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On the Evolution of a 1 $M_\odot$ Star with a Periodically Mixed Core

M. Gabriel, A. Noels, R. Scuflaire and A. Boury
Institut d'Astrophysique, Université de Liège

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Summary. To solve the neutrino problem, Dilke and Gough have suggested that the vibrational instability of g^+ modes of non radial oscillation may be the cause of recurrent mixing in the sun. Supposing this to be correct, the evolution of the sun is completely different from the standard one. Unmixed solar models are stable when older than $3 \cdot 10^9$ years. It is therefore necessary to check whether in the modified evolution, instabilities still exist at the solar age. They do, provided that the mass fraction of the mixed core is large enough. However, the neutrino flux at its minimum during a thermal pulse occurring at the solar age remains too high. Constraints imposed by ice age records are also discussed.

Key words: stellar evolution — stellar stability — solar neutrinos

I. Introduction

Fowler (1972) suggested that the failure to detect solar neutrinos in Davis' experiment might be due to the sun being in a transient phase of thermal imbalance caused by a fast mixing of the core. The reduction in the neutrino flux in such a situation was confirmed by Ezer and Cameron (1972) and by Rood (1972). Dilke and Gough (1972) suggested that this mixing is caused by the vibrational instability of solar models against g^+ modes of non radial oscillations.

Stability analysis (Christensen-Dalsgaard *et al.*, 1974a, b; Noels *et al.*, 1974; Boury *et al.*, 1975; Shibahashi *et al.*, 1975) have shown that a 1 $M_\odot$ star evolving without any mixing in radiative regions is unstable at ages between $\sim 2.4 \cdot 10^8$ and $\sim 3 \cdot 10^9$ years.

If Dilke and Gough's suggestion is correct, the evolution of the star becomes completely different from the standard one as soon as it becomes unstable. Of course, we do not know for how long a star must have been vibrationally unstable before the mixing is triggered. We shall assume here that mixing takes place each time an unstable model in thermal equilibrium is encountered. If, after such a mixing, the star remains vibrationally unstable, we may expect that mixing will not die out and that the subsequent evolution will be that of a star with a continually mixed central core. In that event there will be no other thermal pulse. On the contrary, if, after the mixing, the star becomes stable again, it will evolve in the standard way until instability reappears. In that event the star will undergo recurrent pulses.

This second possibility is the most likely because stability analysis of standard models reveals that a necessary, but not sufficient, condition for instability is that the ^{3}He abundance be close to its equilibrium value in the nuclear burning region. Since ^{3}He is burnt at the beginning of a pulse it is necessary to synthesize enough fresh ^{3}He before unstable models can reappear in the evolutionary sequence.

If a 1 $M_\odot$ star older than $2.4 \cdot 10^8$ years is submitted to recurrent mixings and thermal pulses, one must show that the star is still in that phase at age $4.5 \cdot 10^9$ years in order for Dilke and Gough's suggestion to be significant in the case of the solar neutrino problem.

The shape of the eigenfunctions also influences the stability and therefore the time interval between pulses. If that interval decreases, less fresh ^{3}He is accumulated, the thermal pulses become less violent and the minimum neutrino flux is less deep. If that interval becomes short enough it is even possible that the neutrino flux always remains larger than the maximum value acceptable from Davis' observations.

As early as 1950, Öpik suggested that the terrestrial ice ages are caused by the sun going through phases which cannot be represented by static models. The idea reappeared with Fowler's suggestion. It is therefore interesting to check whether the time interval between successive mixings, i.e., in our assumption, the time necessary for the models to become unstable again after having been mixed, is compatible with the time interval between two major glaciations which seems to have been of the order of 2 to $3 \cdot 10^8$ years over the last $3 \cdot 10^9$ years.

The present work is not an attempt to produce neutrino fluxes below the observational upper limit. We nevertheless give, as an indication, the values yielded by our models and discuss the constraints posed by the observations.

II. Numerical Calculations

The models have been computed with the same physical input as in Boury *et al.* (1975) and with initial chemical composition $X=0.7417$, $Y=0.2383$, $Z=0.02$. This composition reproduces the sun at an age of $4.5\cdot 10^9$ years if the sun evolves in the standard way. Therefore it is inappropriate for the evolution considered here. If Fowler's idea is correct the proper composition is difficult to determine since the sun is presently undergoing a thermal pulse and we do not know accurately the solar luminosity L_S outside the thermal pulse at an age of about $4.5\cdot 10^9$ years. It is however possible to rescale the ages of our models with enough accuracy in the following way.

