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EXTRAIT

Sobolev type line profiles in case of wind density perturbations modulated by non radial pulsations

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ASTROPHYSIQUE

Sobolev type line profiles in case of wind density perturbations modulated by non radial pulsations

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Summary. — We have studied the modifications induced on the P Cygni line profiles of an outwards accelerating wind by density fluctuations modulated by non radial pulsations.

1. INTRODUCTION

During the past few years, observations have drawn the attention on the importance of non radial pulsations in massive stars and their possible link with some characteristics of the mass loss of these objects [Abbott *et al.*, 1986]. Until now, detailed numerical simulations of the perturbations of a line profile by non radial pulsations have been limited to photospheric lines. This problem is relatively simple in the sense that it is a two dimensional one: the observed variations are produced by the time dependent pattern of the velocity field on the surface of the star.

Due to the growing evidence that a link between non radial pulsations and mass loss variations could exist [Abbott *et al.*, 1986] and also an observational indication that a coupling between wind and photospheric variations could indeed exist [Baade, 1986], we have decided to investi-

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gate the signature on P Cygni profiles of perturbations of the wind density modulated by non radial pulsations. Until now, numerical simulations of perturbations of P Cygni profiles by variations of the wind density have been limited to cases where the perturbation is spherically symmetric [for example, Prinja and Howarth, 1985] or axially symmetric [Rumpl, 1980].

We present here, a method to handle this problem when the wind has no longer spherical nor axial symmetry. We have tackled the problem in the frame of a linear theory. We start with a classical Sobolev type line profile produced by a spherically symmetric wind accelerating outwards. As a working hypothesis we assume that the density perturbations of the wind have the same geometrical and temporal pattern as non radial pulsations. The resultant variations of the line profile have been calculated from the linearized equations of the problem. In this first attempt, we have limited our investigations to only one model.

2. Stationary model

A model of Castor and Lamers [1979] has been chosen as our stationary model. The expanding envelope is described by two functions, the expansion velocity $v(r)$ and the optical depth $\tau_{\rm rad}(r)$, where r is the distance from the centre of the star. In the adopted model, the expansion velocity is given by

$$v = v_0 + v_1 (1 - \mathrm{R}/r)^{1/2}$$

where $v_0 = 0.01\ v_\infty$
$v_1 = 0.99\ v_\infty$
R is the radius of the star
v_∞ is the terminal velocity of the wind

This classical velocity law is an increasing function of r, so that the Sobolev approximation may be used. This means that a photon going through the envelope interacts with the absorbing ions at only one place of its trajectory, located at a distance r from the center of the star, which depends on the frequency of that photon.

The optical depth of the envelope, viewed by a photon moving radially is given by

$$\tau_{\rm rad} = \frac{\pi e^2}{mc} f \lambda_0 n_i \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

where f is the oscillator strength, λ_0 the wavelength at rest of the considered line, n_i the density of the absorbing ions and $\mathrm{d}v/\mathrm{d}r$ the velocity gradient. All these quantities are computed at the point r where the photon under consideration interacts with matter. We have not tried to compute the density of the ions from the ionization equilibrium. Instead, following Castor and Lamers [1979], we have adopted for $\tau_{rad}(r)$ the law

$$\tau_{rad} = C(1 - v/v_{\infty})$$

with C = 2.04.

Both adopted laws $v(r)$ and $\tau_{rad}(r)$ correspond to the case $\mathscr{I} = 1$ of figure 8B of Castor and Lamers [1979].

The profiles were computed using the method developed by Castor (1970), neglecting desexcitation by collisions. We also made the approximation

$$\beta_c/\beta = W = [1 - \sqrt{1 - (R/r)^2}]/2$$

Both approximations are justified by the preliminary character of the present investigation and the arbitrariness of the laws $v(r)$ and $\tau_{rad}(r)$. The source function is given by $S = W\, I_c$ where I_c is the intensity coming out of the photosphere, assumed to be independent of the frequency in the neighbourhood of the line. In a future work we plan to include in our perturbation treatment a more accurate expression for the escape probability.

3. Computation of the perturbed profile

As a working hypothesis we assume that the stellar oscillation affects the wind only through a modulation of the density at the base of the wind and has no effect on the velocity law. This last assumption is valid if the wind is accelerated by optically thin lines: in that hypothesis the acceleration of the denser regions will be the same as the quiet wind. In this first approach of the problem, this simplifying hypothesis is necessary: the use of non monotonic velocity laws and the study of shocks formation are beyond the scope of the present paper. This hypothesis is also supported by the following estimation of the ratio between the maximal oscillation velocity v_{osc} and v_{∞}. If σ is the angular frequency of the considered oscillation mode and ΔR the variation of the radius, we have

$$v_{osc} = \sigma \Delta R = \alpha \frac{\Delta R}{R} \sqrt{\frac{GM}{R}}$$

where α is a numerical factor depending on the mode. For example, for the modes found unstable by Noels and Scuflaire [1986], with periods around 4 hours, α is comprised between 1 and 1.7. If the approximative relation between v_∞ and the escape velocity v_e [Abbott, 1978] holds

$$v_\infty = 3 v_e = 3 \sqrt{\frac{2GM}{R}}$$

we have

$$v_{osc}/v_\infty = \frac{\alpha}{3\sqrt{2}} \frac{\Delta R}{R}$$

With $\alpha = 1.5$ and $\Delta R/R = 0.1$, we obtain $v_{osc}/v_\infty = 0.035$. Hence, it is reasonable to assume that the velocity of the base of the wind due to the non radial pulsation has only a negligible effect on the main part of the velocity curve of the wind.

