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Stability of a 1 $M_\odot$ star with decreasing gravitational constant*

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Summary. The evolution of a $1 M_\odot$ star is computed in the Multiplicative Creation Hypothesis with a solar age of 4.6 10^9 yr and an age of the Universe of 18 10^9 yr. The stability towards non-radial pulsations is studied. An instability with respect to $l=1$, g_1, and g_2 modes appear at the beginning of the MS phases. It lasts up to about the present Sun, which is marginally unstable. The duration of the unstable phase decreases if the age of the Universe is increased.

Key words: G variable – solar evolution and stability

I. Introduction

During the seventies, the consequences for stellar evolution of Dirac's Large Number Hypothesis were extensively discussed (Chin and Stothers, 1975, 1976; Vandenberg, 1976, 1977; Maeder, 1977a). It was shown that while the Additive Creation Hypothesis is untenable, the Multiplicative Creation Hypothesis (MCH) satisfies all the tests. It even predicts a smaller neutrino flux, about one half of the standard model's. It also meets the classical cosmological tests (Canuto and Lodenquai, 1977; Maeder, 1977b).

The question is now: What about the stability of the Sun in the Multiplicative Creation Hypothesis? The classical evolution leading to the standard solar model crosses a vibrationally unstable phase towards non-radial pulsations (Christensen-Dalsgaard et al., 1974; Boury et al., 1975; Shibahashi et al., 1975; Saio, 1981). The adiabatic results show that this phase starts at an age of about 2.4 10^8 yr and lasts about 3 10^9 yr. With a non-adiabatic analysis, Saio (1981) even finds that the present sun is still unstable. In fact, all non fully convective stars less massive than the Sun show the same instability during some fraction of their main sequence phase (Noels et al., 1976). In order to analyse the stability of the Sun in the MCH frame, we have evolved a solar model with this hypothesis. The main properties of the models investigated here are described in Sect. II. An important parameter for this type of instability is the central condensation. In fact, its behaviour with time favours such an instability during a time even longer than in the standard case. The results of the stability analysis are discussed in Sect. III.

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* The recent and sudden death of Dr. Arsène Boury has prevented him from participating in the elaboration of the manuscript. Any errors have to be attributed to the other authors

II. Models

An evolution starting at the end of the Hayashi phase has been followed with $G \propto t^{-1}$, $M \propto t^2$ supposing an age of the Universe of 18 10^9 yr, and a solar age of 4.6 10^9 yr. The physics of the models is the same as in Boury et al. (1980) except that now the opacities are obtained from the Astrophysical Opacity Library (Huebner et al., 1977). Taking $Z=0.02$, $X=0.687$ provides at the solar age a model whose luminosity and radius discard by a few per cent from the solar values. This accuracy is good enough for our purpose. Our X-value is smaller than that obtained by Maeder (1977a) by about 0.05. This is due to the new opacities which are about 20% larger than Cox (1971)'s values.

Table 1 gives a few properties of the models. The final model is very close to Maeder's results. The luminosity is 6% lower, the central temperature T_c is 0.8% higher while the central density is 10% lower. One important factor for the stability analysis is the variation of the central condensation with time. At the solar age it is 20% lower than for the standard Sun discussed in Boury et al. (1975). Therefore we may immediately expect that the unstable phase will last longer in these models than for a standard solar evolution.

III. Stability analysis

The stability of the models given in Table 1 has been studied for the first g modes of spherical surface harmonic order l equal to 1, 2, 3, and 4. The equations and the method of computation are the same as in Boury et al. (1975).

For $l>1$, all the modes in all the models are stable.

For $l=1$, only the g_1 and g_2 modes become unstable.

The results for these modes are given in Table 2. It gives for each modes the period P, the location, in fraction of the radius, x_1 and x_2 and the absolute value of the extrema of $(\delta p/p)$, R_1, and R_2 supposing a normalization of $(\delta p/p)=1$ at the surface and the e-folding time σ'^{-1} (a negative sign means instability).