Consider a dimensionless model characterized by the pair of parameters C and D (Schwarzschild, 1958) and with an average hydrogen abundance $\bar{X}_i$ in the mixed core and X_e in the envelope. To that dimensionless model corresponds a set of physical models with the same ratio $\bar{X}_i/X_e$. More exactly this assumes that all models have the same distribution of the quantities i, j and l defined in Eq. (13.3) of Schwarzschild (1958). Conversely, the pair (C, D) is only a function of $\bar{X}_i/X_e$. Therefore, the luminosity is a function of the luminosity L_{MS} on the ZAMS and of $\bar{X}_i/X_e$. As $\bar{X}_i/X_e$ does not vary by more than $\sim 15\%$ in the evolution considered, one can write approximately

$$L \simeq L_{\mathrm{MS}}(\bar{X}_i/X_e)^{-n} \quad (n>0)\,. \tag{1}$$

Since the thermal relaxation phases last less than a tenth of the interpulse time, one has, within a few percent:

$$\frac{d\bar{X}_i}{dt} = -\frac{L}{M(\mathrm{core})6.3\cdot 10^{18}} = -\frac{L_{\mathrm{MS}}}{M(\mathrm{core})6.3\cdot 10^{18}}\left(\frac{\bar{X}_i}{X_e}\right)^{-n}.$$

Thus, (2)

$$t = \frac{6.3\cdot 10^{18}\,M(\mathrm{core})X_e}{(n+1)L_{\mathrm{MS}}}\left[1-\left(\frac{\bar{X}_i}{X_e}\right)\right]^{n+1}$$

and

$$\frac{Lt}{X_e} = \frac{6.3\cdot 10^{18}\,M(\mathrm{core})}{n+1}\left[1-\left(\frac{\bar{X}_i}{X_e}\right)\right]^{n+1}\left(\frac{\bar{X}_i}{X_e}\right)^{-n}. \tag{3}$$

M(core) stands for the mass of the mixed core where all reactions take place. Therefore Lt/X_e is constant for all models having the same dimensionless structure. As the range of possible X_e values is no more than two or three per cent wide (Bahcall *et al.*, 1968), we can say that Lt is constant to a good accuracy. A change in Z would only modify L_{MS}, leaving our rescaling method valid.

Given a value of L_S, let t_1 and L_1 be the age (in years) and the luminosity of the model in our sequence which, outside a thermal pulse, satisfies the condition

$$L_1 t_1 = 4.5\cdot 10^9 L_S\,. \tag{4}$$

If we multiply the age of any model of the computed sequence by $\alpha = \frac{4.5\cdot 10^9}{t_1} = \frac{L_1}{L_S}$ we obtain approximately the age of the model having the same distribution of dimensionless variables and thus the same mass concentration but whose chemical composition is such that for $t=4.5\cdot 10^9$ years, $L=L_S$.

If we take $L_S=L_\odot$, we find that in our computed sequence, the equality (1) is fulfilled at $t_1=4.838\cdot 10^9$ years and $\alpha=0.93$; but if we choose $L_S=1.2\,L_\odot$ we then obtain $t_1=5.457\cdot 10^9$ years and $\alpha=0.825$.

Boury *et al.* found that in their standard evolutionary sequence the first unstable model was $8.67\cdot 10^8$ years old (see their model n°2). We have assumed that up to that age the evolution proceeds in the usual way, then at that time the central core is "rapidly" mixed. We have chosen a constant mass fraction of the mixed core $q_c=0.83$. This value of q_c corresponds approximately to the mass fraction at the node of the radial displacement for the g_1, $l=1$ mode. The star undergoes then a phase of thermal readjustment. Such phases have been previously studied by Ezer and Cameron (1972), Rood (1972) and Ulrich and Rood (1973) and our purpose is not to describe them in detail. Table 1 gives data concerning selected models. A roman numerical represents the number of the thermal pulse. As can be seen three pulses have been computed. Within a pulse, arabic numerals 1, 2 and 3 stand respectively for the model which has to be mixed, for that with minimum surface luminosity and for the first model for which the nuclear reactions produce at least 90% of the surface luminosity in the phase of increasing luminosity. The first model found to be unstable after a pulse is given number 4. The age of any model 1 is referred to as t_M and that of others is given as t_M plus the time span since mixing started. The ages given are the computed values without any rescaling coming from composition effects. After the initial increase in the nuclear energy generation the models become stable again because the ^{3}He abundance is then below its equilibrium value for $r/R \gtrsim 0.1$ which reduces the destabilizing influence of the nuclear reactions, and also because the decrease in $\varrho_c/\bar{\varrho}$ and the mixing have modified the shape of the eigenfunctions (see discussion in Boury *et al.*, 1975). When ^{3}He has again approached its equilibrium abundance in the nuclear burning region the model is still stable. It is necessary for the μ-gradient to reach a large enough value to make the models unstable again (model 4).

