Astron. Astrophys. 143, 206–208 (1985)

ASTRONOMY AND ASTROPHYSICS

The rapidly oscillating Ap stars

M. Gabriel[1], A. Noels[1], R. Scuflaire[1], and G. Mathys[2]

[1] Institut d'Astrophysique de l'Université de Liège, 5 avenue de Cointe, B-4200 Liège, Belgium
[2] Institut für Astronomie, ETH Zentrum, Venusstrasse 17, CH-8092 Zürich, Switzerland

Received March 2, accepted September 21, 1984

Summary. Frequency separation $\Delta\nu$ between consecutive modes have been computed for rapidly oscillating stars. It is shown that $\Delta\nu$ is a good indicator of the star's position in the main sequence band. These results are applied to Kurtz observations of HD 101065 and HD 24712. The problem of HD 83368 is also discussed.

Key words: Ap stars – stellar pulsations – non radial oscillations

1. Introduction

Rapid oscillations with period between 6 and 14 min, have been discovered by Kurtz (1980, 1982, 1983a, b) and Kurtz and Seeman (1983) among cool Ap stars. Several properties of these stars have been explained by Kurtz (1982) with the oblique pulsator model. These oscillations can only be high overtone p-modes ($n>10$), because of their short period and of low l. As in that range the frequency spectrum is dense, (the frequency difference between successive modes is less than 100 μHz) the knowledge of a few frequencies brings no information concerning the structure of the stars. However, even when only 2 *successive* such frequencies are observed, their separation $\Delta\nu$ provides a piece of information which helps to locate the star in the H–R diagram. Two stars, HD 101065 and HD 24712 showing several successive frequencies have been detected by Kurtz. Therefore we have computed curves of constant $\Delta\nu$ in the H–R diagram for A stars and we have applied our results to these two stars.

For HD 83368, Kurtz has also found 2 frequencies whose ratio is exactly 2. He interprets them as dipolar and quadrupolar modes whose radial harmonic order are in the ratio 2. We have checked this suggestion and shown that it is not verified by numerical calculations.

These discussions are presented in Sect. 3 on the basis of numerical results given in Sect. 2.

In this work, the computation of the models as well as their frequencies is done neglecting the magnetic field. It therefore rests on the hypothesis that the magnetic field of Ap stars does not introduce significant differences in the models and in their periods of oscillation.

2. The frequencies

A few frequencies have been computed using our usual numerical codes (see Boury et al., 1975) for models of 1.5, 1.75, and 2 $M_\odot$ in the main sequence phases. The models have an initial chemical composition given by $X=0.7$, $Z=0.02$. Some of their properties are given in Table 1. The second column gives the sequence number of the model, columns 6 and 7 give respectively the central hydrogen abundance X_c and the central concentration $\varrho_c/\bar{\varrho}$. The position of the models in the H–R diagram is also given in Fig. 1.

Table 1. Properties of the models

$M/M_\odot$	N	$L/L_\odot$	$R/R_\odot$	$T_{\rm eff}$	X_c	$\varrho_c/\bar{\varrho}$
2	1	20.00	1.533	9880	0.6465	87.57
	2	22.23	1.797	9368	0.4309	149.5
	3	24.46	2.297	8487	0.0602	475.7
	4	25.20	2.297	8553	0.0283	544.9
1.75	1	11.46	1.396	9008	0.6832	87.72
	2	12.81	1.626	8583	0.4592	149.1
	3	13.70	1.894	8090	0.2387	277.1
	4	14.12	2.092	7753	0.0914	459.4
	5	14.51	2.126	7741	0.0402	561.4
1.5	1	6.010	1.317	7893	0.6907	96.98
	2	6.909	1.513	7628	0.4319	173.6
	3	7.492	1.842	7035	0.1184	433.0
	4	7.697	1.897	6996	0.0512	554.8

For each models, 3 pairs of successive frequencies have been computed. The first two pairs are dipolar ($l=1$) modes, while the modes of the last pair (ν_e and ν_f) are quadrupolar ($l=2$). The first pair (ν_a and ν_b) has periods of about 12 min while the periods of the other two are around 6 min. The frequencies are given in Table 2. N is the sequence number of the models already introduced in Table 1. The n (columns 3, 5, 7, 9, 11, and 13) give the radial overtone number of the modes. Columns 15 and 16 give respectively the average frequency separation computed from the 2[nd] and 3[rd] pairs of modes and the same quantity in dimensionless unit $\Delta\nu_{SD}=\overline{\Delta\nu}(R^3/GM)^{1/2}$. The last column gives $2\nu_a$ or $2\nu_b$ according to the one which is closest to either ν_e or ν_f.

