NON RADIALLY PULSATING WOLF-RAYET STARS

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A few non radial modes, some of which are trapped modes, can be amplified in H-burning shell models coming from the evolutionary sequence of a star of initial mass 100 $M_\odot$ losing mass through stellar wind. The periods range from half an hour to a few hours. The duration of this instability phase is rather short, of the order of a few e-folding times, which are of a few thousand years.

Some WR stars, particularly WN stars, show a variability in emission lines with periods in a narrow range of a few hours. Vreux (1985), Vreux et al. (1985) and Vreux (1986) have emphasized that variability, suggesting that it could be due to non radial oscillations.

It is well accepted that some WR stars can be formed from massive stars evolving with mass loss. Radial oscillations have been shown to be amplified in such models after the H-burning shell phase, if the structure of the star is close enough to a homogeneous helium star, more massive than the critical mass of such stars, of the order of 16 $M_\odot$ (Noels and Gabriel 1981, Noels and Gabriel 1984, Maeder 1985). The periods however seem too short, less than one hour, to explain the variability observed in WR stars. Longer periods can be found in non radial oscillations but the problem is to find favourable conditions to amplify them, i.e. to obtain a vibrational instability.

We have analysed models extracted from the evolution of a 100 $M_\odot$ star (Noels and Gabriel 1981) whose H-R diagram is given in figure 1. Some properties of the selected models are given in Noels and Scuflaire (1986).

In the vibrational stability analysis, the perturbation of any variable f is written in the form

$$\delta f(r,\theta,\varphi,t) = C\,\delta f(r)\, P_\ell^m(\theta,\varphi)\cos(m\varphi-\sigma t)\, e^{-\sigma' t}$$

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P_ℓ^m is the associated Legendre polynomial of degree ℓ and order m, σ the adiabatic angular frequency and σ' the damping coefficient whose expression is

$$\sigma' = - \frac{\int \frac{\delta T}{T} \delta \varepsilon \, dm - \int \frac{\delta T}{T} \delta \left(\frac{1}{\rho} \operatorname{div} \vec{F} \right) dm}{2 \sigma^2 \int |\delta r|^2 \, dm} = - \frac{E_N - E_F}{D}$$

A positive value of σ' means a damping of the oscillation while a negative value means an amplification and a vibrational instability. E_N comes from the nuclear terms and has always a destabilizing effect. E_F comes from the flux terms, its main contribution arises from the external layers and it has generally a damping effect.

So, a necessary condition for a mode to be amplified is to have a large amplitude in a nuclear burning region and a rather small amplitude outside.

In the case of non radial oscillations, the amplitude of $\frac{\delta P}{P}$ goes to zero at the center, so core burning models can be discarded at once. Figure 2 shows the first g^+ modes for l = 1 in the case of model 2 which is a H-core burning model. We can see that the amplitudes are large outside the nuclear burning region and all the non radial modes are damped.

We find more favourable conditions in H-shell burning models, and two different kinds of situation arise.

1. Low l, low order modes

Figure 3 shows that for model 3, the amplitude of the g_1^+ mode for l = 1 is very large in the H-burning shell, and rather small in the external layers. The amplification term E_N dominates the damping term E_F and this mode can reach a finite amplitude. The g_2^+ and g_3^+ modes have a node in side the H-burning shell, which lowers E_N and these modes are damped (Noels and Sculfaire, 1986). Model 4 is also vibrationally unstable towards the g_2^+ mode for which the second extremum is just inside the nuclear burning shell. Due to the sharp increase in the central condensation, $\frac{\rho_c}{\rho}$, the amplitude near the surface becomes too large in the following models and stability is restored at model 5. The periods obtained here are of the order of 4 hours which comes closer to the observed value. The amplification time, $\frac{1}{|\sigma'|}$, is of the order of a few thousands years, about 10 times shorter than the whole duration of the unstable phase, so this instability is rather mild.

