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The Solar Structure and the Low l Five-minute Oscillation. II

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Summary. Models of solar envelope have been computed with different values of Y, Z, and l/H (respectively abundances of helium and heavy elements and ratio of the mixing-length to the pressure scale height in the convective zone). These parameters are fitted so that the model gives the frequencies of solar oscillations observed by Grec et al. (1980) at the geographic south pole. We find that the observations are compatible with the usually accepted solar abundances but they require a convective zone somewhat deeper than the one found in standard solar models.

Key word: solar oscillation

1. Introduction

The observations of Grec et al. (1980) at the geographic south pole have given a detailed spectrum of solar oscillations with frequencies close to 3 mHz corresponding to oscillatory modes with low values of the spherical harmonics degree l.

A three parameters family of solar envelope models has been computed. The first two parameters describe the chemical composition: Y is the abundance of helium and Z the abundance of heavy elements. The third parameter is $\alpha = l/H$, the ratio of the mixing-length to the pressure scale-height in the convective zone. The frequencies of oscillation have been computed for modes p_{10} to p_{30} and for $l = 0$ to 4. Then the parameters Y, Z and α were chosen so that the computed spectrum matches the observed one as well as possible (least square fit).

2. The Models

Envelope models were computed as described in Scuflaire et al. (1981) for 3 chemical compositions and 5 values of $\alpha = l/H$. The 3 compositions are given in Table 1. The first one is the same as in Scuflaire et al. (1981). The values of α also are the same as in this paper and range from 1.85 to 5. The characteristics of the models with the first composition have already been given in this paper. Tables 2 and 3 give the characteristics of the models with the two other compositions.

For each model the frequencies of radial and nonradial oscillations of low l values ($l = 0$ to 4) were computed in the range

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Table 1. Chemical compositions used for the models. X, Y, and Z are respectively the abundances of hydrogen, helium and heavy elements

N°	X	Y	Z
1	0.7417	0.2383	0.0200
2	0.7417	0.2403	0.0180
3	0.7600	0.2200	0.0200

Table 2. Some characteristics of models with chemical composition No. 2. α is the ratio of the mixing-length to pressure scale height in the convective zone, r, ϱ, and T respectively the radius, the density and the temperature at the basis of the convective zone and m_c the mass of the convective zone

α	$r\,(R_\odot)$	$m_c\,(M_\odot)$	ρ (g cm^{-3})	$T\,(10^6$ K)
1.85	0.739	0.017	0.131	1.92
2.4	0.700	0.037	0.275	2.33
3	0.674	0.060	0.434	2.60
4	0.649	0.092	0.658	2.89
5	0.631	0.119	0.842	3.09

Table 3. Same as Table 2 but for models with chemical composition No. 3

α	$r\,(R_\odot)$	$m_c\,(M_\odot)$	ρ (g cm^{-3})	$T\,(10^6$ K)
1.85	0.749	0.012	0.095	1.80
2.4	0.703	0.030	0.220	2.26
3	0.677	0.049	0.357	2.54
4	0.650	0.078	0.562	2.85
5	0.634	0.101	0.719	3.03

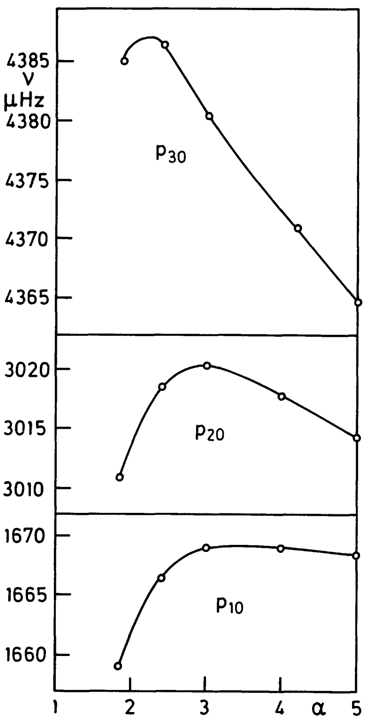

Fig. 1. Frequencies of three $l=2$ modes as function of α for chemical composition n° 1