In these conditions the density of the wind close to its base (specifically close to the point where $v = 0.01\, v_\infty$) is written

$$\rho_p(\theta', \phi', t) = \rho_{p0} + \delta\rho_p(\theta', \phi', t)$$

where the index p indicates quantities close to the base of the wind, the index 0 refers to unperturbed quantities and the prefix δ refers to small Eulerian (i.e. local) perturbations.

$$\delta\rho_p(\theta', \phi', t)/\rho_{p0} = \varepsilon \sqrt{4\pi}\, Y_{lm}(\theta', \phi') e^{-i\sigma t}$$

ε is a small parameter

Y_{lm} is a spherical harmonics of degree l that we write

$$Y_{lm}(\theta', \phi') = \frac{e^{im\phi'}}{\sqrt{2\pi}} \Theta_{lm}(\cos\theta')$$

with

$$\Theta_{lm}(x) = \sqrt{\frac{(2l+1)\,(l-m)!}{2(l+m)!}}\, P_l^m(x) \quad \text{if } m \geqslant 0$$

$P_l^m(x)$ is an associated Legendre function. For negative m, $\Theta_{lm}(x)$ is defined by

$$\Theta_{lm}(x) = (-1)^m \Theta_{l,\,-m}(x)$$

θ', ϕ' are angular coordinates in an inertial system having its origin at the center of the star and its z' axis coinciding with the rotation axis of the star.

σ is the angular frequency of the considered oscillation mode. This frequency depends on three integer indices l, m and k. The index k numbers the modes associated to the same l and m. When there is no rotation, σ does not depend on m and this frequency is $(2l + 1)$-fold degenerated. Let σ_0 be the frequency when there is no rotation. When the rotation is not too fast, the frequency is given by

$$\sigma = \sigma_0 + \beta m \Omega$$

where β is a factor often close to the unity and Ω the angular velocity of rotation [see Ledoux, 1951].

The equation of continuity allows us to follow the evolution of the photospheric perturbation through the envelope.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 v) = 0$$

$$\rho_0(r) = \frac{\mathrm{R}^2 v_p}{r^2 v(r)} \rho_{p0}$$

$$\delta\rho(r, \theta', \phi', t)/\rho_0(r) = \delta\rho_p(\theta', \phi', t - t'(r))/\rho_{p0}$$

$t'(r)$ is the time spent by an element of matter from the photosphere to reach the point of coordinate r. The expression given by Prinja and Howarth [1985] contains a mistake. The right expression is

$$t'(r) = \int_{\mathrm{R}}^{n} \frac{\mathrm{d}r'}{v(r')} = \frac{\mathrm{R}}{v_\infty} \left\{ \frac{(w_1 - w_0 w) w r}{(w_1 - w_0)\mathrm{R}} - \frac{w_1}{2} \ln(1 - w) \right.$$

$$\left. + \frac{w_1}{2(w_1 - w_0)^2} \ln(1 + w) - \frac{2 w_0 w_1^2}{(w_1 - w_0)^2} \ln \frac{w_0 + w_1 w}{w_0} \right\}$$

with

$$w = \sqrt{1 - \mathrm{R}/r}$$

$$w_0 = v_0/v_\infty$$

$$w_1 = v_1/v_\infty$$

In the following, we shall omit the index 0 indicating unperturbed quantities.

Let us choose another coordinate system Oxyz. The origin coincides with the centre of the star, the z axis is pointing towards the observer and the rotation axis of the star (Oz' axis) lies in the Oxz plane. Let θ and ϕ

be the angular coordinates associated with this system and θ_0 the angle between the z and z' axes (figure 1).

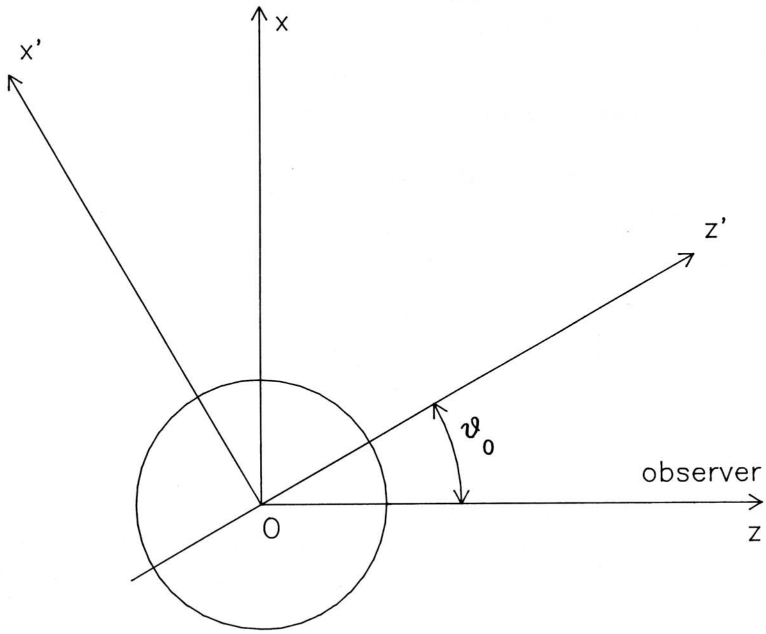

FIG. 1. — Relative orientation of the axes. The y and y' axes coincide and are orthogonal to the plane of the figure.

Let us consider a ray propagating towards the observer. Let $p = \sqrt{x^2 + y^2}$ be the distance between this ray and the centre of the star. The intensity $I(\lambda)$ at the location of the observer is given by (Castor, 1970)

$$I(\lambda) = \begin{cases} I_c e^{-\tau} + S(1 - e^{-\tau}) & \text{if } p < R \\ S(1 - e^{-\tau}) & \text{if } p > R \end{cases}$$

where

$$\tau = \frac{1 + \sigma}{1 + \sigma\mu^2}\,\tau_{\text{rad}}$$

$$\sigma = \frac{\mathrm{d}\ln v}{\mathrm{d}\ln r} - 1$$

$$\mu = z/r$$

All these quantities must be evaluated at the only point of the ray where matter and radiation of wavelength λ can interact. The flux received by the observer is given by

$$F(\lambda) = \iint I(\lambda)\, dx\, dy = \int_0^\infty p\, dp \int_0^{2\pi} I(\lambda)\, d\phi$$

Outside of the line, the flux is given by

$$F_c = \pi R^2 I_c$$

so that the flux in the line compared to the continuum has the following expression

$$F(\lambda)/F_c = \frac{1}{\pi R^2} \int_0^\infty p\, dp \int_0^{2\pi} \frac{I(\lambda)}{I_c}\, d\phi$$