Curves of constant frequency separation are given for $\Delta\nu=100$, 90, 75, and 60 μHz in Fig. 1. They are almost parallel of the ZAMS. Therefore the value of $\Delta\nu$ gives no information concerning the mass of the star but it is a good indicator of its position in the MS band. From the first order asymptotic formula for ν (Tassoul, 1980) one gets

$$\overline{\Delta\nu}=\left(2\int_0^R\frac{dr}{c}\right)^{-1}.$$

Send offprint requests to: M. Gabriel

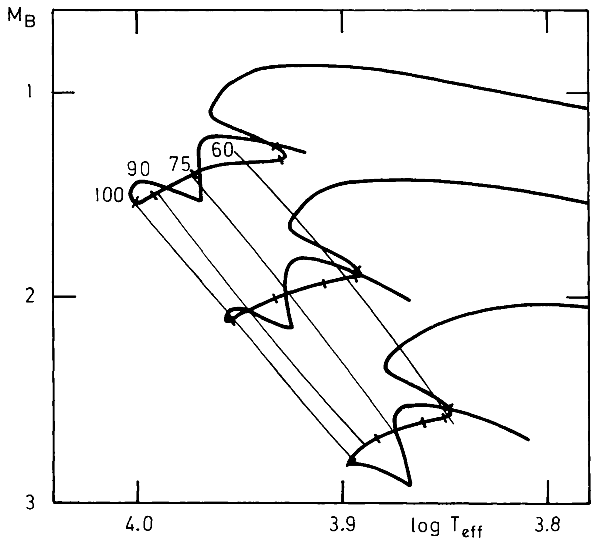

Fig. 1. H–R diagram of stars with 1.5, 1.75, and 2 $M_\odot$ (thick lines). The thin lines give curves of constant $\Delta\nu$ for $\Delta\nu = 100$, 90, 75, and 60 μHz

Introducing a dimensionless sound speed c^* defined by $c = (GM/R)^{1/2}\, c^*$ we obtain, if $x = r/R$

$$\overline{\Delta\nu} = \left(\frac{GM}{R^3}\right)^{1/2} \left(2\int_0^1 \frac{dx}{c^*}\right)^{-1} = \left(\frac{GM}{R^3}\right)^{1/2} \Delta\nu_{SD}.$$

We may expect that $R^{3/2}$ increases more rapidly than $\Delta\nu_{SD}$ (leading to a decrease of $\overline{\Delta\nu}$) as the stars evolve through the main sequence band. This is indeed verified by the numerical results.

As the mass fraction in the convective core varies from 0.076 for 1.5 $M_\odot$ to 0.132 for 2 $M_\odot$, models may not be obtained from one another by a homologous transformation, for ZAMS stars of different masses. Because of the variation of the hydrogen profile in the course of the evolution, the same situation arises for models having the same mass but different ages. Thus, we do expect c^* and therefore $\Delta\nu_{SD}$ to depend on the model considered. However, the variations of $c^*(x)$ from one model to the other compensate to give values of $\Delta\nu_{SD}$ nearly constant as it varies by less than 5% for the models considered here.

Though the sun has no convective core and a convective envelope taking $\Delta\nu = 135.5$ μHz, for the sun, we get $(\Delta\nu_{SD})_\odot = 0.216$. Therefore for main sequence stars with mass between 1 and 2 $M_\odot$ it is possible to take $\Delta\nu_{SD} = 0.205$ with an accuracy better than 10%.

3. Comparison with observations

We will apply the results of the preceding section to the observations of HD 101065 and HD 24712 which are so far the only stars pulsating in at least 2 successive modes. In HD 83368, Kurtz has found 2 frequencies whose ratio is exactly 2; we will also discuss his interpretation of that observation.