The E' are most easily defined by the formula

$$\sigma' = -\frac{E_N - E_F + E_{\varepsilon_2}}{\sigma^2 \int |\delta r|^2 dm}$$

$$E_N = \int \frac{\delta T}{T} \delta \varepsilon dm$$

$$E_F = \int \frac{\delta T}{T} \delta\left(\frac{1}{\varrho} \boldsymbol{\nabla} \cdot \boldsymbol{F}\right) dm$$

$$E_{\varepsilon_2} = \int (\Gamma_3 - \delta/3) \frac{\delta\varrho}{\varrho} \delta\left(\varepsilon_2 + \frac{1}{\varrho} \overline{\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} p}\right) dm,$$

Table 1. Properties of the models

Model	$M/M_\odot$	Age (yr)	$L/L_\odot$	Lg $T_{\rm eff}$	$\varrho_c/\bar{\varrho}$	T_c (10^6 K)	ϱ_c	X_c
1	0.5669	1.52(8)	0.322	3.7312	30.98	12.62	87.96	0.6798
2	0.5713	2.05(8)	0.328	3.7318	31.48	12.61	88.16	0.6768
3	0.5990	5.31(8)	0.358	3.7346	33.89	12.72	91.13	0.6577
4	0.6431	1.03(9)	0.407	3.7388	37.43	12.97	94.51	0.6274
5	0.7244	1.92(9)	0.506	3.7457	45.06	13.45	101.5	0.5712
6	0.7818	2.51(9)	0.584	3.7501	51.41	13.80	107.2	0.5316
7	0.8404	3.10(9)	0.673	3.7541	59.34	14.18	113.8	0.4898
8	0.9017	3.69(9)	0.774	3.7579	69.32	14.58	121.7	0.4458
9	0.9650	4.28(9)	0.890	3.7615	81.97	15.03	131.3	0.3987
10	1	4.60(9)	0.959	3.7633	90.50	15.29	137.1	0.3720

Table 2. Properties of the oscillations

Model	Mode	$P_{\rm (min)}$	R_1	x_1	R_2	x_2	E_N	E_F	$E_{\varepsilon 2}$	σ'^{-1}(yr)
1	g_1	90.27	1.898	0.2474	–	–	3.270(34)	3.364(34)	7.125(32)	1.478(9)
	g_2	127.6	5.559	0.1713	4.994	0.4664	3.188(34)	3.371(34)	1.553(32)	6.269(7)
2	g_1	89.98	1.884	0.2466	–	–	3.614(34)	3.504(34)	7.305(32)	-1.062(8)
	g_2	126.7	5.219	0.1706	4.603	0.4562	3.433(34)	3.473(34)	1.712(32)	5.256(8)
3	g_1	89.08	1.815	0.2256	–	–	5.026(34)	4.280(34)	8.727(32)	-2.758(7)
	g_2	123.8	4.258	0.1297	3.424	0.4239	4.338(34)	4.031(34)	2.522(32)	-3.551(7)
4	g_1	87.93	1.746	0.1973	–	–	7.339(34)	5.698(34)	1.129(33)	-1.588(7)
	g_2	120.4	3.637	0.1131	2.536	0.3887	5.722(34)	4.945(34)	3.967(32)	-1.815(7)
5	g_1	84.72	1.686	0.1412	–	–	1.326(35)	9.180(34)	1.896(33)	-9.250(6)
	g_2	113.5	3.087	0.0757	1.749	0.3346	8.144(34)	6.648(34)	7.811(32)	-1.400(7)
6	g_1	81.73	1.658	0.1007	–	–	1.882(35)	1.244(35)	2.975(33)	-7.588(6)
	g_2	105.9	2.767	0.0739	1.478	0.2851	9.651(34)	8.064(34)	1.203(33)	-1.669(7)
7	g_1	78.20	1.535	0.0900	–	–	2.562(35)	1.667(35)	5.253(33)	-6.971(6)
	g_2	104.6	2.543	0.0522	1.292	0.2547	1.153(35)	1.008(35)	1.877(33)	-2.247(7)
8	g_1	74.38	1.253	0.0674	–	–	3.366(35)	2.308(35)	1.219(34)	-7.380(6)
	g_2	100.4	2.344	0.0497	1.136	0.2097	1.452(35)	1.337(35)	3.136(33)	-3.242(7)
9	g_1	70.40	0.8688	0.0640	–	–	4.312(35)	3.587(35)	3.864(34)	-1.037(7)
	g_2	95.95	2.115	0.0471	0.9874	0.1906	1.904(35)	1.846(35)	5.562(33)	-5.410(7)
10	g_1	68.27	0.6299	0.0621	–	–	4.918(35)	5.562(35)	9.309(34)	-4.278(7)
	g_2	93.43	2.005	0.0417	0.8940	0.1758	2.244(35)	2.238(35)	8.002(33)	-8.218(7)

where all symbols have the same meaning, as in Boury et al. (1975) and in Ledoux and Walraven (1958).