The perturbed equations write

$$\delta\tau/\tau = \delta\tau_{rad}/\tau_{rad} = \delta\rho/\rho$$

With our simplification in the computation of S, the perturbation of $I(\lambda)$ is reduced to

$$\delta I(\lambda)/I_c = I_\rho(\lambda)\delta\rho/\rho$$

with

$$I_\rho(\lambda) = \begin{cases} (W-1)\tau e^{-\tau} & \text{if } p < R \\ W\tau e^{-\tau} & \text{if } p > R \end{cases}$$

$$\delta I(\lambda)/I_c = \varepsilon\sqrt{4\pi}\, I_\rho(\lambda) Y_{lm}(\theta', \phi') e^{-i\sigma[t - t'(r)]}$$

The perturbation of the flux can now be written

$$\delta F(\lambda)/F_c = \frac{2\varepsilon}{R^2\sqrt{\pi}} \int_0^\infty p dp \int_0^{2\pi} I_\rho(\lambda) Y_{lm}(\theta', \phi') e^{-i\sigma[t - t'(r)]}\, d\phi$$

Let us express the spherical harmonics $Y_{lm}(\theta', \phi')$ in terms of spherical harmonics in the θ, ϕ variables [see for instance, Nikiforov and Ouvarov, 1983]

$$Y_{lm}(\theta', \phi') = \sum_{m'} D^l_{mm'} Y_{lm'}(\theta, \phi)$$

The coefficients $D^l_{mm'}$ depend on the relative orientation of both axes systems, i.e. of θ_0 in the present case.

When integrating on the ϕ variable, only the term with $m' = 0$ gives a non zero contribution. The coefficient D^l_{mo} is given by

$$D^l_{mo} = \sqrt{\frac{2}{2l+1}}\, \Theta_{lm} (\cos \theta_0)$$

It comes

$$\delta F(\lambda)/F_c = A(F_1(\lambda) \cos \sigma t + F_2(\lambda) \sin \sigma t)$$

with

$$A(l, m, \theta_0) = \varepsilon \sqrt{2}\, \Theta_{lm} (\cos \theta_0)$$

$$F_1(\lambda, \sigma, l) = 2 \int_0^\infty I_\rho(\lambda) P_l (\cos \theta) \cos [\sigma t'(r)] p \, dp/R^2$$

$$F_2(\lambda, \sigma, l) = 2 \int_0^\infty I_\rho(\lambda) P_l (\cos \theta) \sin [\sigma t'(r)] p \, dp/R^2$$

We note that

$$\max_{\theta, \phi} \left| \frac{\delta\rho(r, \theta, \phi)}{\rho(r)} \right| = \varepsilon \sqrt{2} \max_{\theta} |\Theta_{lm} (\cos \theta)|$$

so that, for a favourable angle θ_0, A is precisely equal to the left member of the above relation.

$$\max_{\theta_0} |A(l, m, \theta_0)| = \max_{\theta, \phi} \left| \frac{\delta\rho(r, \theta, \phi)}{\rho(r)} \right|$$

The index m as well as the angle θ_0 between the line of sight and the rotation axis appear only in a multiplicative factor in the expression of the flux perturbation. So they influence only the strength of the perturbation but not its shape. Figure 2 shows how is varying the ratio $A(\theta_0)/A_{max}$ in the case $l = 2$.

Note that in the linear theory developed above, the determination of the perturbation of the line profile requires only about twice as much computation as in the case of a stationary spherical wind, for a given value of l.

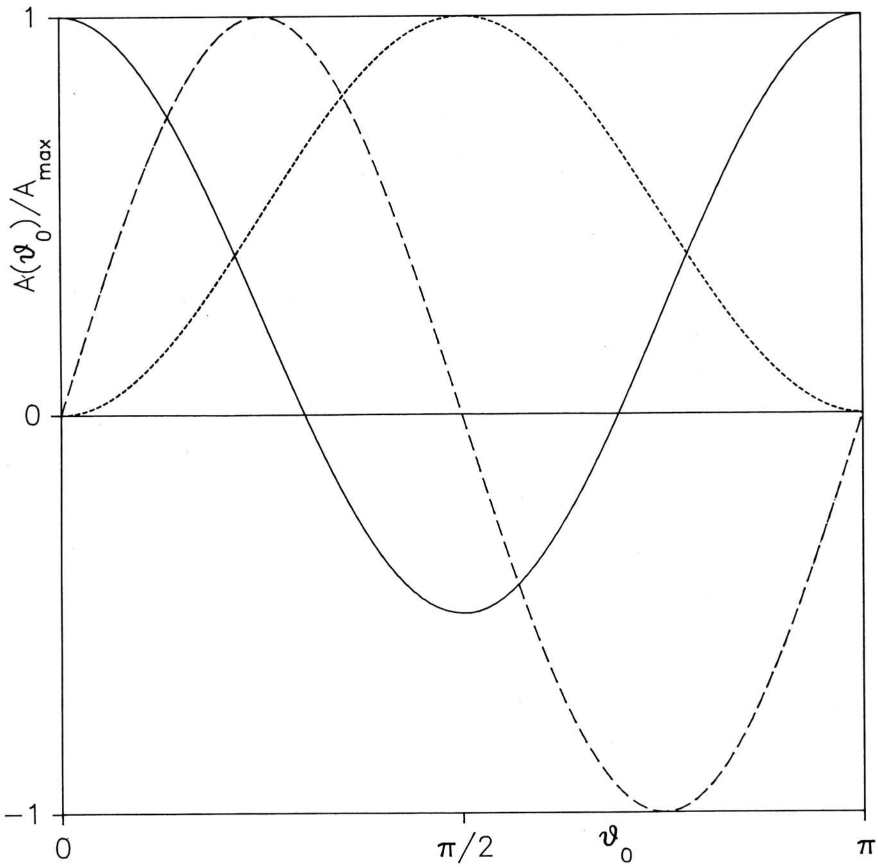

FIG. 2. — The "visibility" factor of the perturbation as a function of the angle θ_0 between the line of sight and the rotation axis of the star, for $l = 2$, $m = 0$ (solid line), $m = 1$ (dashed line) and $m = 2$ (dotted line).