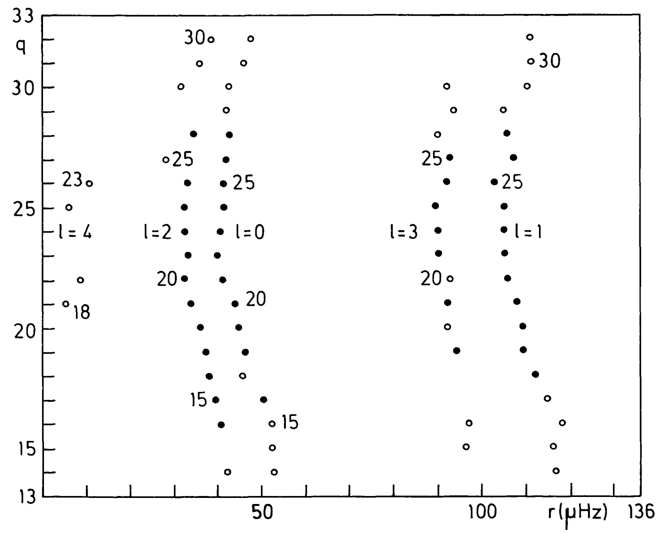

Fig. 2. Representation of the observed spectrum in the coordinates q, r (see text, Sect. 3). The filled circles represent well identified frequencies, the identification of frequencies represented by open circles is less certain

from 1.6 and 4.5 mHz (i.e. from mode p_{10} to p_{30}). In the numbering of the modes, we have omitted the chromospheric mode which is of a different nature and is not observed. Its frequency is almost independant of l and is about 3.44 mHz. Proceeding in this way the frequency of the mode p_n is in first approximation a linear function of n (for a given l). In order to reduce computation cost, we have computed only one frequency out of five and obtained the others with a spline interpolation. We have verified for one model that the error brought about by this process is less than 1 μHz.

To obtain the oscillation frequencies for arbitrary values of Y, Z, α we have interpolated the table of computed frequencies. With only three chemical compositions we could only linearly interpolate in the variables Y and Z. Figure 1 shows that the frequencies do not depend linearly on α, so the interpolation in α has been carried out with the five points Lagrange's formula. We shall note $\nu_{ln}^c\,(Y, Z, \alpha)$, or more briefly $\nu_i^c\,(Y, Z, \alpha)$, the frequency obtained in this manner for the given l and mode p_n.

The computational error ε_i^c on ν_i^c will be taken equal to 1 μHz in the following discussion. It may however be higher when the chemical composition is somewhat distant from those used in the computation. This value does not include the "physical error" resulting from the uncertainties in the input physics (equation of state, opacity) for which we have no reasonable estimation.

3. The Observations

The observations are those obtained by Grec et al. (1980). In the power spectrum we have retained 41 well-marked peaks which can be identified without ambiguity. We discarded a number of peaks on the borders of the observed range, the identification of which is less certain. We shall note ν_{ln}^0 or ν_i^0 the corresponding frequencies. They are given without asterisk in Table 4. These frequencies agree very well with the latest values given by Grec et al. (1981). This table gives also $\Delta\nu_i$, the "half-width" of the peaks. These $\Delta\nu_i$ may have no physical meaning. They are defined as the standard deviation from the central frequency but may result from noise or from the erroneous inclusion in one single peak of faint unidentified neighbouring frequencies.

As the ν_{ln}^0 for a given l are almost equidistant (separation of about 136 μHz) they are at best represented by two coordinates q, r defined by

$$\nu_i^0 = aq_i + r_i$$

with $a = 136$ μHz, q_i is an integer and r_i is comprised between 0 and a. Figure 2 gives such a picture for the observed spectrum (filled circles).

4. Comparison

For each observed mode, we define a distance between the observed and the computed frequency:

$$d_i(Y, Z, \alpha) = |\nu_i^c(Y, Z, \alpha) - \nu_i^0| - \varepsilon_i^c - \varepsilon_i^0 .$$

The error on the observed frequency ε_i^0 has been taken equal to the half-width (see previous section). Of course, if this expression gives a negative result, d_i is taken equal to zero. For a model with parameters Y, Z, α a mean distance is defined by

$$d(Y, Z, \alpha) = \sum_{i=1}^{N} [[d_i(Y, Z, \alpha)]^2/N]^{1/2} .$$

The best models are those which minimize d.

