NONRADIAL OSCILLATIONS OF SOLAR MODELS WITH AN INITIAL DISCONTINUITY IN HYDROGEN ABUNDANCE

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ABSTRACT

Solar models are calculated with low central hydrogen abundance. The stability of these models is investigated. The eigenspectrum is computed and compared with the SCLERA observations of solar oscillation.

1. INTRODUCTION

In an attempt to solve the solar neutrino problem, Faulkner, Da Costa and Prentice (1975) followed a suggestion of Prentice (1973) and constructed models of the sun in which the initial hydrogen content in a small central region was much smaller than that in the rest of the star. Although the models which give the observed solar luminosity at the present solar age yield neutrino fluxes that are too large, they are interesting because they exhibit oscillatory modes connected with the discontinuity in density that is associated with the discontinuity in chemical composition. Moreover, the possible observations of the oscillation spectrum of the sun by the SCLERA group (Brown, Stebbins and Hill 1976, 1978) could permit comparison between various solar models (Scuflaire et al. 1976; Christensen-Dalsgaard and Gough 1976; Hill and Caudell 1979).

2. MODELS AND OSCILLATION PERIODS

Following Faulkner, Da Costa and Prentice (1975), an evolutionary sequence was computed by the Henyey method of a 1 $M_\odot$ star of heavy element abundance $Z = 0.02$ and of initial hydrogen abundance $X = X_c = 0.1$ in the region $m(r)/M_\odot \leq 0.03$ and $X = X_s$ elsewhere. The value of X_s necessary to fit the luminosity at evolutionary age 4.7×10^9 years to the present solar luminosity was found to be 0.7813. The evolutionary sequence was constructed with a ratio of mixing length ℓ/H to the pressure scale height equal to 1.5; this ratio had to be adjusted to 2.15 in order to match the present value of the solar radius to within less than 1 percent. A second sequence with $X_c = 0$ for $m(r)/M_\odot \lesssim 0.03$ and $X_s = 0.794$ elsewhere was also calculated. The behavior of the models of this second sequence being qualitatively the same as that of the models with $X_c = 0.1$, no precise adjustments of X_s and ℓ/H were made to achieve a precise fit with the present sun.

The properties of the models tested for vibrational instability are listed in Table 2 where x_D, X_{Di} and X_{DO} respectively represent the non-dimensional distance of the discontinuity to the center of the star and the hydrogen abundance on the inner and outer sides of the discontinuity. Models 1, 2, 3, 7, and 8 correspond to the approach to the main sequence; Model 6 corresponds to the present sun.

The integration of the fourth order differential system of nonradial adiabatic oscillations was then performed following the scheme given in Boury et al. (1975). The fourth column of Table 2 gives the periods of the modes g_1 through g_5 for the horizontal wavenumber $\ell = 1$. In the fourth column of Table 3, we list for $\ell = 1$ to 10 the periods of the modes associated with the discontinuity in density. These modes have a very large amplitude in a narrow layer centered on the discontinuity. With respect to solar seismology, Table 4 provides a list of periods of Model 6 corresponding to the present sun; this allows for a comparison with the SCLERA periods. It is immediately seen that the predicted spectrum is much more compact than the observed spectrum. This compactness comes from the high central condensation of the star due to the very low central abundance of hydrogen. In the present state of observations, models with the assumed distribution of hydrogen do not pass the test of solar seismology.

3. VIBRATIONAL STABILITY

The damping coefficient $\sigma'_{k,\ell}$ relative to the k mode associated with the ℓth harmonic is written, as usual, in the following form (Boury et al. 1975):

$$\sigma'_{k,\ell} = -\frac{1}{2}\left[\frac{\int_0^{Ma}\left(\frac{\delta T}{T}\right)_{k,\ell}\delta\varepsilon\,dm - \int_0^{Ma}\left(\frac{\delta T}{T}\right)_{k,\ell}\delta\left(\frac{1}{\rho}\,\mathrm{div}\,\vec{F}\right)dm + \int_0^{Ma}\left(\Gamma_3-\frac{5}{3}\right)\frac{\delta\rho}{\rho}\,\delta\left(\varepsilon_2+\frac{1}{\rho}\underset{\sim}{V}\cdot\underset{\sim}{\nabla}p\right)dm}{\sigma^2\int_0^{M}|\underset{\sim}{\delta r}|^2\,dm}\right]_{k,\ell}$$