4. RESULTS AND DISCUSSION

In the computations described in the preceding section, the angular frequency of the oscillation appears only through the dimensionless variable $\sigma t'(r)$. As a natural unit for t' is R/v_∞, it is convenient to introduce the dimensionless parameter

$$\omega = R\sigma/v_\infty$$

which measures the frequency of the stellar pulsation in a natural unit associated with the wind.

Figure 3 shows the line profile of the unperturbed model. As usual, the horizontal scale gives the wavelength measured from the center of the line, with the half-width taken as unit.

$$\Delta\lambda/\Delta\lambda_{\infty} = (\lambda - \lambda_0)c/\lambda_0 v_{\infty}$$

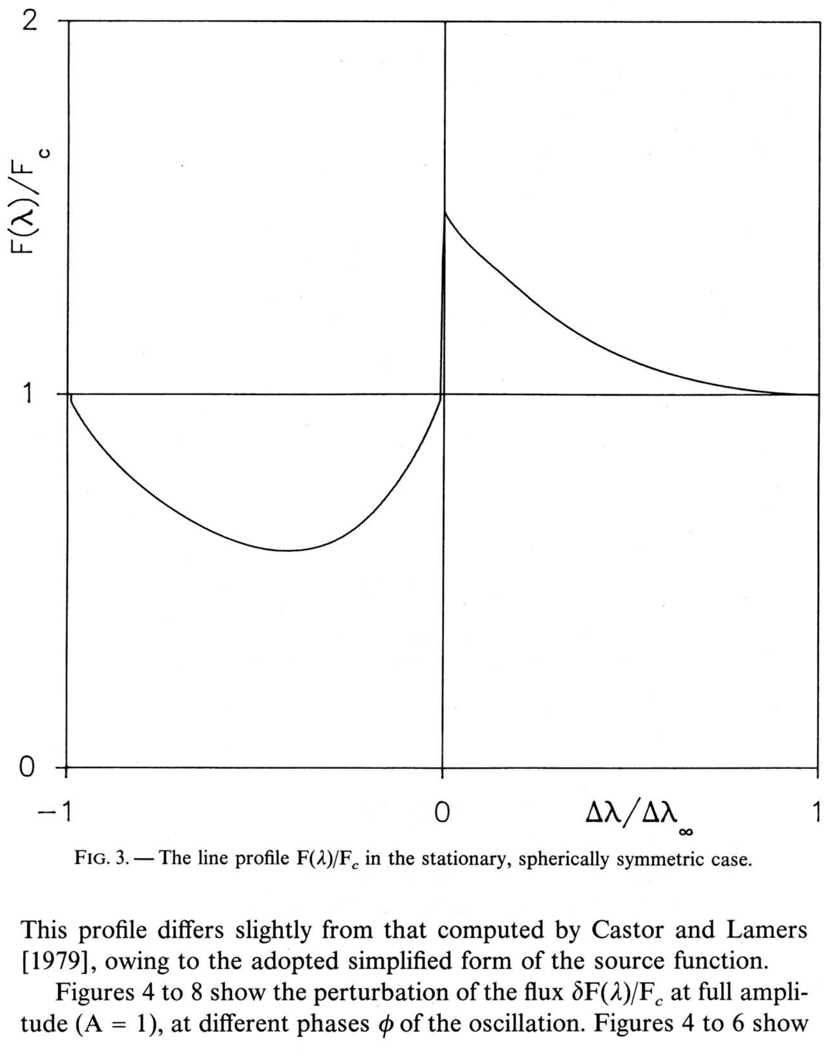

FIG. 3. — The line profile $F(\lambda)/F_c$ in the stationary, spherically symmetric case.

This profile differs slightly from that computed by Castor and Lamers [1979], owing to the adopted simplified form of the source function.

Figures 4 to 8 show the perturbation of the flux $\delta F(\lambda)/F_c$ at full amplitude ($A = 1$), at different phases ϕ of the oscillation. Figures 4 to 6 show