When an unstable model is found, it is mixed ($\mathrm{I}_4 \equiv \mathrm{II}_1$) and a new thermal pulse is computed. The proper mixing time is of course unknown and to check its influence it has been strongly varied in the first two mixings. The mixings are made in three steps which

Table 1. Data concerning selected models. The first column gives the sequence number (see text). The others give respectively the age, the central hydrogen abundance X_c, the central temperature T_c and density ϱ_c, the mass concentration $\varrho_c/\bar{\varrho}$, the logarithm of the effective temperature T_e and the surface luminosity L

Model	Ages (years)	X_c	T_c(°K)	ϱ_c	$\varrho_c/\bar{\varrho}$	$\log T_e$	L(erg.s^{-1})
I_1	8.67(8)	0.6806	1.348(7)	93.45	50.91	3.7508	2.914(33)
I_2	t_M+6(6)	0.7343	1.239	58.32	26.89	3.7286	2.122(33)
I_3	t_M+27.07(6)	0.7331	1.320	84.54	45.25	3.7493	2.881(33)
I_4	t_M+266.07(6)	0.7152	1.342	89.07	48.24	3.7514	2.920(33)
II_1	1.133(9)	0.7152	1.342	89.07	48.24	3.7514	2.920(33)
II_2	t_M+2.702(6)	0.7320	1.243	58.88	25.97	3.7275	2.041(33)
II_3	t_M+15.89(6)	0.7313	1.320	83.62	47.26	3.7500	2.967(33)
II_4	t_M+252.21(6)	0.7190	1.346	89.42	48.98	3.7519	2.958(33)
M_1	4.5487(9)	0.7001	1.412	85.99	51.70	3.7579	3.317(33)
III_1	4.7660(9)	0.6799	1.414	95.76	61.27	3.7605	3.546(33)
III_2	t_M+2.051(6)	0.6976	1.366	75.76	42.21	3.7503	2.947(33)
III_3	t_M+9.445(6)	0.6970	1.398	92.52	61.11	3.7600	3.610(33)
III_4	t_M+351.33(6)	0.6649	1.428	97.92	64.43	3.7612	3.639(33)
M_2	6.149(9)	0.6820	1.432	89.18	58.76	3.7620	3.673(33)
NM 1	6.449(9)	0.6500	1.454	100.8	71.70	3.7652	3.962(33)
NM 2	6.749(9)	0.6180	1.473	105.6	77.95	3.7658	4.082(33)

Numbers in parentheses indicate the power of 10 which multiplies the preceding number.

in the first case cover a total of $4\cdot10^6$ years and in the second pulse a total of $2.1\cdot10^5$ years. As a consequence the second thermal pulse is stronger than the first as shown by the lower minimum luminosity. However the luminosity minimum remains well defined in the first pulse. This shows that the mixing need not be extremely fast to give a neat thermal pulse. Moreover the mixing time does not seem to significantly influence the time interval between two successive mixings. In both cases it is of the order of $2.5\cdot10^8$ years.

Now model II_4 should be mixed. However it would be very expensive to compute in such a way the evolution up to the solar age. Therefore we have used the same procedure as Ulrich and Rood (1973). The star is evolved up to an age of $4.5487\cdot10^9$ years (see model M_1) continually mixing the inner core of mass q_c. The mixing in the radiative region is then turned off until, $2.17\cdot10^8$ years later, it becomes vibrationally unstable (slowly mixed models are stable). This stage corresponds to model III_1 whose core is mixed in $1.6\cdot10^5$ years starting a new thermal pulse.

As can be seen from Table 1 the pulse at $4.8\cdot10^9$ years is less violent than the first one since the luminosity drops only by 20%. This is because, during the evolution, the ^{3}He abundance decreases steadily. At q_c, $X_{^3\mathrm{He}}=4.7\cdot10^{-4}$ in model I_1 but in model III_1, $X_{^3\mathrm{He}}=8.1\cdot10^{-5}$.

