

# *Driving mechanism and energetic aspects in $\gamma$ Doradus stars*

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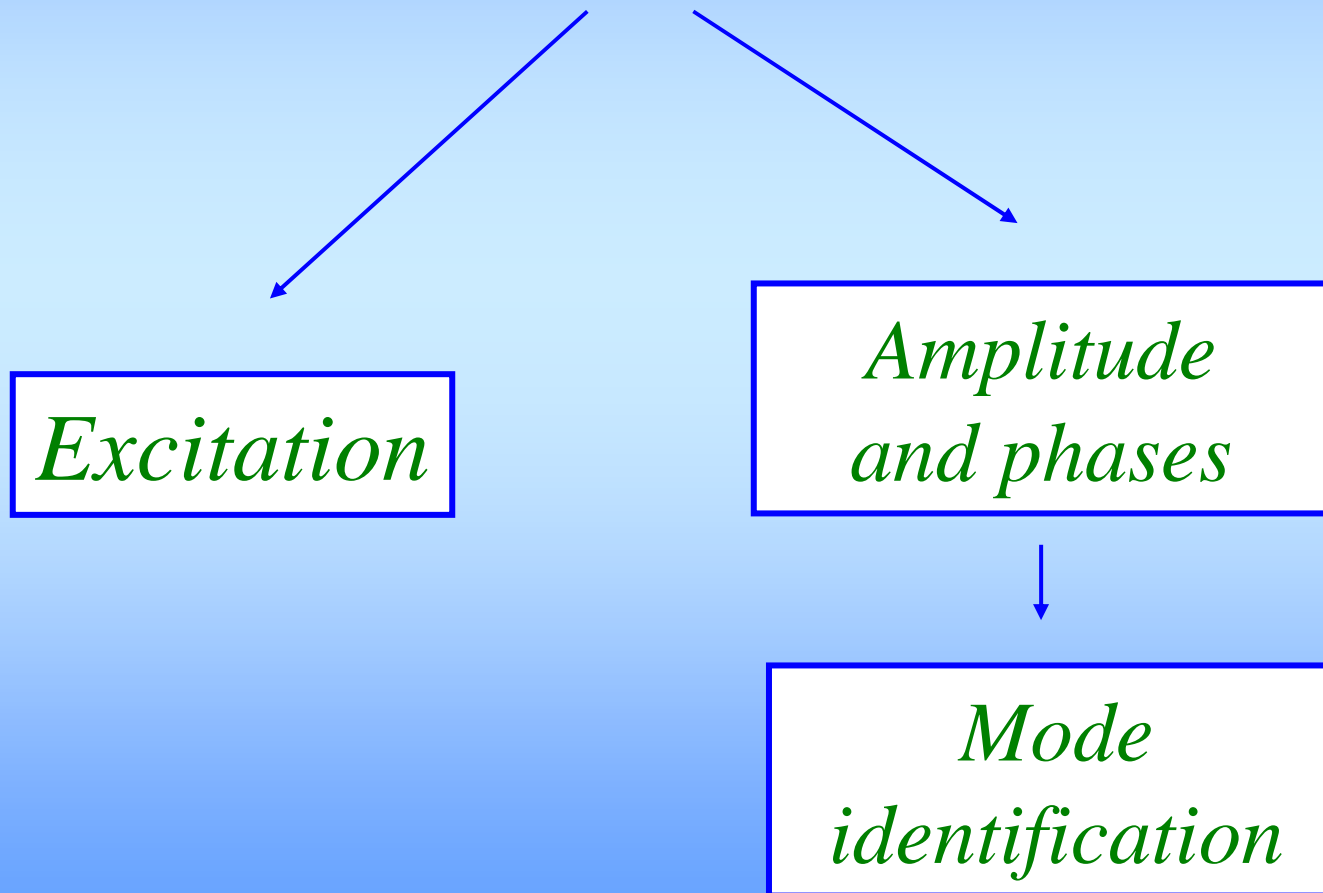
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*R. Scuflaire*