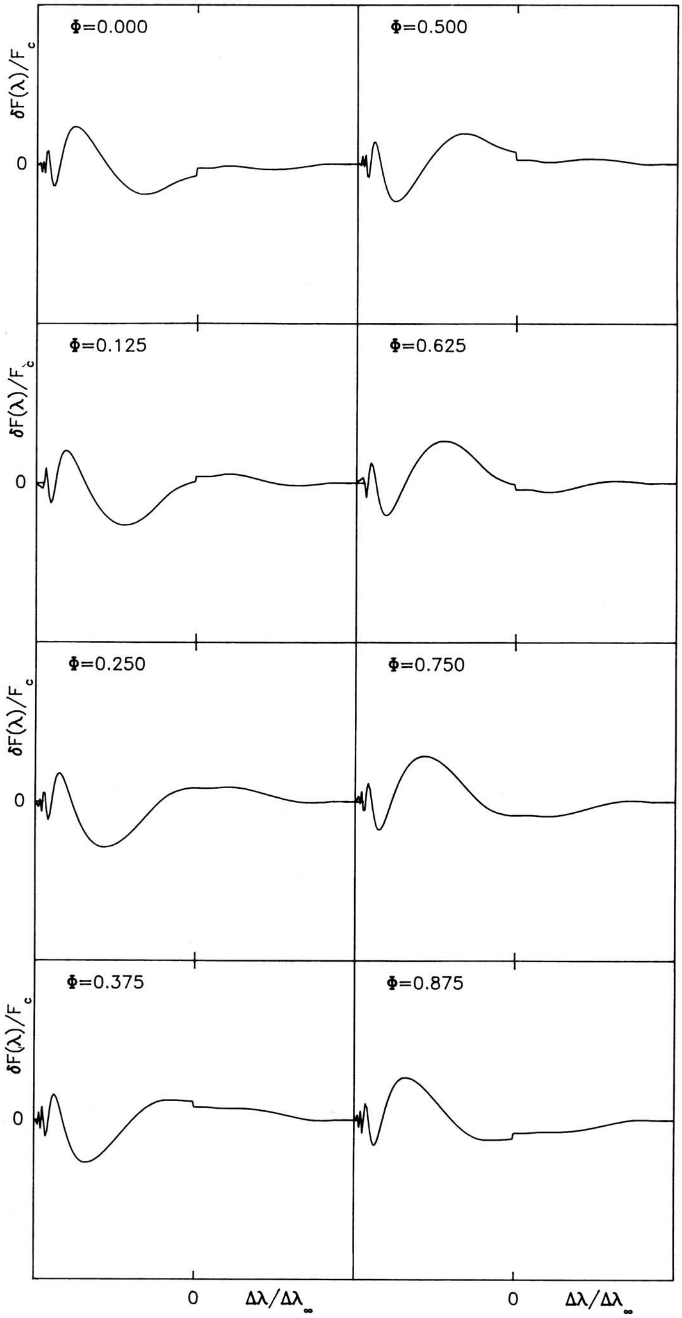

FIG. 4. — $\delta F(\lambda)/F_c$ for $l = 0$, $\omega = 1$ and $A = 1$.

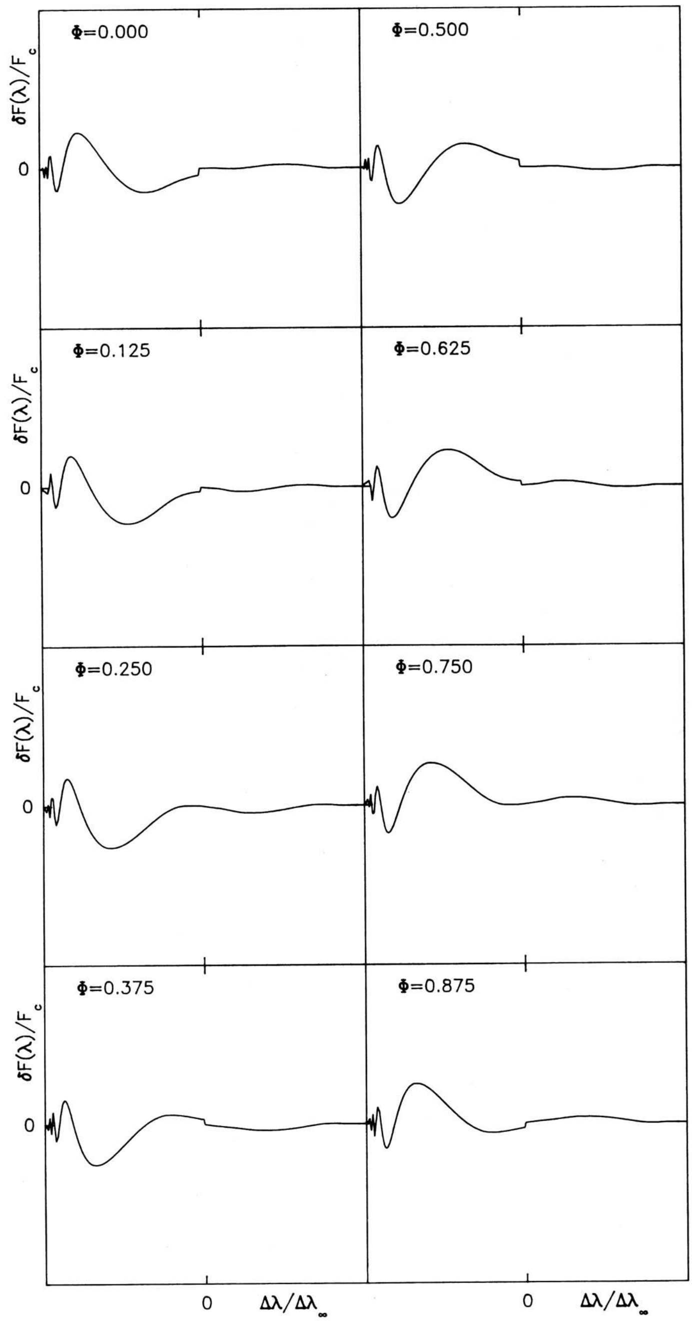

FIG. 5. — $\delta F(\lambda)/F_c$ for $l = 1$, $\omega = 1$ and $A = 1$.