3.1. HD 101065

According to Kurtz (1980), this star is near the main sequence with $T_{\rm eff} = 7400$ K. It shows pulsations in 4 frequencies, one at 1315 μHz and a triplet at the average frequency at 1370 μHz. Kurtz suggests that the oscillations are quadrupolar ($l = 2$) and he interprets the triplet as produced by rotational splitting.

If the two frequencies at 1315 and 1370 μHz are two successive modes with the same l, $\Delta\nu = 55 \pm 4$ μHz, and HD 101065 is at the end, or just after the main sequence phases. Using also the effective temperature given above one gets a mass between 1.5 and 1.7 $M_\odot$.

For HD 24712, Shibahashi (1984) has suggested, on the basis of the rotational splitting, that the oscillations should be associated with values of l differing by one. If we make the same assumption for HD 101065, we find $\Delta\nu = 110$ μHz. Using the same $T_{\rm eff}$, this places this star almost on the ZAMS with a mass $\lesssim 1.5\, M_\odot$.

3.2. HD 24712 (HR 1217)

This star is a cool Ap star. Its effective temperature, according to Bonsack (1979) is between 7350 and 7500 K. Its oscillations were studied by Kurtz (1982) who gives frequencies for 2 triplets and by Kurtz and Seeman (1983) who give 6 frequencies (see Table 3).

To obtain $\overline{\Delta\nu}$ it is necessary to know if all 6 frequencies correspond to the same l or not. According to Kurtz and Seeman the 6 periods are successive radial overtones with the same l. This gives $\overline{\Delta\nu} = 34.7 \pm 4$ μHz. Because a triplet structure was observed by Kurtz (1982) for the modes f_1 and f_2, Shibahashi (1983) suggested that f_1, f_2, and f_6 are dipolar modes while f_3, f_4, and f_5

Table 2. Frequencies of the models (expressed in μHz)

$M/M_\odot$	N	n_a	ν_a	n_b	ν_b	n_c	ν_c	n_d	ν_d	n_e	ν_e	n_f	ν_f	$\overline{\Delta\nu}$	$\overline{\Delta\nu}_{\rm SD}$	$2\nu_{a/b}$
2	1	14	1418.4	15	1509.5	28	2682.3	29	2773.7	29	2812.9	30	2904.4	91.4	0.19548	2836.8
	2	18	1400.1	19	1469.4	35	2636.2	36	2709.8	37	2816.7	38	2889.9	73.7	0.20022	2800.2
	3	27	1439.6	28	1491.1	50	2637.2	51	2689.2	53	2819.2	54	2871.3	52.1	0.20434	2879.2
	4	26	1387.0	27	1438.5	51	2687.4	52	2739.3	53	2817.3	54	2869.3	52.0	0.20402	2877.0
1.75	1	13	1429.1	14	1528.2	25	2602.3	26	2702.8	27	2847.1	28	2948.2	100.8	0.20019	2858.2
	2	17	1451.4	18	1528.1	32	2648.3	33	2728.8	33	2764.1	34	2845.2	80.8	0.20185	2902.8
	3	21	1410.7	22	1475.3	41	2717.3	42	2782.8	41	2747.2	42	2812.8	65.5	0.20588	2821.4
	4	24	1393.3	25	1449.9	47	2668.8	48	2773.5	50	2861.0	51	2917.0	55.4	0.20189	2999.8
	5	25	1415.6	26	1471.1	48	2658.9	49	2713.0	51	2848.6	52	2903.5	54.5	0.20360	2831.2
1.5	1	12	1346.3	13	1446.9	25	2653.9	26	2757.4	26	2801.6	27	2905.5	103.7	0.20386	2893.8
	2	16	1424.5	17	1506.1	31	2678.0	32	2762.6	31	2799.9	32	2884.5	84.6	0.20495	2849.0
	3	21	1380.4	22	1444.0	41	2663.3	42	2726.9	43	2819.7	44	2882.2	63.1	0.20520	2888.0
	4	22	1384.8	23	1446.4	43	2675.9	44	2735.9	45	2823.3	46	2881.2	59.0	0.20044	2892.8