$$= -\left[\frac{E_N - E_F + E_2}{2\sigma^2\int_0^{M}|\underset{\sim}{\delta r}|^2\,dm}\right]_{k,\ell} \qquad (1)$$

where all the terms are expressed in terms of the adiabatic solution. The third integral in equation (1) expresses the influence of the mechanical effects of convection. $\vec{V}$ is the mean velocity of turbulence and ε_2 stands for the rate per unit mass of dissipation of turbulent kinetic energy into heat (Ledoux and Walraven 1958; Gabriel et al. 1975). All other symbols have their usual meaning. Table 2 gives the

Table 1. Properties of the Models

a) Sequence $X_c = 0.1$, $X_s = 0.7813$

Model Number	ℓ/H	Age (years)	x_D	X_{Di}	X_{Do}	T_c	ρ_c	L	$\rho_c/\bar{\rho}$
1	1.5	4.44(7)	0.06150	0.09987	0.7796	1.392(7)	290.8	2.637(33)	168.3
2	1.5	4.98(7)	0.06508	0.09983	0.7791	1.374(7)	295.9	2.612(33)	170.4
3	1.5	5.75(7)	0.06514	0.09978	0.7785	1.358(7)	302.3	2.622(33)	175.4
4	1.5	8.68(8)	0.05720	0.09640	0.7182	1.343(7)	346.3	2.840(33)	220.3
5	1.5	4.70(9)	0.04652	0.07604	0.4190	1.509(7)	471.9	3.750(33)	419.1
6	2.15	4.70(9)	0.04977	0.07582	0.4168	1.513(7)	472.5	3.809(33)	342.1

b) Sequence $X_c = 0$, $X_s = 0.794$

Model Number	ℓ/H	Age (years)	x_D	X_{Di}	X_{Do}	T_c	ρ_c	L	$\rho_c/\bar{\rho}$
7	1.5	4.83(7)	0.05736	-	0.7919	1.275(7)	507.4	2.546(33)	302.2
8	1.5	5.63(7)	0.06268	-	0.7911	1.275(7)	506.2	2.526(33)	300.3
9	1.5	9.03(8)	0.05152	-	0.7234	1.282(7)	543.3	2.703(33)	351.6
10	1.5	2.50(9)	0.04809	-	0.5951	1.334(7)	589.2	2.994(33)	429.0

Table 2. Periods of Adiabatic Oscillation and Vibrational Stability Results:

g-modes of ℓ = 1

Mode	Model	ω^2	P(s)	E_N	E_F	E_{ε_2}	σ'^{-1} years*
			a) Sequence $X_c = 0.1$, $X_s = 0.7813$				
g_1	1	7.8780	3.222(3)	6.621(33)	2.005(37)	6.702(36)	3.617(5)
	2	7.8665	3.217(3)	7.055(33)	1.845(37)	6.676(36)	1.728(5)
	3	7.8640	3.230(3)	7.551(33)	1.839(37)	6.132(36)	1.655(5)
	4	7.9046	3.360(3)	1.425(34)	2.267(37)	7.793(36)	1.627(5)
	5	12.696	3.144(3)	5.622(35)	4.512(36)	9.943(35)	9.657(5)
	6	10.425	3.133(3)	5.143(35)	3.720(36)	7.724(35)	1.031(6)
g_2	1	3.2561	5.012(3)	4.805(35)	1.260(36)	1.414(31)	1.080(6)
	2	3.5710	4.774(3)	4.082(35)	1.222(36)	1.278(31)	1.139(6)
	3	3.9404	4.562(3)	3.436(35)	1.204(36)	1.412(31)	1.185(6)
	4	5.8776	3.912(3)	1.818(35)	1.246(36)	5.114(32)	1.289(6)
	5	10.754	3.416(3)	5.943(35)	6.990(36)	2.263(36)	4.350(5)
	6	8.9907	3.374(3)	4.383(35)	8.810(36)	2.953(36)	3.979(5)
g_3	1	2.1934	6.107(3)	2.070(35)	2.123(35)	1.497(33)	1.129(9)
	2	2.2158	6.061(3)	2.243(35)	2.205(35)	4.203(33)	-6.698(7)
	3	2.2795	5.998(3)	2.466(35)	2.311(35)	3.963(33)	-3.576(7)
	4	3.3686	5.167(3)	4.163(35)	3.769(35)	8.044(33)	-1.719(7)
	5	7.9828	3.965(3)	4.918(34)	3.789(37)	1.202(37)	6.913(4)
	6	7.5896	3.672(3)	1.267(35)	1.708(37)	6.405(36)	1.726(5)
g_4	1	1.3049	7.917(3)	2.148(35)	2.775(35)	1.497(33)	5.712(6)
	2	1.3302	7.823(3)	2.022(35)	2.912(35)	1.523(33)	4.067(6)
	3	1.3971	7.662(3)	1.563(35)	8.474(35)	5.851(32)	5.277(5)
	4	2.0993	6.545(3)	7.504(34)	1.308(36)	1.104(32)	4.267(5)
	5	5.0533	4.984(3)	3.404(35)	1.077(36)	9.896(34)	1.791(6)
	6	4.1532	4.964(3)	3.044(35)	7.788(35)	3.196(34)	2.274(6)
g_5	1	1.1075	8.594(3)	1.913(35)	1.175(36)	1.008(31)	2.912(5)
	2	1.2376	8.110(3)	1.553(35)	1.154(36)	4.795(31)	3.214(6)
	3	1.3607	7.764(3)	1.674(35)	5.898(35)	9.837(32)	8.492(5)
	4	1.9029	6.875(3)	1.419(35)	2.713(35)	3.585(33)	3.791(6)
	5	4.4616	5.3040(3)	1.615(35)	2.424(36)	4.345(33)	4.431(5)
	6	3.6414	5.301(3)	1.424(35)	2.147(36)	1.499(33)	4.363(5)

* A negative signs means instability.

Table 2. Cont.

Mode	Model	ω^2	P(s)	E_N	E_F	E_{ε_2}	σ'^{-1} years*
			b) Sequence $X_c = 0$, $X_s = 0.794$				
g_1	7	12.016	2.645(3)	8.246(33)	1.420(33)	1.120(33)	2.179(6)
	8	11.501	2.700(3)	9.665(33)	1.587(37)	6.319(32)	1.876(6)
	9	13.506	2.602(3)	1.229(34)	1.850(36)	4.443(33)	1.840(6)
	10	16.174	2.522(3)	2.259(34)	2.024(36)	7.014(34)	2.009(5)
g_2	7	7.8233	3.280(3)	9.341(33)	1.125(37)	6.111(36)	3.772(5)
	8	7.8062	3.278(3)	1.022(34)	1.739(37)	5.568(36)	1.690(5)
	9	7.8625	3.411(3)	2.077(34)	1.775(37)	6.621(36)	3.131(5)
	10	8.1235	3.559(3)	1.738(35)	2.059(37)	6.793(36)	1.436(5)
g_3	7	4.6159	4.272(3)	2.205(33)	1.600(36)	2.180(31)	7.385(5)
	8	4.3704	4.380(3)	2.927(33)	1.652(36)	3.393(31)	6.805(5)
	9	5.1462	4.216(3)	1.169(34)	2.061(36)	6.063(32)	6.271(5)
	10	6.9676	3.843(3)	5.438(35)	3.287(36)	9.102(35)	9.375(5)
g_4	7	2.4960	5.808(3)	4.135(33)	4.497(35)	3.301(33)	2.040(7)
	8	3.0155	5.273(3)	5.231(35)	4.776(35)	2.856(33)	-1.617(7)
	9	4.3921	4.564(3)	5.694(35)	5.628(35)	1.355(34)	-4.828(7)
	10	6.1229	4.100(3)	1.723(34)	1.897(36)	4.739(33)	4.449(5)
g_5	7	2.3755	5.953(3)	2.503(33)	1.536(36)	2.708(33)	4.021(5)
	8	2.2827	6.063(3)	8.904(32)	1.813(36)	1.128(31)	3.232(5)
	9	2.6827	5.839(3)	8.637(32)	2.203(36)	2.045(31)	3.037(5)
	10	3.3025	5.582(3)	2.358(35)	6.273(35)	9.362(33)	4.030(5)