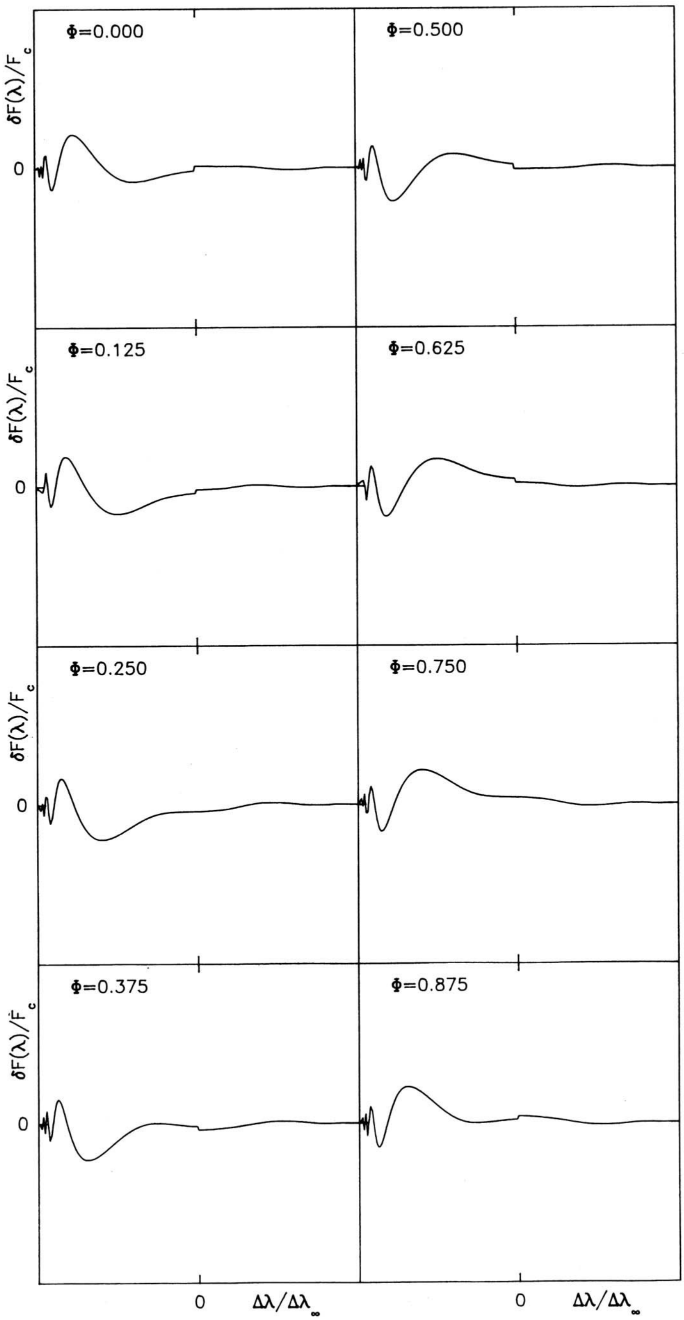

FIG. 6. — $\delta F(\lambda)/F_c$ for $l = 2$, $\omega = 1$ and $A = 1$.

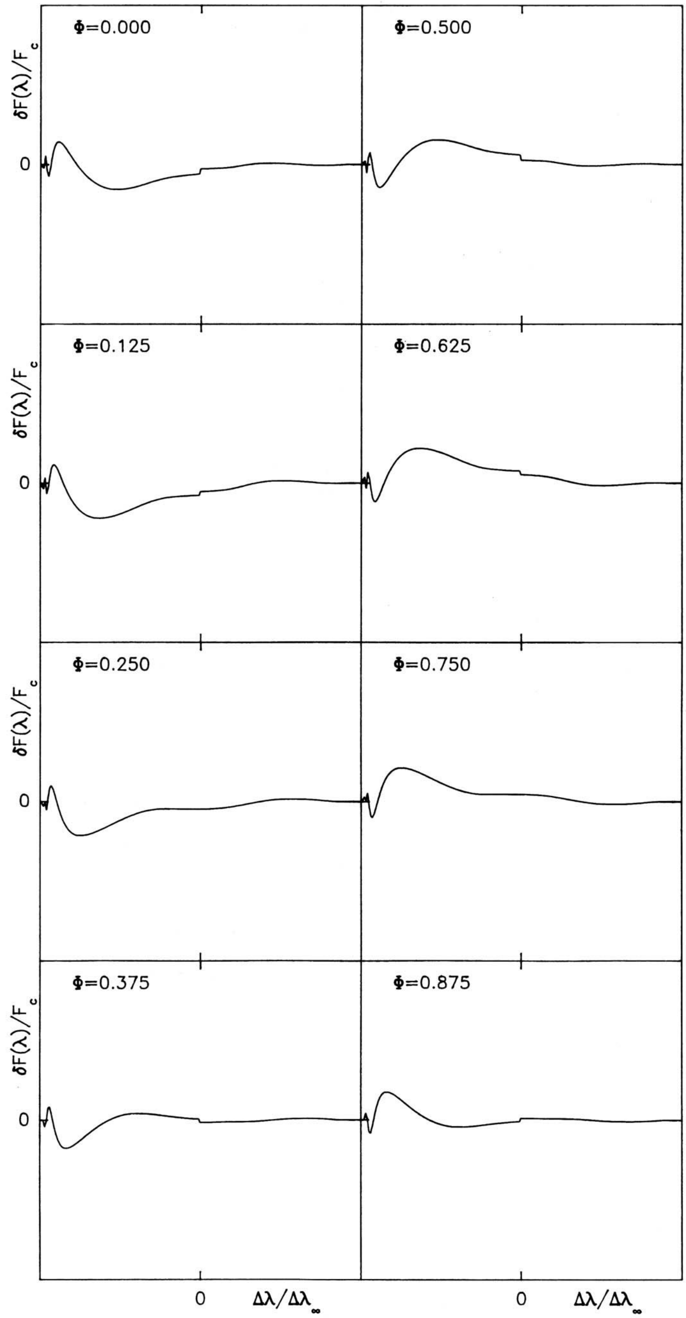

FIG. 7. — $\delta F(\lambda)/F_c$ for $l = 2$, $\omega = 0.5$ and $A = 1$.