If we do not impose any constraint on the three parameters, d is minimum (0.40 μHz) for $Y = 0.271$, $Z = 0.0254$, $\alpha = 2.976$ ($X = 0.7036$). This minimum spreads out broadly along a plane (in Y, Z, α space) whose equation is

$$Z = 0.0795\,Y + 0.0039 .$$

Table 4. The observed frequencies and the half-widths are given in μHz in the form $\nu_i^c \pm \Delta\nu_i$. The asterisks indicate frequencies the identification of which is less certain

n	l=0	l=1	l=2	l=3	l=4
12			1946.8±1.0 *		
13	1957.3±3.3 *	2021.2±0.5 *		2136.7±2.2 *	
14	2092.4±1.9 *	2156.9±1.1 *	2217.0±0.5	2273.5±1.1 *	
15	2228.9±1.1 *	2294.6±0.5 *	2351.9±0.5		
16	2363.0±0.5	2427.1±2.3 *	2486.6±1.8		
17	2494.2±0.5 *	2560.7±1.1	2621.7±1.7	2678.9±0.5	
18	2630.9±0.5	2693.9±1.1	2756.1±1.2	2812.5±2.4 *	2861.7±3.4 *
19	2765.2±1.2	2829.7±2.1	2890.1±1.9	2948.7±0.5	3001.2±2.0 *
20	2900.4±1.5	2964.6±1.5	3025.3±2.1	3085.7±2.7 *	
21	3034.0±2.0	3098.3±2.2	3161.9±2.3	3218.9±1.3	
22	3168.5±0.5	3234.0±1.9	3297.0±2.6	3354.7±2.8	3406.6±2.4 *
23	3305.2±0.5	3369.7±2.4	3432.7±2.8	3490.0±0.5	3546.9±0.9 *
24	3441.9±0.5	3505.7±2.6	3569.6±2.5	3629.1±1.6	
25	3577.8±1.1	3639.5±2.7	3700.7±2.0 *	3765.8±1.0	
26	3714.8±1.3	3780.1±0.9	3842.8±1.8	3898.6±1.7 *	
27	3851.0±1.1	3914.4±2.2		4038.0±3.9 *	
28	3986.2±5.7 *	4049.9±2.4 *	4112.6±3.5 *	4173.1±1.7 *	
29	4123.5±1.9 *	4191.4±4.8 *	4252.4±2.1 *		
30	4262.6±1.9 *	4328.1±1.0 *	4391.1±2.8 *		
31	4400.0±1.1 *	4463.8±1.8 *			

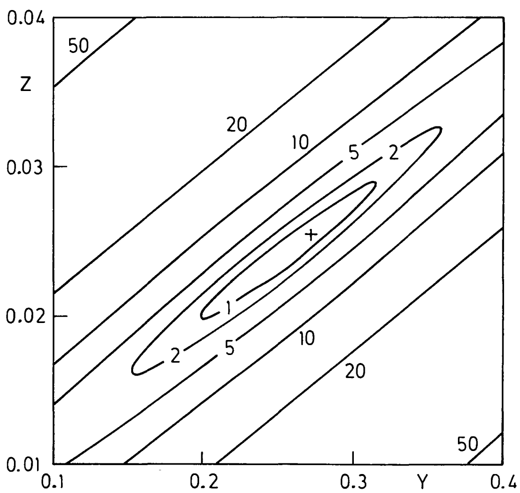

Fig. 3. Curves d = constant (expressed in μHz) in the Y, Z plane for $\alpha = 2.976$. The cross indicates the position of the minimum of d

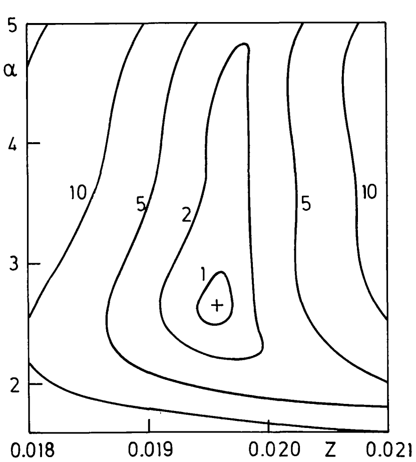

Fig. 4. Curves d = constant (expressed in μHz) in the Z, α plane when $Z/X = 0.025$. The cross indicates the position of the minimum of d

As far as this relation is satisfied, d varies little and remains less than 1 μHz when Y varies from 0.19 to 0.32 and α from 2.5 to 3.8. Figure 3 gives contours d = constant in the plane $\alpha = 2.976$.