*A. Noels*

*M. Gabriel*

# *Driving mechanism and energetic aspects in $\gamma$ Doradus stars*

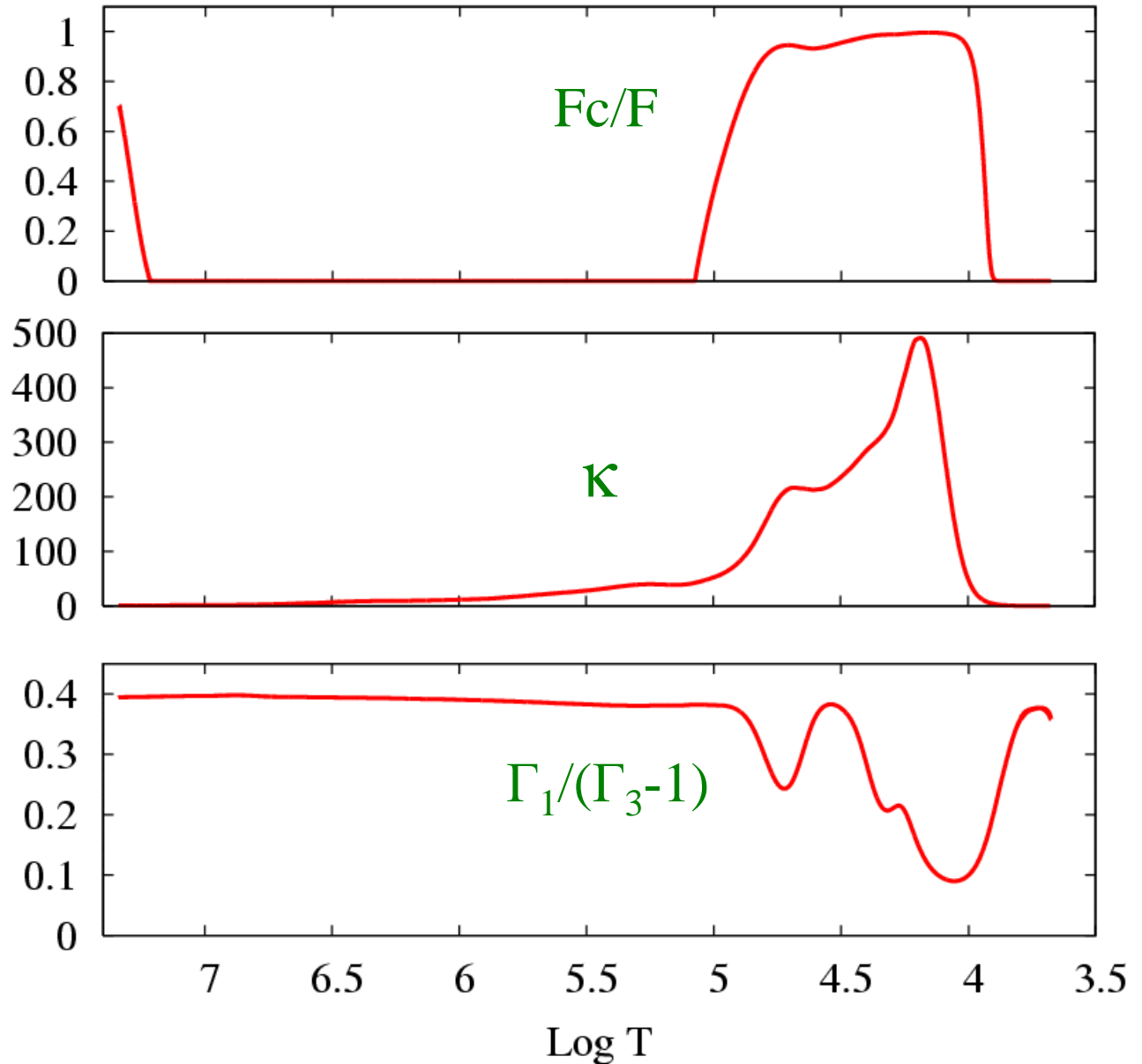


## Internal physics:

- 1 convective core
- 1 convective envelope

He and H partial ionization zones are inside the convective envelope

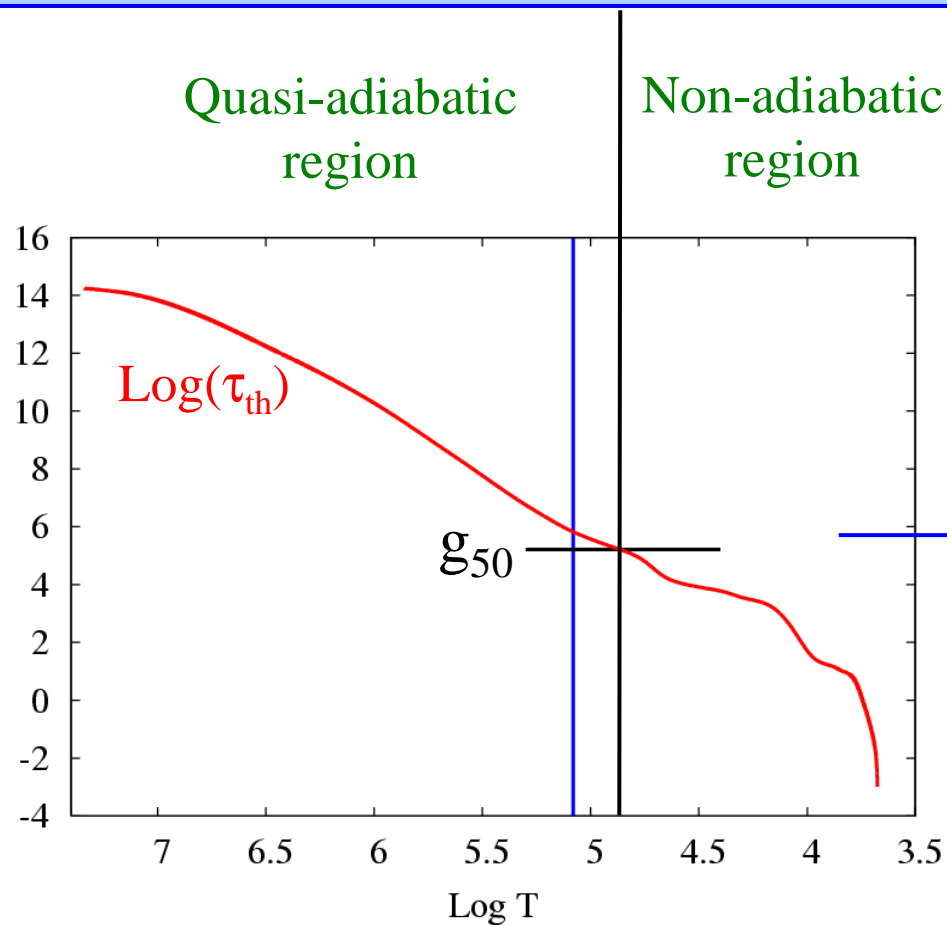
## Convection and partial ionization zones



## $\gamma$ Doradus stars

## Driving mechanism

Main driving occurs in the transition region where the thermal relaxation time is of the same order as the pulsation periods



For a solar calibrated mixing-length, the transition region for the g-modes is near the convective envelope bottom.

$$\delta S \neq 0$$

Coupling between

- the dynamical equations and
- the thermal equations

$\gamma$  Doradus

Driving mechanism

Flux blocking at the base of the convective envelope

Motor thermodynamical cycle

$$\delta r(r, \theta, \phi, t) = \delta r(r) Y_1^m(\theta, \phi) e^{\sigma_i t} e^{i\sigma_r t}$$

