kinds of solar models with central condensation lower than the present sun ($\varrho_c/\bar{\varrho}=110$) can reproduce the observed periods. Observations at lower frequencies would also be useful.

Vibrational Stability

Most of the modes whose eigenvalues are given in Table 1 have been checked for vibrational stability using the same method as in Boury *et al.* (1975). As could be expected from the shape of the eigenfunctions all the modes are stable. The e-folding time τ is given in Table 4 for a few modes of two models.
The values given in Table 4 are computed neglecting the mechanical effects of convection (see Gabriel *et al.*, 1974) as our poor knowledge of time-dependent convection makes them difficult to estimate even with regard to their sign.
Calculations with the formulas of Gabriel *et al.* (1974) using various values of the perturbation of the mixing length indicate that these effects are probably unable to destabilize the normal modes.
The ratio of the flux term to the nuclear one appearing in the numerator of the coefficient of vibrational stability is at least 100 and grows rapidly with the degree of the overtone. Therefore, if one wants with Hill *et al.* (1974) to call upon energy transport by normal modes to solve the solar neutrino problem, it is necessary to introduce a new destablizing mechanism more efficient by at least a factor 100 than the usual perturbation of the nuclear energy generation rate to drive at least one mode.
Presently, it seems more likely to admit that the observed oscillations are stable normal modes which are excited in the convective zone or by solar activity (Wolff, 1972). The short values for the e-folding time and their relative values exclude the possibility of damped modes excited by the rapid mixing of the core of model M_1. In such an event we could not explain the amplitude ratio of the order of 1 for the 48 and the 30 min oscillations and any reasonable initial amplitude would have completely died out in 10^6 years.

Table 4. e-folding times (in years) for modes f to p_4 of the standard sun ($\ell=2$) and of Model M_4($\ell=0$ and 2)

Model	ℓ	f or p_0	p_1	p_2	p_3	p_4
S	2	7.51 (4)	2.62 (4)	2.52 (3)	4.17 (2)	1.13 (2)
M_4	0	1.52 (5)	2.09 (4)	2.88 (3)	6.03 (2)	1.36 (2)
M_4	2	3.52 (5)	3.30 (4)	3.43 (3)	7.63 (2)	1.55 (2)

Conclusions

Comparison of the observed normal mode periods with theoretical ones for a standard solar model and for models of a mixed sun shows that:
1) The quadrupolar modes ($\ell=2$) probably fit the observations best.
2) The fitting is also very good for radial modes, except for the 48 min period.
3) Radial modes and quadrupolar modes for the standard sun predict a period of about 43 min. The mixed

models predict no mode at that period for quadrupolar oscillation.

This is the main difference between the predictions of the two kinds of models. Improvement of the observations will perhaps allow a choice between the two types of model to be made.

4) The modes which fit the observations are p modes and are stable with regard to the usual vibrational stability analysis.

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