The frequencies of the model which minimizes d have helped us to confirm the identification of a few more frequencies in the observed spectrum. They are marked with an asterisk in Table 4 and represented in Fig. 2 by open circles.

Figure 3 clearly shows that we must resort to independent information about the solar composition if we want to specify it unambigously. Photospheric spectra allow the determination of the Z/X ratio. Values cited for this ratio are 0.026 (Pagel, 1979), 0.0245 (Cameron, 1980) and 0.0259 (Grevesse, 1981). We have taken $Z/X = 0.025$ and we are left with two independent variables Z and α. $d(Z, \alpha)$ reaches a minimum (0.86 μHz) for $Z = 0.0196$ ($X = 0.784$, $Y = 0.1964$ and $\alpha = 2.64$. Figure 4 gives the curves d = constant in the (Z, α) plane.

5. Conclusions

The accuracy of the observations reached by Grec et al. (1980) has allowed us to determine the solar composition provided the ratio Z/X is fixed independently. This composition is very close to the usually adopted one (Cameron, 1980, recommends $X = 0.772$, $Y = 0.209$, $Z = 0.019$).

It is also in good agreement with the composition deduced by Gabriel et al. (1982) from the observations made by Claverie et al. (1981). This agreement however reflects only the fact that both sets of observations are in good agreement and that the same models were used in both studies.

The model which gives the better agreement with the observations (with $Z/X = 0.025$) has a rather deep convective envelope whose mass equals $0.036\,M_\odot$ and whose inner limit is located at $r = 0.69\,R_\odot$ from the centre. Let us recall that our standard model gives $m_c = 0.014\,M_\odot$ and $r = 0.74\,R_\odot$. The present result is compatible with the inequality deduced by Rhodes et al. (1977) which gives $0.011 < m_c/M_\odot < 0.095$ and $0.62 < r/R_\odot < 0.75$. It is also in good agreement or compatible with the results of other studies. Berthomieu et al. (1980) fix the base of the convective zone at $r/R_\odot = 0.71$ with $\alpha = 2.5$ and Rhodes and Ulrich (1980) give $r/R_\odot = 0.73$. In future work we intend to study the influence of the input physics.

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References

Berthomieu, G., Copper, A.J., Gough, D.O., Osaki, Y., Provost, J., Rocca, A.: 1980, in *Nonradial and Nonlinear Stellar Pulsation*, eds. H.A. Hill and W.A. Dziembowski, Springer, Berlin, Heidelberg, New York, p. 307

Cameron, A.G.W.: 1980, preprint n° 1357, Harvard-Smithsonian Center for Astrophysics

Claverie, A., Isaak, G.R., McLeod, C.P., van der Raay, H.B., Roca Cortes, T.: 1981, *Nature* **293,** 443

Gabriel, M., Scuflaire, R., Noels, A.: 1982, *Astron. Astrophys.* **110,** 50

Grec, G., Fossat E., Pomerantz, M.: 1980, *Nature* **288,** 541

Grec, G., Fossat E., Pomerantz, M.: 1981, Latest results from the south pole observations (preprint)

Grevesse, N.: 1981, *Ciel et Terre* **97,** 193

Pagel, B.E.J.: 1979, in *Les éléments et leurs isotopes dans l'univers*, p. 261 (22ème colloque international d'astrophysique, Liège, 1978)

Rhodes, E.J., Ulrich, R.K.: 1980, *Bull. Amer. Astron. Soc.* **12,** 475

Rhodes, E.J., Ulrich, R.K., Simon, G.W.: 1977, *Astrophys. J.* **218,** 901

Scuflaire R., Gabriel, M., Noels, A.: 1981, *Astron. Astrophys.* **99,** 39