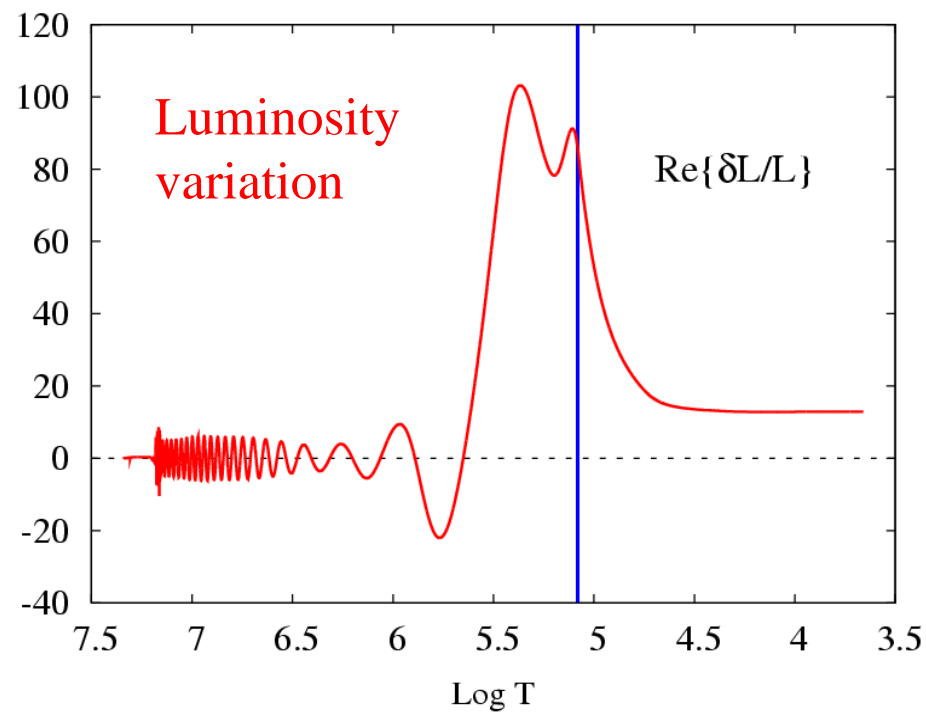
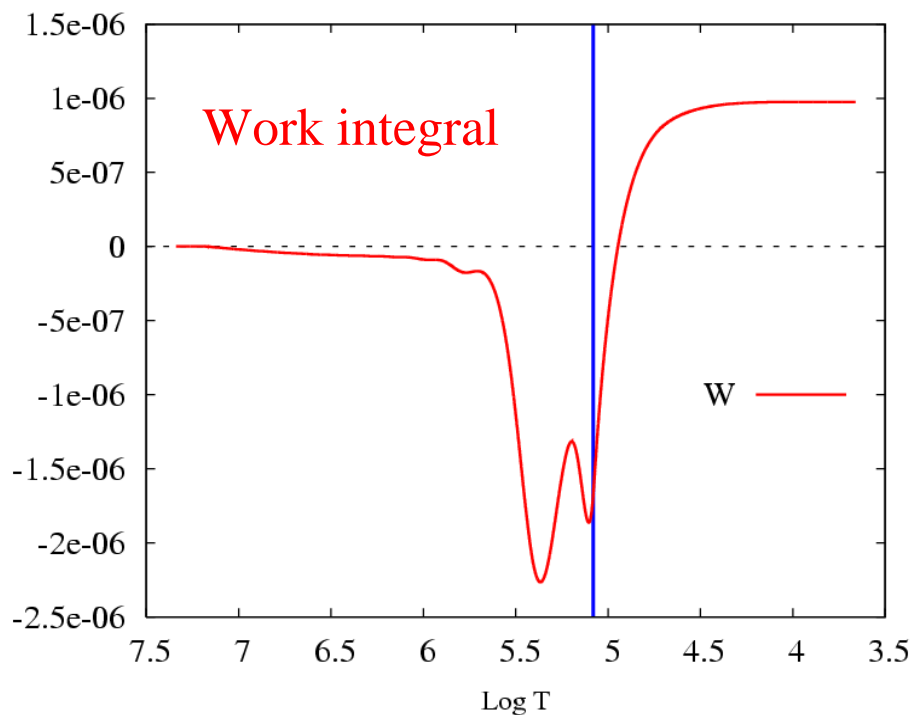
$$\sigma_i = \frac{-1}{2\sigma_r^2} \frac{\int_0^M \frac{\delta T}{T} \frac{d\delta L}{dm} dm}{\int_0^M \delta r^2 dm}$$

# $\gamma$ Doradus

# Driving mechanism

Flux blocking at the base of the convective envelope

Motor thermodynamical cycle



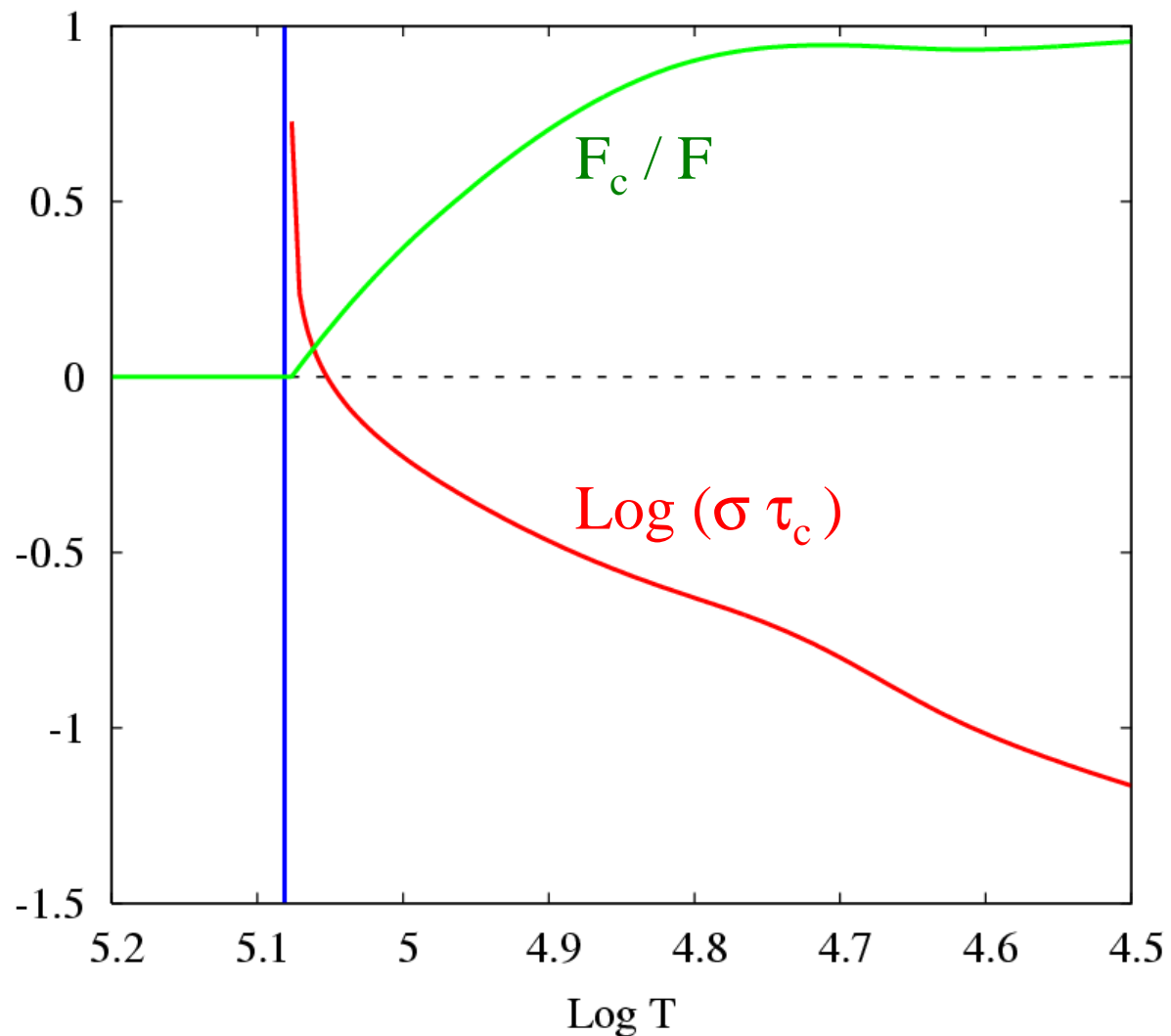
# $\gamma$ Doradus

## Driving mechanism

$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
 $\alpha = 2$   
Mode  $(l=1, g_{50})$

