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**«Oscillations as a Probe of the Sun's Interior»**

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INFLUENCE OF OPACITIES, PARTITION FUNCTIONS AND HYDROGEN DIFFUSION ON  
THE 5 MIN. SOLAR OSCILLATIONS.

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ABSTRACT : Several accuracy tests of the frequencies computation have been performed. They show that for a given model, our frequencies have errors lower than  $2 \mu\text{Hz}$ . Several tests have also been made on the Henyey code. They show that the use of the same opacity formula as Bahcall et al. (1983) allows to reproduce their model with a good accuracy. Our model has however frequencies systematically  $12 \mu\text{Hz}$  larger than found by Ulrich and Rhodes. A model with diffusion of H and He has also been studied. Its frequencies are nearly the same as those of the standard model.

I. INTRODUCTION

The first comparison of the 5-minutes solar oscillation periods with the theoretical predictions (see Christensen-Dalsgaard and Gough 1980, 1981, Ulrich and Rhodes 1982, 1983, Shibahashi and Osaki 1981, Shibahashi et al 1983, Scuflaire et al. 1981, 1982, Gabriel et al. 1982) show a fairly good agreement, better than 1%, but also a significant discrepancy.

We first thought that improvements in the physics could reduce the small gap between theory and observations. Models computed by Shibahashi et al. (1983) taking the electrostatic corrections into account and using the Plank-Larkin approximation of the partition functions (Ulrich, 1982) did provide a very significant improvement. As the correct expression for the partition functions is questionable we resumed the same work using the same equation of state but another

form of the partition functions. They were defined by

$$B_i = \sum_i g_i \exp(-\epsilon_i/kT)$$

where the summation extends over all states such that

$$|\epsilon_i| < Z e^2/R_D$$

So defined, the partition functions are step functions and have to be smoothed out. This was done using for  $B_i$  the continuous function defined by the lowest value of  $B_i$  at each discontinuity.

These models give results very different from these of Shibahashi et al. (1983) but rather close to those found by Ulrich and Rhodes (1983).

Facing this, we wondered if at least part of the differences between these frequencies could be explained by numerical problems. Two tests were made to check the accuracy of our frequency calculations. Firstly we computed high order overtones for the homogeneous models. The relative error did not exceed  $10^{-5}$ . Secondly, Shibahashi's program was used to compute the frequencies for a standard solar model. The values obtained agreed within 2  $\mu$ Hz with those given by our code.

More details on this work can be found in Noels et al. (1983).

## II. INFLUENCE OF THE MODEL CALCULATION ON THE 5-MINUTES FREQUENCIES

The next step was to check the influence of the model.

First we recomputed the structure of the sun, for a given  $X(m)$  profile, using the usual atmospheres grid as surface boundary conditions and integrating towards the interior with a Runge-Kutta type method. The model obtained differed by less than  $10^{-3}$  from that found by the Henyey code.

Secondly we brought in two important modifications in the program.

So far we had used an envelope routine from Henyey (see Henyey et al. 1965) which uses the electronic pressure  $p_e$  as one of the variables and a second order integration scheme. It was replaced by a routine  $\&np$  instead of  $p_e$  and a fourth order integration method. The physics was kept unchanged but great care was paid to obtain accurate adiabatic coefficients. Although it is very difficult to check their accuracy, their internal consistency can be tested in the following way. Given the derivatives of the energy  $E$  and of the pressure computed numerically,  $C_p$  and  $C_v$  are obtained from

$$C_p = \left( \frac{\partial E}{\partial T} \right)_p - \frac{p}{\rho T} \left( \frac{\partial \&np}{\partial \ln T} \right)_p \quad (1)$$

$$C_V = \left( \frac{\partial E}{\partial T} \right)_\rho = \left( \frac{\partial E}{\partial T} \right)_p + \frac{1}{T} \left( \frac{\partial E}{\partial \ln p} \right)_T \left( \frac{\partial \ln p}{\partial \ln T} \right)_\rho \quad (2)$$

but they must also verify

$$C_p - C_V = \frac{p}{\rho T} \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_p \left( \frac{\partial \ln p}{\partial \ln \rho} \right)_T \quad (3)$$

We satisfy eq.(3) with an accuracy better than  $10^{-3}$  throughout the model.