Table 1 also shows that the time interval between two successive pulses is now $3.5\cdot10^8$ years thus increasing with evolution. This results from the following changes in the eigenfunctions. As $\varrho_c/\bar{\varrho}$ increases, the ratio of the surface amplitude to the amplitude in the central burning region increases and for a given $\varrho_c/\bar{\varrho}$ this ratio is larger here than in the standard evolution. This of course is a stabilizing factor whose weight increases with time. On the other hand when a model is mixed the node of $\delta p/p$ and also its minimum move outward. This, as explained in Boury *et al.*, has a stabilizing effect. At the end of a thermal pulse the node of δr remains close to q_c. Later on as a μ-gradient builds in the core the opposite evolution appears, the node of $\delta p/p$ moves inward. This favours instability. The higher $\varrho_c/\bar{\varrho}$ is, the larger the inward displacement of the node must be to produce instability. Consequently the time between two successive pulses increases.

Model III_4 is about the solar age but since L_S is uncertain the time scaling factor α is also inaccurate. If $L_S=1.2\,L_\odot$ ($\alpha=0.825$) the rescaled age of model III_4 would be 10% lower than the sun's. Therefore we have considered model M_2 in the sequence with a continually mixed core which is $6.149\cdot10^9$ years old. We have turned off the mixing and we have computed models NM 1 and NM 2 (see Table 1). Model NM 1 is stable while model NM 2 is unstable. From linear interpolation the instability appears $5\cdot10^8$ years after mixing stopped. Model NM 2 should therefore be mixed starting a new thermal pulse. For $\alpha=0.825$, model NM 2 is $5.57\cdot10^9$ years old.

This clearly shows that it is possible to find models evolving in the way suggested by Dilke and Gough and which still undergo unstable phases well after the solar age.

Neutrino fluxes are given in Table 2 for models in the third computed pulse. As can be seen, the minimum flux of 0.98 SNU is still larger than or uncomfortably close to the upper bound of ~1 SNU given by Davis and Evans (1973) and Evans *et al.* (1974). The drop in the neutrino flux is much smaller than in the second pulse where it goes down to 0.33 SNU. This is to be expected, since the available ^{3}He progressively decreases.

Table 2. Neutrino fluxes, in SNU, during the third computed pulse (sequence III). Time after the start of the pulse is given in the first column. The next four columns give the neutrino flux from ^{8}B, ^{7}Be, the pep reaction and all reactions including the CNO cycle. The last three columns give the surface luminosity, the central temperature and the ^{3}He abundance at the centre

$t(y)$	^{8}B	^{7}Be	pep	Total	L(erg.s^{-1})	T_c	X_3
0.00	1.775	0.348	0.235	2.473	3.546(33)	1.415(7)	3.389(−5)
2.78(5)	6.269	1.162	0.173	7.658	3.441(33)	1.407(7)	2.368(−4)
5.16(5)	3.266	0.709	0.146	4.161	3.258(33)	1.391(7)	1.847(−4)
1.09(6)	1.664	0.470	0.132	2.245	3.061(33)	1.378(7)	1.327(−4)
2.05(6)	0.804	0.242	0.137	1.210	2.947(33)	1.366(7)	8.121(−5)
2.85(6)	0.595	0.210	0.148	0.980	2.991(33)	1.361(7)	6.106(−5)
3.98(6)	0.611	0.224	0.166	1.031	3.165(33)	1.362(7)	4.701(−5)
5.58(6)	0.792	0.263	0.187	1.279	3.398(33)	1.372(7)	4.274(−5)
4.11(8)	2.143	0.440	0.240	2.897	3.639(33)	1.424(7)	3.168(−5)

The fluxes calculated here are not however directly comparable with observations as our models do not have the appropriate chemical composition. However the minimum neutrino flux scales pretty much as the minimum luminosity. If the ratio of the minimum luminosity to the luminosity just before mixing is not affected by a change in composition we have to expect a minimum neutrino flux of 1.08α S.N.U. This value could however be affected by the choice of the mixing time and by the exact way mixing take place. Recently Ulrich (1975) performed a similar calculation for a model with a higher luminosity out of pulse; he mixes $0.75\,M_\odot$ in $10^6\,y$ and obtains a minimum neutrino flux close to ours. This shows that it is indeed difficult to obtain neutrino fluxes low enough.