Role of  
time-dependent  
convection

$\tau_c$  : Life-time of  
convective elements  
 $\sigma$  : Angular frequency



# Convection – pulsation interaction: Work integral

Radiative luminosity

Convective luminosity

$$\begin{aligned}
 W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p}{\rho} \right\} dm \\
 &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_R + \delta L_c)}{dm} \right\} dm \\
 &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p_t}{\rho} \right\} dm \\
 &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p_t}{\rho} \right\} dm
 \end{aligned}$$

Turbulent pressure

Turbulent kinetic energy dissipation



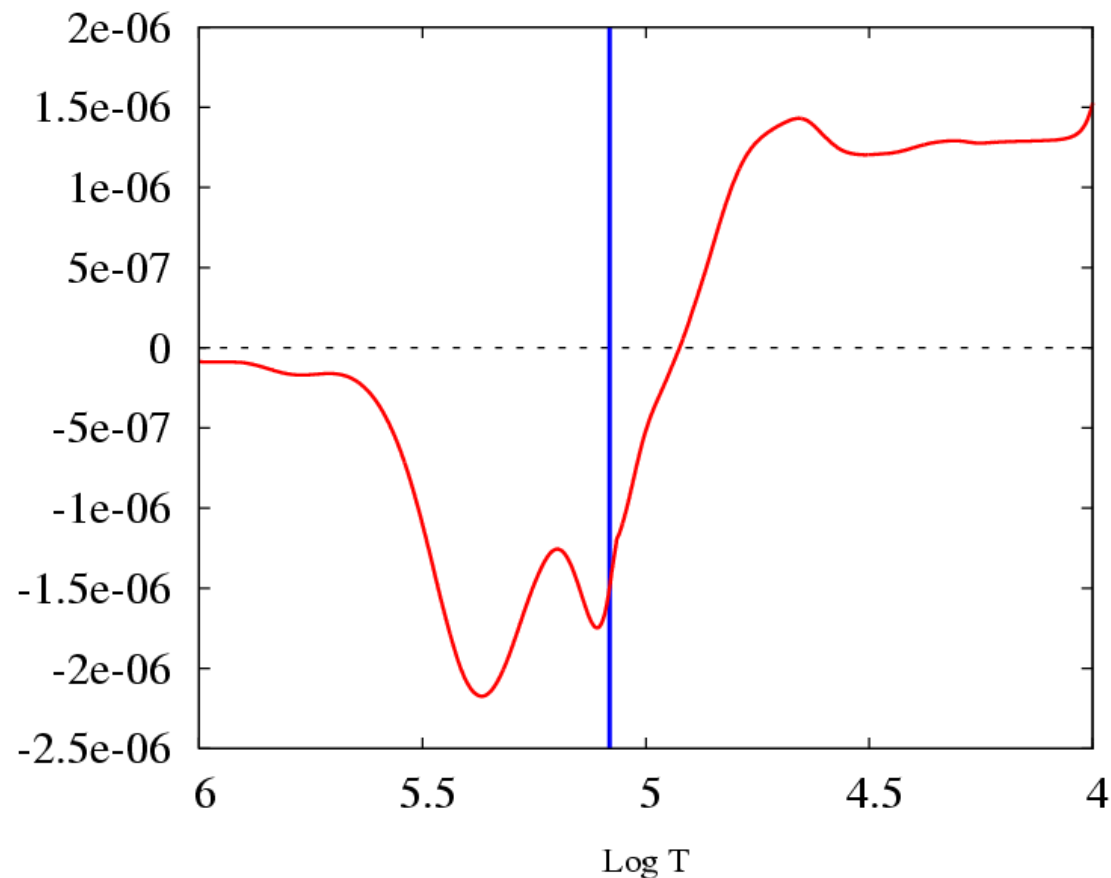
# $\gamma$ Doradus

## Driving mechanism

$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
 $\alpha = 2$   
Mode  $(\ell=1, g_{50})$

$W_{\text{FRr}}$ : Radial radiative  
flux term

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



# $\gamma$ Doradus

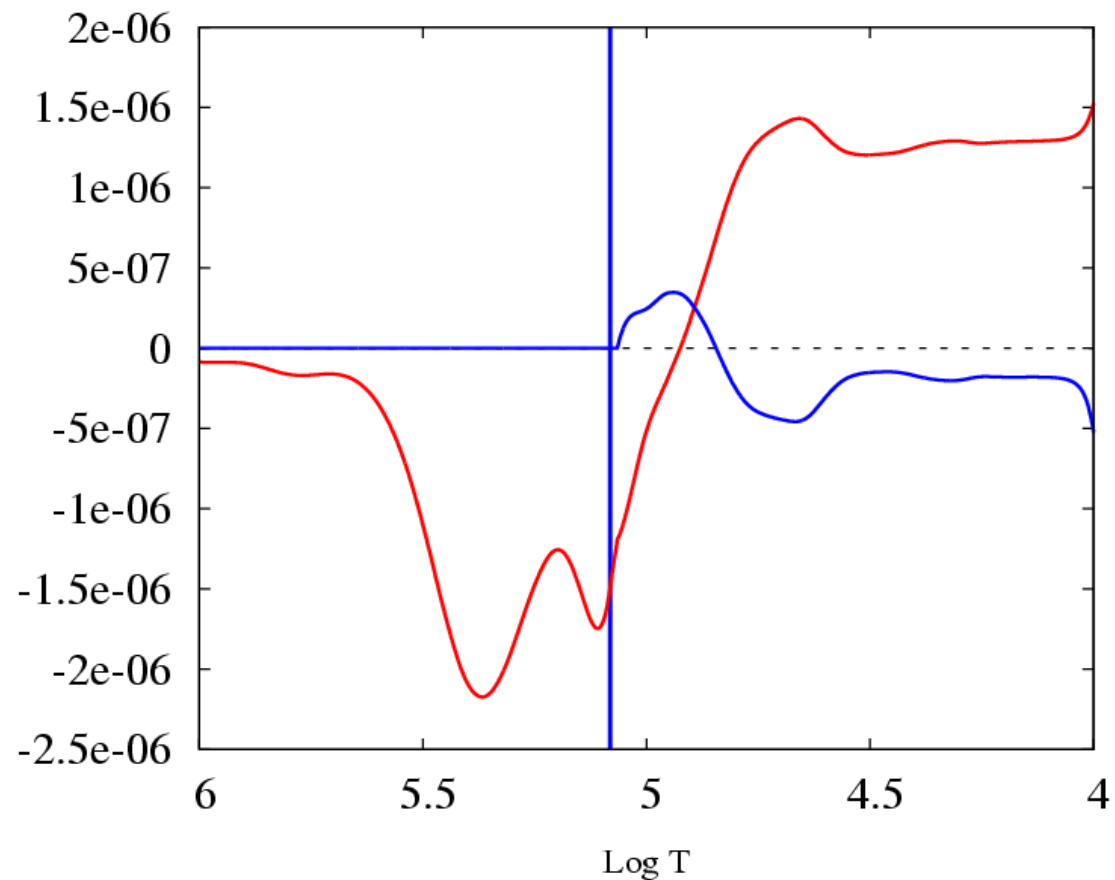
## Driving mechanism

$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
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Mode  $(\ell=1, g_{50})$