We also obtained from Huebner (1982) the interpolation formula used as opacity data by Bahcall et al.(1982) which replaced the tables formerly used.

With these two modifications, a new standard solar model was computed with  $Z = 0.018$ . Some of its properties are given in the first line of table I. The third line gives Bahcall et al.(1982) results. It can be seen that the agreement between the two models is now good. Test calculations show that nearly all the differences with our previous results come from the use of the opacity interpolation formula.

The theoretical echelle spectrum for that model is given in fig. 1 and 3. The frequencies are increased by 10 to 20  $\mu\text{Hz}$  compared to our previous results. However for low  $\ell$  they practically coincide with the values of Ulrich and Rhodes (1983) if the latter are increased by 12  $\mu\text{Hz}$ . The shift is somewhat larger for  $\ell=10$  (14  $\mu\text{Hz}$ ) and 20 (16  $\mu\text{Hz}$ ).

To test the influence of the partition functions in the frequencies, a model was computed using the Plank-Larkin approximation for the partition functions (but not taking into account the scattering states which according to Ulrich and Rhodes produce a downwards shift of about 5  $\mu\text{Hz}$ ). Frequencies for different  $\ell$  and  $11 < n < 33$  show a decrease by about 1  $\mu\text{Hz}$ . This leaves an unexplained difference between our results and those of Ulrich and Rhodes of about 6  $\mu\text{Hz}$  which could be due to a difference of  $2 \cdot 10^{-3}$  in the sound speeds.

### III. INFLUENCE OF HYDROGEN DIFFUSION

The Shibahashi et al.(1983) models, as well as the envelope models considered by Scuflaire et al.(1982) and Gabriel et al.(1982), when compared in the (U,V) plane with a standard solar model, look like a solar model about 20% older. As this age seems large, though may be not impossible, we first searched for a way to make the sun look older. We introduced diffusion of H and  $H_e$  using the same theory as Stringfellow et al.(1983).

The properties of the model are given in the second line of Table I. The properties of the convective envelope differ only very slightly from these of the standard sun.

TABLE Ia : Properties of the models

$X_I$	$X_s$	L	R	age
0.7275	0.7275	3.858(33)	6.96(10)	4.65(9)
.7296	0.7608	3.860(33)	6.96(10)	4.60(9)
Bahcall et al.	0.7322			4.7 (9)

TABLE Ib : Properties of the models

$X_I$	$\rho_c$	$T_c$	$X_c$	$d_e$ (km)	$\rho_e$	$T_e$
0.7275	157.2	15.45(6)	.36709	1.916(5)	0.1667	2.059(6)
.7296	157.5	15.46(6)	.35695	1.926(5)	0.1663	2.055(6)
Bahcall et al.	156.3	15.30	.3545	1.88 (5)	0.15	2.0 (6)