2. Moderately high l, trapped modes

Trapped modes are modes which have an oscillatory behaviour in the r variable, in a narrow trapping zone in the star and which are evanescent, with a decreasing amplitude, outside. The local condition for a mode to have an oscillatory behaviour is that its angular frequency σ must be greater or smaller than both the acoustic cut-off frequency σ_a and the gravity cut-off frequency σ_g, given by

$$\sigma_a = \frac{\sqrt{l(l+1)}\,c}{r} \qquad \sigma_g = \sqrt{-\left(\frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln P}{dr}\right)g}$$

where c is the local speed of sound.
Figure 4 shows σ_a^2 for l = 5 and l = 10 and σ_g^2 in model 4. The very high peak of σ_g near the H-burning shell allows modes, for moderately high l, to be trapped in that region if their angular frequency is smaller than the height of the peak of σ_g. For l = 5, the g_6^+, g_5^+, g_4^+, g_3^+, f and p_6 modes are trapped and for l = 10, the g_6^+, g_5^+, g_3^+, g_1^+, p_2 and p_7 modes are trapped (Scuflaire and Noels 1986). Figure 5 illustrates the difference of behaviour between two consecutive p modes. The p_7 mode is trapped in the H-burning shell while the p_6 mode has a negligible amplitude in the nuclear burning shell. Amplification occurs for the p_7 mode while damping is found for the p_6 mode.
The situation is similar but less favourable in model 3 for which however the low order g^+ modes are marginally stable. All the trapped modes are either vibrationally unstable or marginally stable. Their periods range from half an hour to a few hours and their amplification time is of the order of a few thousand years, but here again the duration of the unstable phase is rather short, of the order of only a few e-folding times.

Some non radial modes can be amplified in H-burning shell models, with periods in the range of the observed periods. Such unstable models can only represent WN stars, and maybe even only late type WN stars, as they must still have enough hydrogen in the external layers but this is still in agreement with the observations, showing such variations mostly in WN stars.

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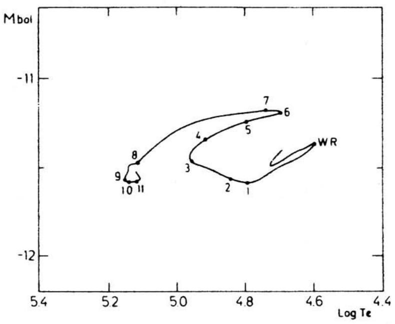

Figure 1

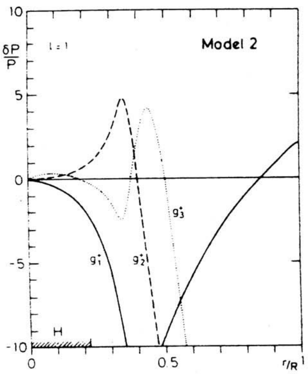

Figure 2

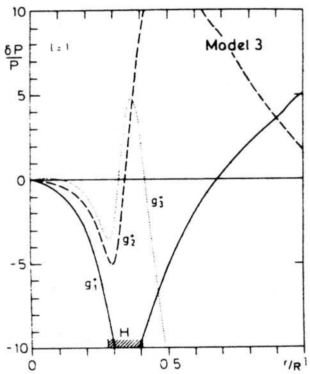

Figure 3

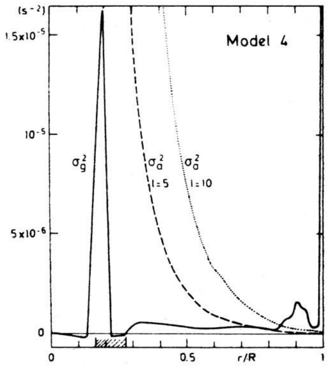

Figure 4

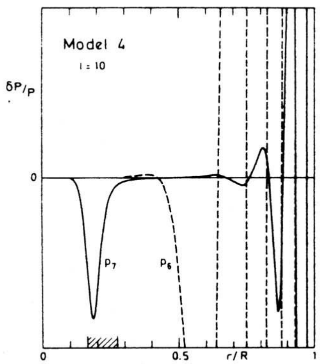

Figure 5