III. Discussion

There are several arbitrary postulates in the present study. Firstly, we assumed that a vibrational instability produces mixing of the core as suggested by Dilke and Gough. This point has been criticized by Ulrich and Rood (1973) and by Ulrich (1974). It is a difficult problem to know what the non-linear effects of the instability are. This question is however beyond the scope of this paper. Secondly we know neither the time scale for the mixing nor the time the star remains unstable before mixing is triggered. These points are not critical for us since we are not interested in the details of pulses. As shown above, the mixing time may be of several millions years and still produce a very well defined pulse. Thirdly the mass fraction q_c of the mixed core is also unknown. The choice of q_c may be critical. If q_c goes to zero the maximum age for which unstable models can be found will decrease to $3\cdot10^9$ years, the value obtained for an unmixed evolution. Therefore at $4.5\cdot10^9$ years thermal pulses can no longer exist if q_c is taken small enough. Because there is no theory to suggest a q_c value and that it could vary with time, because also L_S is uncertain we must limit ourselves to show that there exist q_c values such that thermal pulses still occur at the solar age. For this purpose we had to take a large q_c because the larger q_c, the more slowly $\varrho_c/\bar{\varrho}$ increases and we know from the study of unmixed models that it is the increase of $\varrho_c/\bar{\varrho}$ beyond some critical value which modifies the eigenfunctions in such a way as to finally restore stability. As a matter of fact there are arguments in favour of a large q_c value. Rood's calculations (1972) show that for the sun to produce neutrino fluxes below 1 S.N.U. at its present luminosity, q_c must be larger than 0.5. Our choice has the following motivation. It is known that figures obtained drawing the displacements corresponding to a g^+ mode form in a meridian plane streamlines which look very much like those of meridian circulation in rotating stars and further, for $l=1$, there is one more cell than nodes of the radial displacement. It is tempting to think that in non linear motions the material would progressively mix in each cell. If mixing is connected with the g_1^+ mode, the position of the node of the radial displacement would fix in a natural way the value of q_c.

Let us now consider the possible connection of thermal pulses with major ice ages on Earth. The period between two major glaciations is generally admitted to be $2.5\cdot10^8$ years. At solar age, instability appears $3.5\cdot10^8$ years after the onset of mixing. As there is a delay between the appearance of the instability and the ensuing mixing, this value gives the strictly minimum time span Δt between two successive pulses at about the solar age, which is too large. The rescaling of the models using a reasonable value of L_S will lead to a smaller $\Delta t=3.5\cdot10^8\alpha$ years with $\alpha<1$. However, if mixing takes place when the amplitude of the oscillations has grown by a factor $\exp(k)$, we find that we must add $3.4\cdot10^7\,k\alpha$ years to Δt before mixing starts. With $k=10$, we already obtain a interpulse time of $6.9\cdot10^8\alpha$ years which is uncomfortably large for any reasonable value of α ($\gtrsim 0.8$).

Another difficulty is presented by paleoclimatological models which predict that a 2.5% decrease in the solar constant causes a drop of 5°K in temperature on Earth. According to Sellers (1969), Gordon and Davis (1974, 1975a and b), Van der Haar and Ellis (1975), such a 5 °K could, with the present atmosphere and location of the continents, produce a "White Earth". Then only mild pulses are acceptable, requiring small q_c values, for which the neutrino flux won't fall below 1 SNU (Ulrich and Rood, 1973; Ulrich, 1974). Moreover for $q_c \lesssim 0.25$ unstable models will probably no longer occur at solar age.

IV. Conclusions

Our calculations show that it is possible to find models evolving in the way suggested by Dilke and Gough (1972) which still undergo unstable phases at the solar age. For this the mixed core must be sufficiently large. The minimum neutrino flux remains very close to or larger than the experimental upper bound and it seems very difficult to obtain lower values, though this cannot yet be excluded. Difficulties appear in the climatic changes that pulses can produce. The interpulse period at solar age seems longer than the interval between major glaciations. The greatest difficulty comes, however, from the fact that pulses deep enough to produce agreement with neutrino observations would have produced complete freezing of the Earth.

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M. Gabriel
A. Noels
R. Scuflaire
A. Boury
Institut d'Astrophysique
Avenue de Cointe, 5
B-4200 Cointe-Ougrée
Belgique