$W_{\text{FRr}}$ : Radial radiative  
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$W_{\text{Fcr}}$ : Radial convective  
flux term

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



# $\gamma$ Doradus

## Driving mechanism

$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
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Mode  $(l=1, g_{50})$

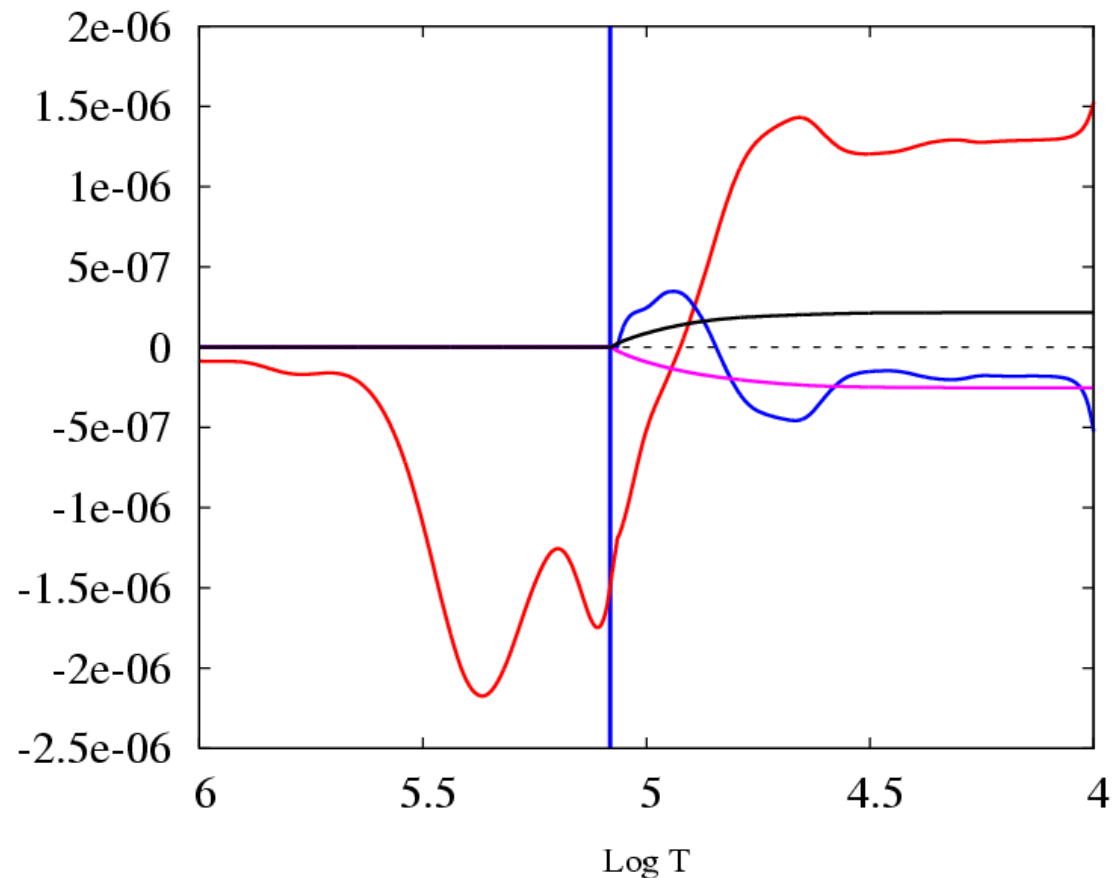
$W_{\text{FRr}}$ : Radial radiative  
flux term

$W_{\text{Fcr}}$ : Radial convective  
flux term

$W_{\text{pt}}$ : Turbulent pressure

$W_{\varepsilon 2}$ : Turbulent kinetic  
energy dissipation

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



# $\gamma$ Doradus

## Driving mechanism

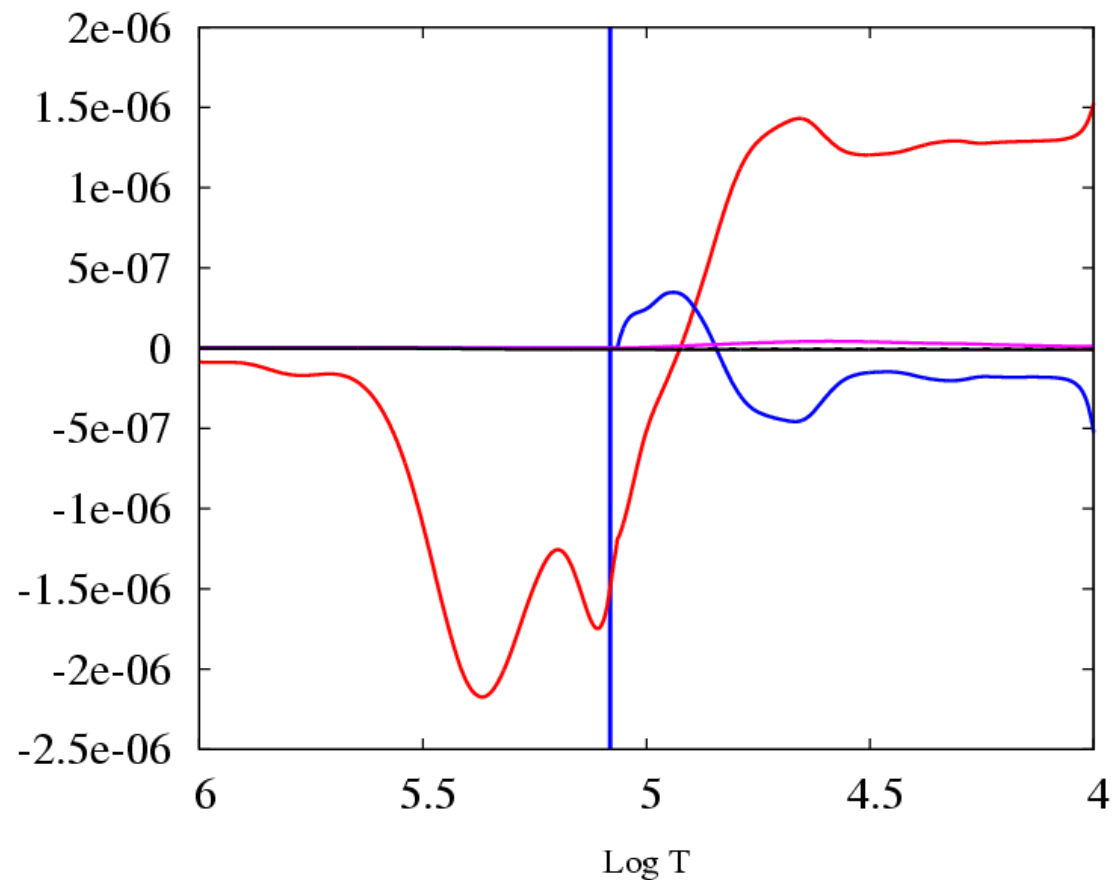
$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
 $\alpha = 2$   
Mode  $(\ell=1, g_{50})$