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* A negative signs means instability.

Table 3. Periods of Adiabatic Oscillations and Vibrational Stability :

Discontinuity Modes

a) Model 1

1	Mode	ω^2	P(s)	E_N	E_F	$E_R^r(D)$	$E_{\varepsilon 2}$	$\dot{\sigma}$-1 years
1	p_1	18.605	2.097(3)	6.094(34)	2.289(38)	8.775(34)	7.990(37)	4.901(4)
	p_2	21.825	1.936(3)	5.488(34)	3.998(38)	1.270(35)	1.462(38)	3.008(4)
2	p_4	69.615	1.084(3)	6.168(35)	9.731(37)	1.390(36)	3.886(37)	3.117(5)
3	p_5	113.65	8.483(2)	1.002(36)	1.788(38)	3.554(36)	7.882(37)	2.958(5)
4	p_6	156.63	7.227(2)	1.228(36)	7.506(37)	6.573(36)	2.386(37)	6.099(5)
5	p_7	198.64	6.417(2)	1.389(36)	8.889(37)	1.058(37)	1.830(37)	8.939(5)
6	p_7	240.26	5.835(2)	1.506(36)	1.726(37)	1.556(37)	5.971(35)	4.209(6)
7	p_8	282.13	5.384(2)	1.651(36)	2.006(37)	2.099(37)	1.187(34)	4.097(6)
8	p_9	323.45	5.029(2)	1.732(36)	2.949(37)	2.762(37)	4.970(35)	3.195(6)
9	p_9	364.68	4.736(2)	1.800(36)	3.431(37)	3.513(37)	1.800(31)	3.022(6)
10	p_{10}	405.86	4.489(2)	1.858(36)	4.427(37)	4.352(37)	1.435(35)	2.601(6)

b) Model 8

1	Mode	ω^2	P(s)	E_N	E_F	$E_R^r(D)$	$E_{\varepsilon 2}$	$\dot{\sigma}$-1 years
1	p_2	260.54	1.794(3)	1.013(35)	3.035(38)	2.280(33)	1.179(38)	3.150(4)
2	p_5	119.35	8.382(2)	6.820(35)	6.243(39)	3.190(35)	2.313(39)	7.852(3)
4	p_9	277.24	5.500(2)	1.714(36)	5.922(39)	3.121(36)	1.714(39)	1.781(4)
8	p_{13}	573.71	3.823(2)	2.729(36)	2.801(39)	1.774(37)	7.096(34)	6.342(6)

Table 4.