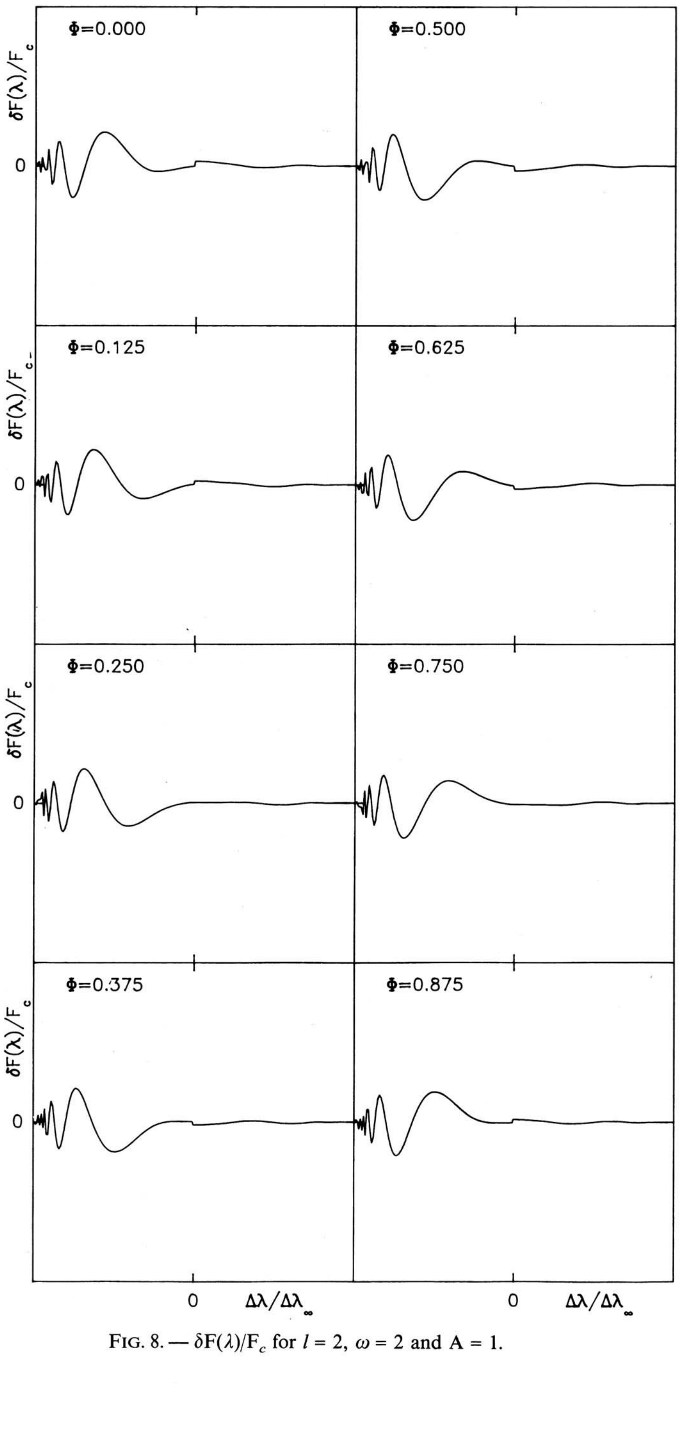

FIG. 8. — $\delta F(\lambda)/F_c$ for $l = 2$, $\omega = 2$ and $A = 1$.

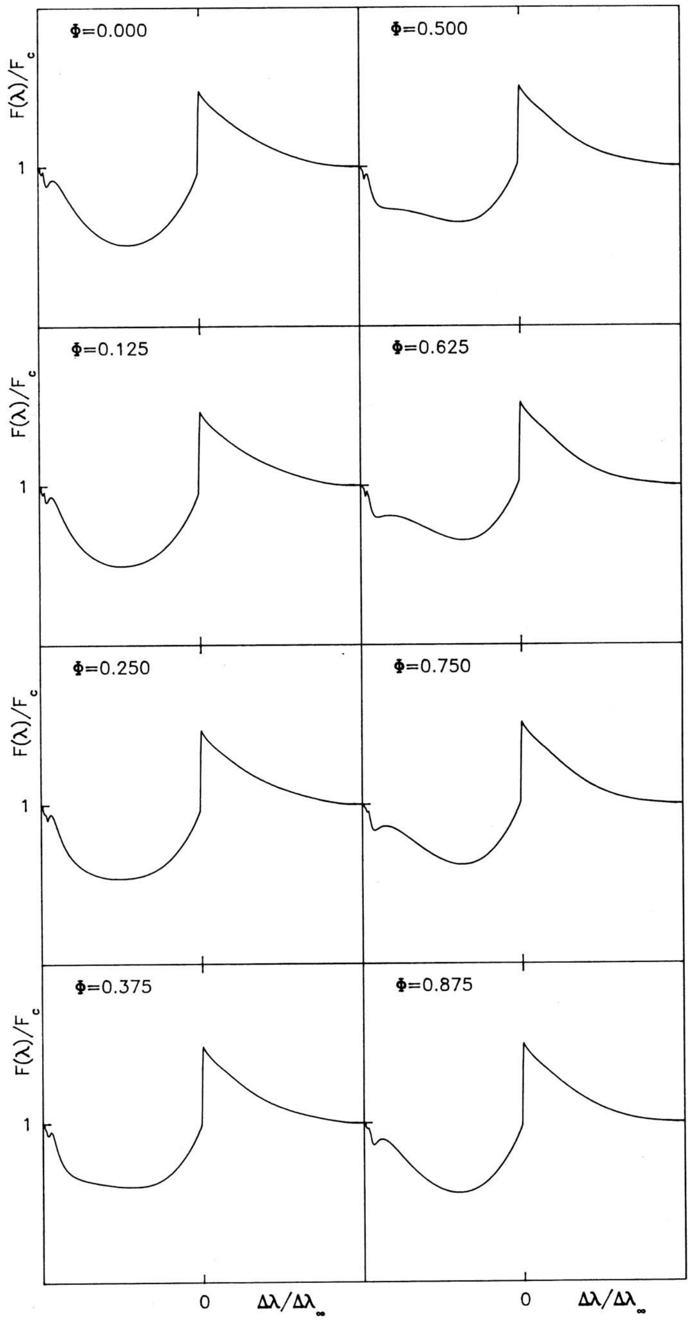

FIG. 9. — The line profile $F(\lambda)/F_c$ for $l = 2$, $\omega = 0.5$ and $A = 0.5$.