$W_{\text{FRr}}$ : Radial radiative  
flux term

$W_{\text{Fcr}}$ : Radial convective  
flux term

$W_{\text{Fh}}$ : Transversal convective  
and radiative flux

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



# $\gamma$ Doradus

# Driving mechanism

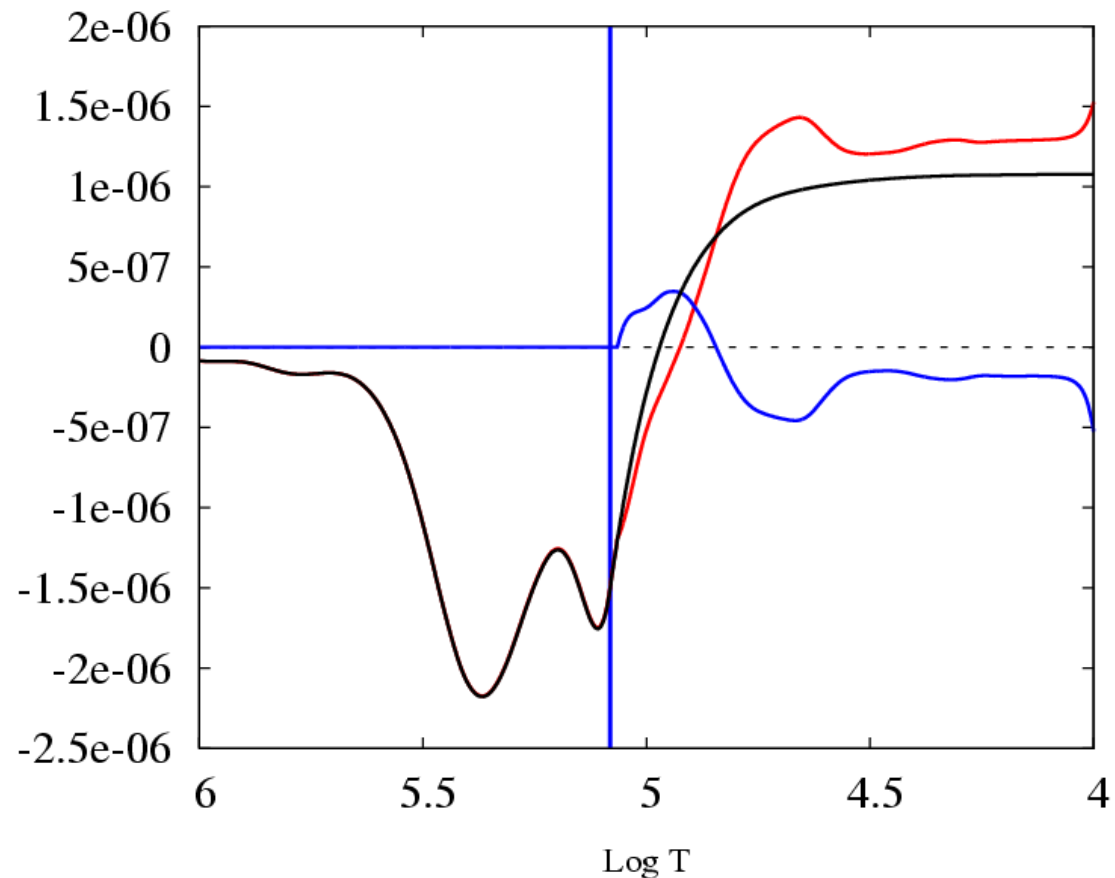
$M = 1.6 M_{\odot}$   
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 $\alpha = 2$   
Mode  $(l=1, g_{50})$