E_n gives the influence of the perturbation of the nuclear reactions and it is always destabilizing. E_F gives the effect of the perturbation of the radiative and convective fluxes. As in most stars this term is stabilizing in solar models. E_n and E_F are known with a good accuracy as the perturbation of the convective flux is known with a good accuracy in the solar type conditions. More delicate is the evaluation of E_{ε_2} which gives the contribution of the mechanical effects of convection. It depends much more on the theory of the interaction between convection and pulsation used.

Table 2 shows that the instability appears very early in the evolution of these models as in the standard evolution. It comes in first through the $l=1$, g_1 mode but very quickly the $l=1$, g_2 mode becomes also unstable. The instability increases up to model 5 for the g_2 mode and up to model 7 for the g_1 mode. Afterward, the stabilizing term E_F grows more rapidly than the other two terms which are destabilizing. The instability is nevertheless still present for the two modes at the solar age, which was not the case for the standard Sun. During the whole unstable phase, the *e*-folding time is much shorter than the evolutionary time. The amplitude of these modes has therefore plenty of time to grow. Non-radial oscillations however are not observed in solar type stars or in less massive ones. One possible reason could be that in these stars, the amplitudes can reach only small values because the differential non-radial motion produces turbulence at a local scale which dissipates the energy of the pulsation.

Due to the uncertainties in the evaluation of E_{ε_2}, the importance of this term on the value of σ' has to be discussed. Models 2 to 9 would remain unstable even if the contribution of E_{ε_2} was stabilizing and in most cases, its value is one order of magnitude smaller than (E_n-E_F). For these models, concluding to instability seems very safe. For model 10, the term E_{ε_2} is necessary to produce the instability and for this model we consider that it is harzardous to conclude in either direction.

The cause of the instability is the same as for the standard solar models. A detailed discussion can be found in Boury et al. (1975). We will reproduce it here only briefly. The oscillations are driven by the nuclear reactions while their energy is dissipated by the radiative and convective fluxes. At the beginning of the main sequence phases, the abundance of He^3 increases and reaches its equilibrium value in a larger and larger mass. As a result, the temperature dependence of the perturbation of the energy generation rate $\delta\varepsilon$ increases from the static value of about 4 to about 10 favouring the destabilizing term E_n. When He^3 reaches its equilibrium in a large enough core, the instability sets in. Of course, the instability appears also because the amplitudes are large enough in the nuclear burning core compared to the outer layers, where the flux is the most efficient. As time goes on, the amplitudes get smaller in the core compared to the surface layers so R_1 and R_2 decrease. This will finally lead to the decrease of the instability. But before model 7 (5 for g_2 modes) is reached, a third effect contributes to the instability. The positions of the extrema move inwards (x_1 and x_2 decrease). This, at first and all other things being equal, increases the mean amplitude in the nuclear energy production zone and enhances E_n. When x_1 comes too close to the center, this effect is reversed. The conjunction of the last two points discussed will finally restore the stability of the star.

Finally one can wonder what influence the supposed age of the universe has on our results. According to Maeder (1977a), $L \propto G^7 M^5 = L_n \Big/ \frac{t}{t_1}$. As $dx/dt \propto LM^{-1}$, $dx/dt = (dx/dt)_n t/t_n$, where t is the cosmic time and the subscript n indicates a value now at $t = t_n$.

The total amount ΔX of hydrogen burned since the birth of the sun at $t_0 = t_n - 4.6\,10^9$ yr is given by

$$\Delta X = -\frac{1}{2}\left(\frac{dX}{dt}\right)_n t_n \left[1 - \left(\frac{t_0}{t_n}\right)^2\right] = -\left(\frac{dX}{dt}\right)_n 4.6\,10^9 \left[1 - \frac{4.6\,10^9}{t_n}\right].$$

Clearly ΔX increases with the age of the Universe and an older Universe would lead to a more evolved solar model i.e. to a model with a larger mass concentration. Such a model would be more stable than found here. As a consequence, the length of the unstable period decreases when the age of the universe increases.

IV. Conclusion

A solar evolution computed in the frame of the Multiplicative Creation Hypothesis supposing a solar age of $4.6\,10^9$ yr and an age of the universe of $18\,10^9$ yr shows the same instability with respect to the $l = 1$, g_1, and g_2 modes as the standard sequence. However the unstable phase lasts longer than in the standard case. In the MCH frame, the present Sun is still marginally unstable. The length of the unstable phase decreases when the age of the Universe is increased.

The e folding time of the instability is of the order of 10^7 yr. Pulsations should therefore be observed in solar type stars or else, the instability should have unknown consequences on the evolution of the models.

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