l=0	l=2	l=4	l=6	l=8	Observed
	70.7 (g_6)		71.1		
65.9	61.2 (g_5)	67.3 (g_{11})	67.4 (g_{16})		66.25
		64.7 (g_{10})	65.2 (g_{15})		
		62.9 (g_9)	61.3 (g_{14})		
	59.4 (g_4)	56.2 (g_8)	58.9 (g_{13})	59.1 (g_{17})	
	51.3 (g_3)	53.0 (g_7)	57.6 (g_{12})	55.2 (g_{16})	
			56.5 (g_{11})	55.0 (g_{15})	
			51.8 (g_{10})	52.9 (g_{14})	
			51.1 (g_9)	51.1 (g_{13})	
41.7	44.8 (g_2)	49.6 (g_6)	46.3 (g_8)	49.6 (g_{12})	44.66
		43.8 (g_5)	42.8 (g_7)	48.5 (g_{11})	
		41.5 (g_4)	42.0 (g_6)	45.5 (g_{10})	
				44.2 (g_9)	
				42.0 (g_8)	
31.4	39.1 (g_1)	38.4 (g_3)	37.1 (g_5)	38.0 (g_7)	39.00
	35.5 (f)	36.6 (g_2)	34.9 (g_4)	37.6 (g_6)	32.1
	32.7 (p_1)		34.3 (g_3)D	33.1 (g_5)	
			31.3 (g_2)	31.4 (g_4)	
				31.3 (g_3)	
24.7	25.4 (p_2)	29.9 (g_1)	26.5 (g_1)	28.3 (g_2)	28.7
	20.8 (p_3)	28.1 (f)	25.3 (f)	24.7 (g_1)	24.8
		27.9 (p_1)	24.7 (p_1)	23.3.(f)	21.0
		22.3 (p_2)	20.3 (p_2)	23.2 (p_1)	
17.4	17.6 (p_4)	18.5 (p_3)	17.1 (p_3)	18.8 (p_2)	19.5
15.0	16.3 (p_5)D	15.9 (p_4)	14.8 (p_4)	16.0 (p_3)	13.3
13.3	15.3 (p_6)	14.0 (p_5)	13.1 (p_5)	13.9 (p_4)	12.1
12.0	13.5 (p_7)	12.8 (p_6)		12.4 (p_5)	11.4
	12.1 (p_8)	11.4 (p_7)D		11.1 (p_6)	10.7
	11.0 (p_9)	11.3 (p_8)		10.2 (p_7)	
	10.1 (p_{10})	10.3 (p_9)			
		9.53(p_{10})	9.46(p_9)	9.40(p_8)	9.9

Periods (in minutes) of model 6 ("present sun") for radial (l=0) and non-radial (l=2,4,6,8) modes. The identification of the modes is given in parentheses. D indicates a discontinuity mode. Last column gives solar periods observed by Brown et al. (1978) in the range 10m-70m.

values of E_N, E_F, E_2, and the e-folding time $1/\sigma'$ for the low order g modes corresponding to $\ell = 1$. A negative sign for $1/\sigma'$ means instability and growth of the oscillation amplitude. For the models with $X_c = 0.1$, the g_3 mode becomes unstable in the approach to the main sequence in Model 2 close to the temporary minimum in the ratio ρ_c/ρ, due to the slight expansion of the central regions accompanying the onset of nuclear reactions. The instability subsists more than 10^9 years, until the central condensation has reincreased enough to produce a corresponding increase in amplitude in the envelope which is large enough to damp the oscillation. The "present sun" is stable. The differences in the results for Models 5 and 6 are due to the difference in radii of the two models, the large difference in their ratios of ℓ_c/ρ, and from the high sensitivity of the eigenvalues and eigenfunctions to ρ_c/ρ.

In the sequence $X_c = 0$, the instability appears in the g_4 mode. Let us recall here that in the standard solar evolution of models less condensed than the present ones, a phase of instability towards the g_2 and g_3 modes occurs (Boury et al. 1975). The modes associated with the discontinuity turn out to be very stable (Table 3). The destabilizing effect of the nuclear energy term is largely overcome by the large perturbation of the temperature gradient, which appears as the radial part $\frac{d\delta L}{dm}$ of the term $\delta(\frac{1}{\rho}\,\mathrm{div}\,\vec{F})$. The seventh column of Table 3 shows the contribution $E_R^r(D)$ of the discontinuity to the integral $\int \frac{\delta T}{T}\,d\delta L$. A steep change in density would have the same stabilizing effect as a strict discontinuity.

In conclusion, the evolution of the sun when starting with a small (or zero) hydrogen abundance in a small central region presents the same instability as the standard evolution towards low-order g nonradial modes for $\ell = 1$; however, the spectrum of the model corresponding to the present solar age is not compatible with the observations.

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