$W_{\text{FRr}}$ : Radial radiative  
flux term

$W_{\text{Fcr}}$ : Radial convective  
flux term

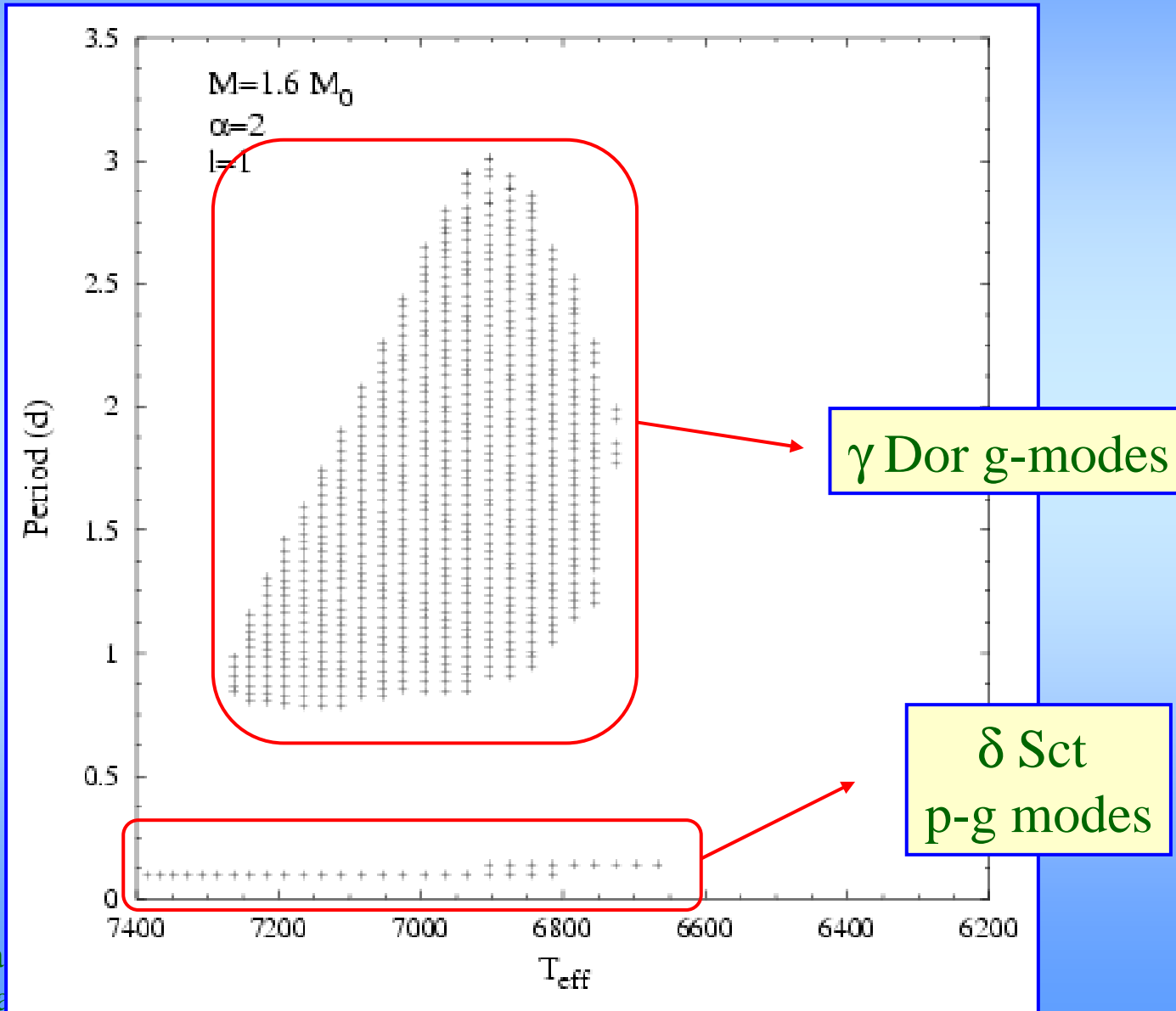
$W_{\text{tot}}$ : Total work

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