Table 3. Frequencies observed by Kurtz and Seeman for HD 24712

	ν(μHz) ±2	$\Delta\nu$(μHz) ±4
f_5	2620.00	
		32.78
f_2	2652.78	
		34.72
f_3	2687.50	
		33.33
f_1	2720.83	
		34.72
f_4	2755.55	
		38.06
f_6	2793.61	

are radial modes. This gives $\overline{\Delta\nu}=68\pm4$ μHz. Indeed f_3, f_4, and f_5 may also contain unresolved contributions from both $l=0$ and $l=2$ if the observation time is too short (see Christensen-Dalsgaard and Gough, 1982). Aslo theory predicts that in the frequency range of the observed modes

$$(\nu_{n-1}+\nu_n)_{l=1}-2(\nu_{n-1})_{l=2}\simeq 10\ \mu\text{Hz},$$
$$2(\nu_n)_{l=0}-(\nu_{n-1}+\nu_n)_{l=1}\simeq 5\ \mu\text{Hz}.$$

In view of Table 3, it seems (though we are at the limit of the accuracy of the frequency measurements) that Shibahashi's suggestion is the good one.

Presently it is impossible to decide unambiguously between these 2 possibilities. The final answer will come only from very accurate frequency measurements. Also we consider the 2 cases.

If we take Kurtz and Seeman suggestion then $\overline{\Delta\nu}\simeq 35$ μHz. Taking $T_{\rm eff}=7400$ K and $\overline{\Delta\nu}_{\rm SD}=0.2$, we find that the star is outside the M.S. band, approximatively 1.6 mag above the ZAMS. The ratio of the probabilities to observe the star there and on the main sequence is about 0.02.

If we take Shibahashi's suggestion, then $\overline{\Delta\nu}\simeq 68$ μHz and HD 24712 can be interpreted as a star of $M\simeq 1.5\,M_\odot$, $R\simeq 1.7\,R_\odot$ about in the middle of the main sequence band if we take $T_{\rm eff}=7400$ K. This value of the radius is in good agreement with the estimate done by Kurtz (1982).

3.3. HD 83368 (HR 3831)

Kurtz (1982) has detected 2 triplets whose period ratio is 2 with an accuracy of $3\,10^{-5}$, the longer period being equal to 11.67 min ($\nu=1428$ μHz).

He suggests that the lower frequency triplet corresponds to a dipole pulsation mode ($l=1$) splitted by rotation and that the higher frequency one is a quadrupole mode. His argumentation is based on the asymptotic formula

$$\nu=\nu_0(n+l/2)$$

which predicts that $2\nu(n,l=1)=\nu(2n,l=2)$.

The comparison of ν_e and ν_f ($l=2$) with the last column of Table 2 shows that $\nu(2n,l=2)-2\nu(n,l=1)>8$ μHz and that the ratio $\nu(2n,l=2)/\nu(n,l=1)$ differs from 2 by more than $5\,10^{-3}$. Kurtz's suggestion may therefore not be accepted. Though Kurtz raised objections to this idea, it seems to us that presently the best explanation is still that the higher frequency triplet is the first harmonic of the low frequency one.

References

Boury, A., Gabriel, M., Noels, A., Scuflaire, R., Ledoux, P.: 1975, *Astron. Astrophys.* **41**, 279

Christensen-Dalsgaard, J., Gough, D.O.: 1982, *Monthly Notices Roy. Astron. Soc.* **198**, 141

Kurtz, D.W.: 1980, *Monthly Notices Roy. Astron. Soc.* **191**, 115

Kurtz, D.W.: 1982, *Monthly Notices Roy. Astron. Soc.* **200**, 807

Kurtz, D.W.: 1983a, *Monthly Notices Roy. Astron. Soc.* **202**, 1

Kurtz, D.W.: 1983b, *Monthly Notices Roy. Astron. Soc.* **205**, 3

Kurtz, D.W., Seeman, J.: 1983, *Monthly Notices Roy. Astron. Soc.* **205**, 11

Bonsack, W.K.: 1979, *Publ. Astron. Soc. Pacific* **91**, 648

Shibahashi, H.: 1984, in *Oscillations as a Probe of the Sun's Interior*, Catania, June 1983 (in press)

Tassoul, M.: 1980, *Astrophys. J. Suppl.* **43**, 469