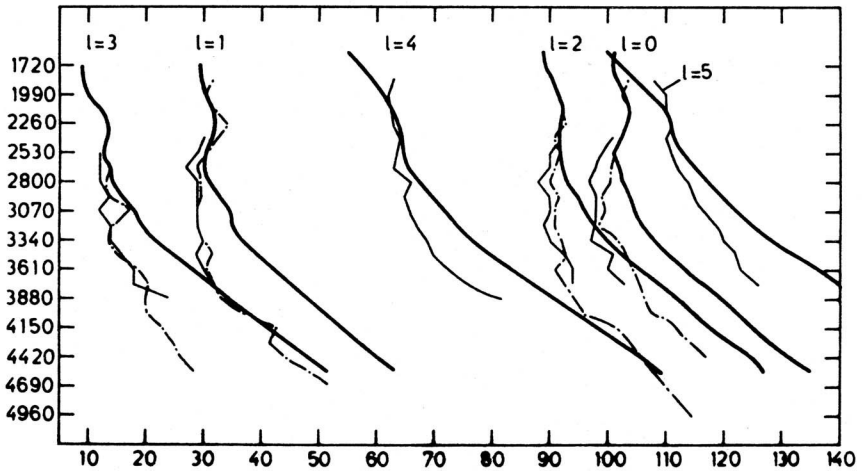


Figure 1. Echelle spectrum ; heavy lines give the theoretical results for our standard solar model, the thin full lines indicate Claverie et al.(1981) observations and the dot dashed lines these of Grec et al.(1982).

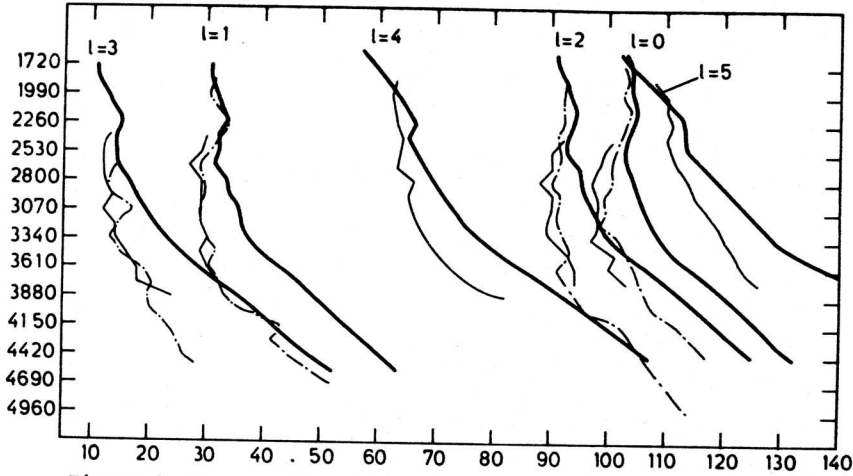


Figure 2. Same as fig. 1 but the model with diffusion.

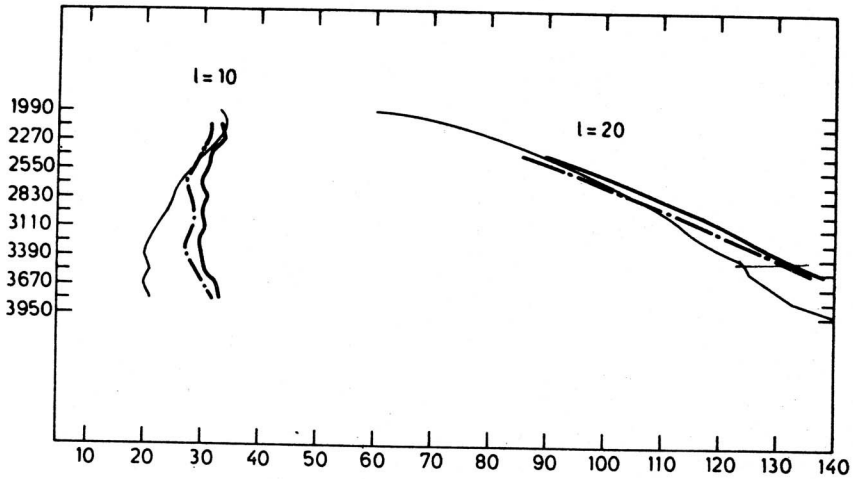


Figure 3. Echelle spectrum for  $l = 10$  and  $20$ ; thin lines correspond to Duvall and Harvey (1983) observations, dot dashed and full heavy lines give the theoretical results for the standard sun and for the model with diffusion.

This echelle spectrum is given in fig. 2 and 3. The frequencies differ only slightly from these of the standard model.

More drastic modifications in the physics of the sun has to be searched for, in order to bring the theoretical frequencies in agreement with observations.

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