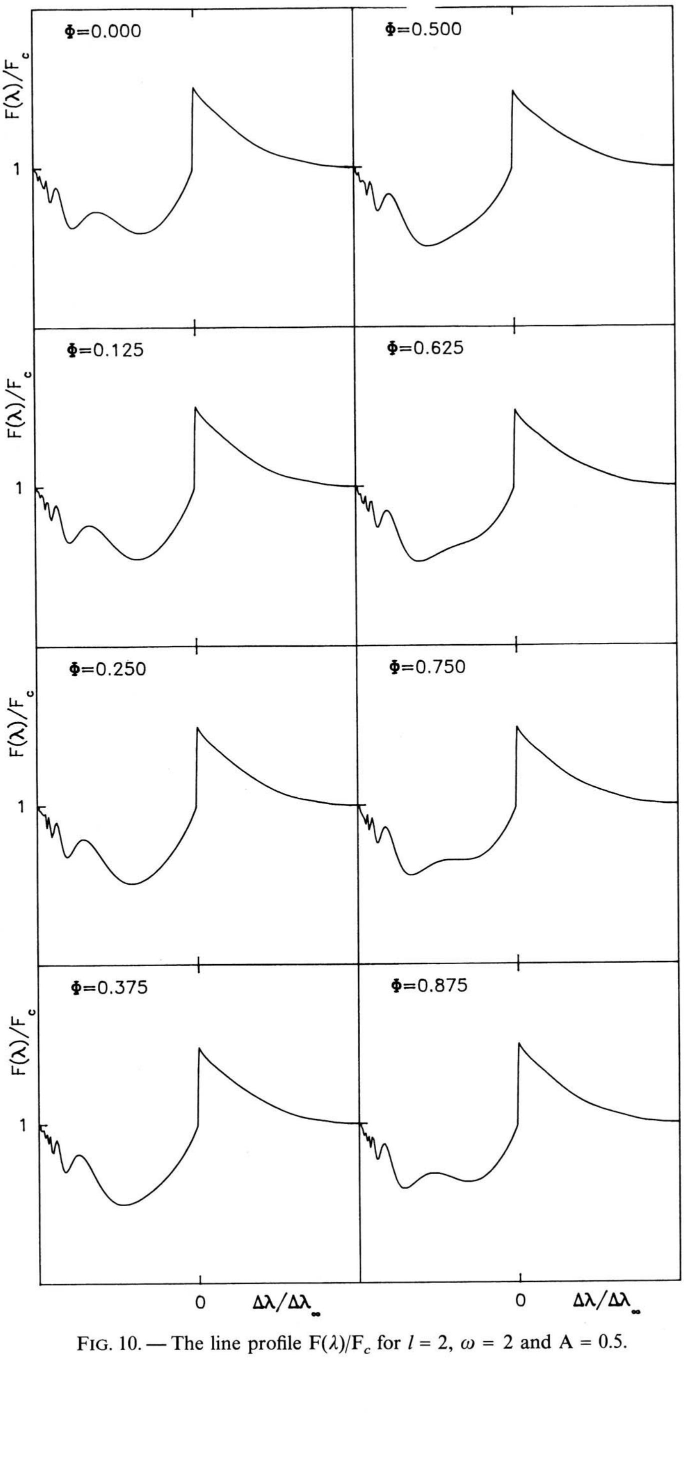

FIG. 10. — The line profile $F(\lambda)/F_c$ for $l = 2$, $\omega = 2$ and $A = 0.5$.

the influence of l on the profile. Comparison of figures 6 to 8 shows the influence of ω. Let us note that, in the case of radial perturbation, near the centre of the line, the perturbation of the flux in the red wing of the absorption lags behind the perturbation in the emission component. The reverse behaviour is observed in the non radial case. In the present state of our computations, it is difficult to say if this must be interpreted as a difference of signatures between radial and non radial perturbations or if this behaviour is dependent on the particular chosen model.

Figure 9 shows the profile resulting from the addition of the perturbation to the unperturbed profile. We have arbitrarily chosen the case

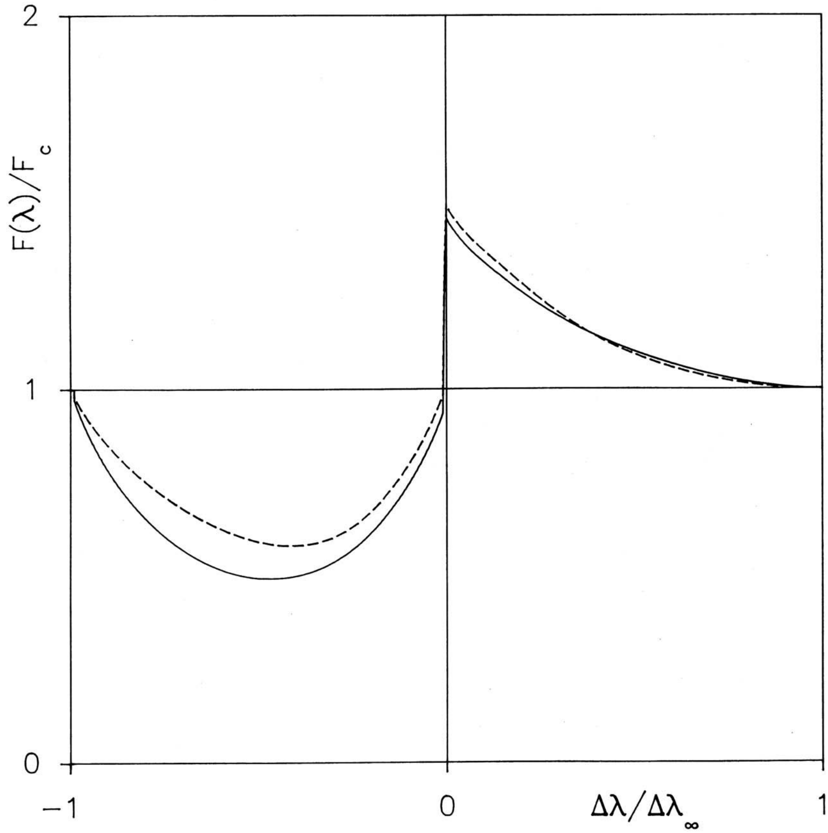

FIG. 11. — The line profile $F(\lambda)/F_c$ resulting from an enhancement of the wind in the equatorial plane ($l = 2$, $\omega = 0$, $A = 0.5$; solid line), compared to the spherically symmetric case (dashed line).

$l = 2$, $\omega = 0.5$ and $A = 0.5$ to illustrate the type of profiles we obtain. As presently our computations are made with only one velocity law and one opacity law, we have not tried to reproduce a given observed profile.

At higher values of ω, as displayed for example in figure 10 ($l = 2$, $\omega = 2$, $A = 0.5$) the perturbations sometimes give the impression of moving absorption features superimposed to a more or less normal P Cygni profile.

The above theory apply also to a stationary deformation of the wind. It is described by $\omega = 0$. Figure 11 shows the perturbation and the profile resulting from an equatorial enhancement of the wind ($l = 2$, $A = 0.5$). The comparison with the profiles computed by Rumpl [1980] must be very cautious, as our theory is valid only for small perturbations.

As a conclusion, we may state that this first tentative to model the variability of ultraviolet P Cygni line profiles of hot stars by a linear theory is encouraging. In the near future we plan to include in our computation of the perturbation a more exact form of the source function and to apply the theory to different velocity and opacity laws. The fitting of computed profiles to observed ones ought to enrich our knowledge of the structure of the wind of mass losing hot stars exhibiting variability in their ultraviolet P Cygni profiles.

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