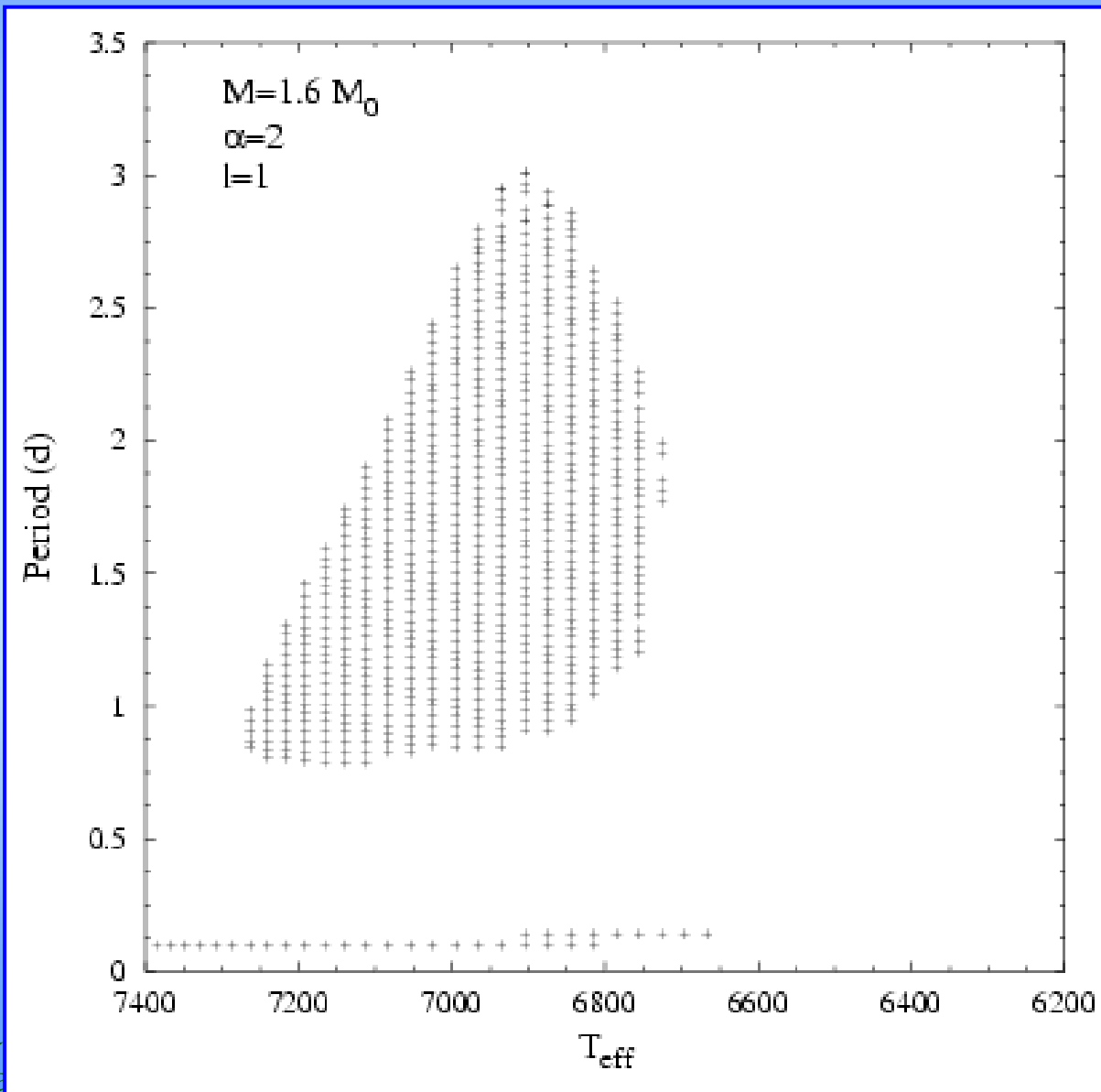
# $\gamma$ Doradus

# Unstable modes



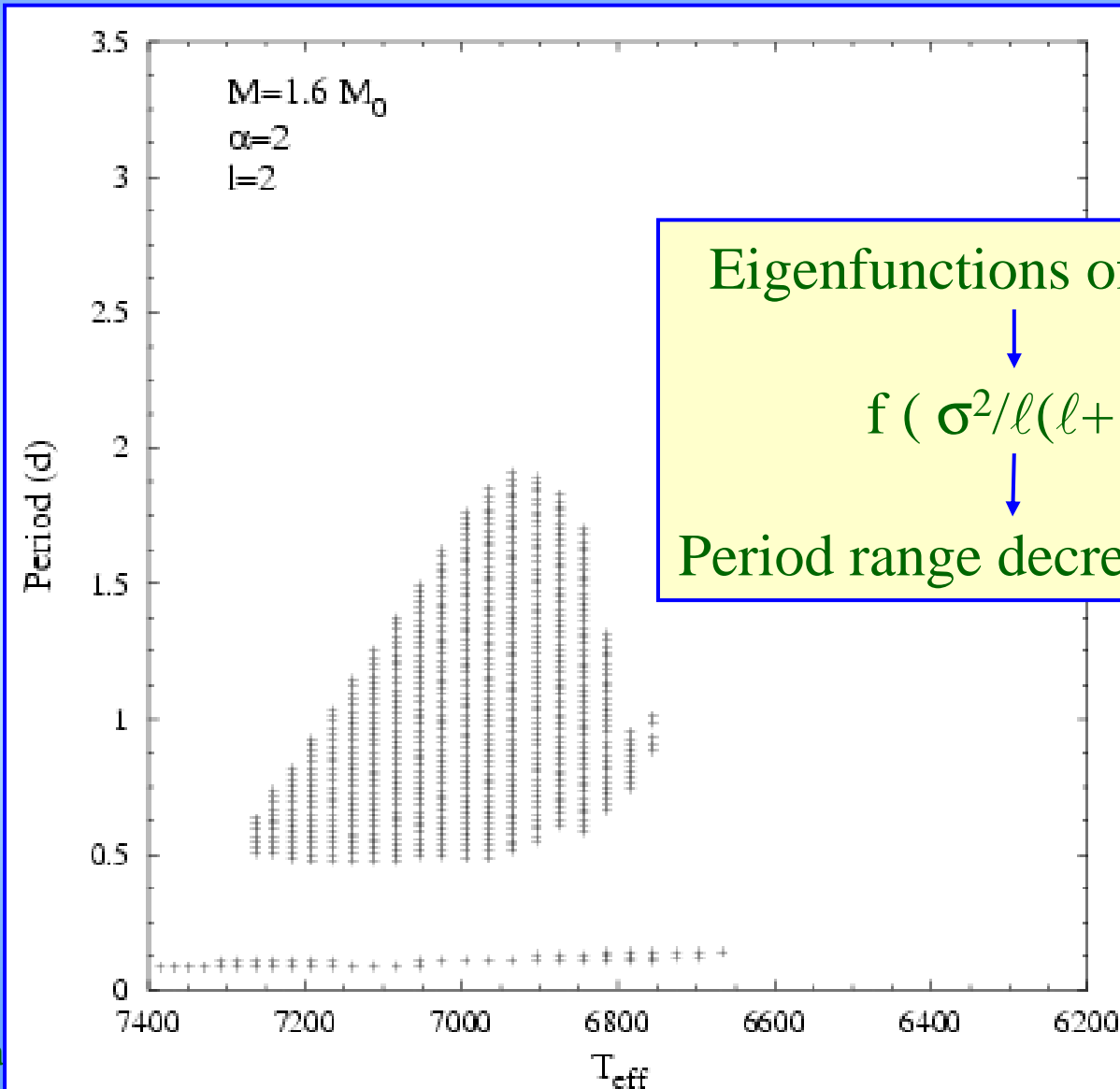
# $\gamma$ Doradus

# Unstable modes



# $\gamma$ Doradus

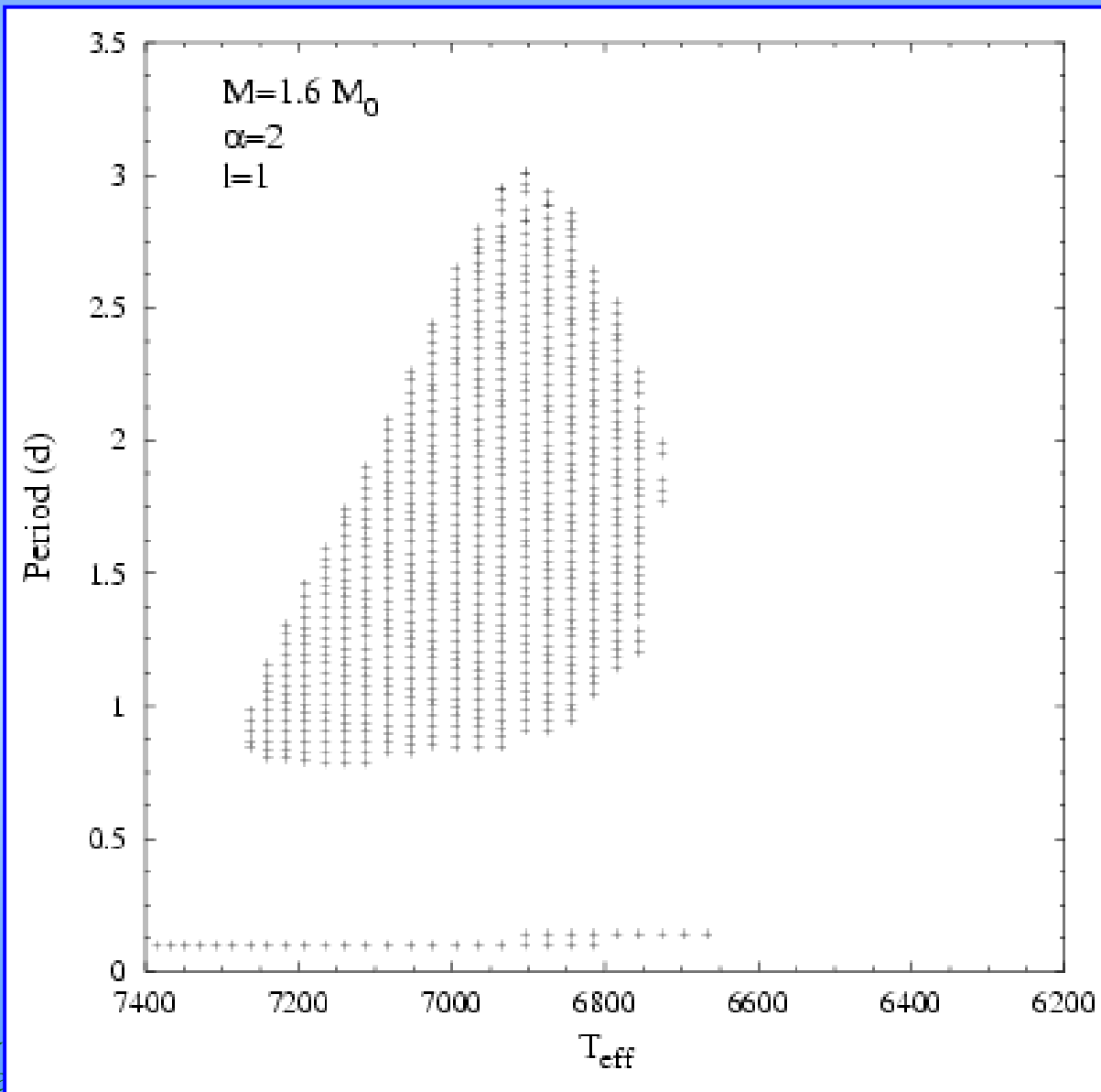
# Unstable modes





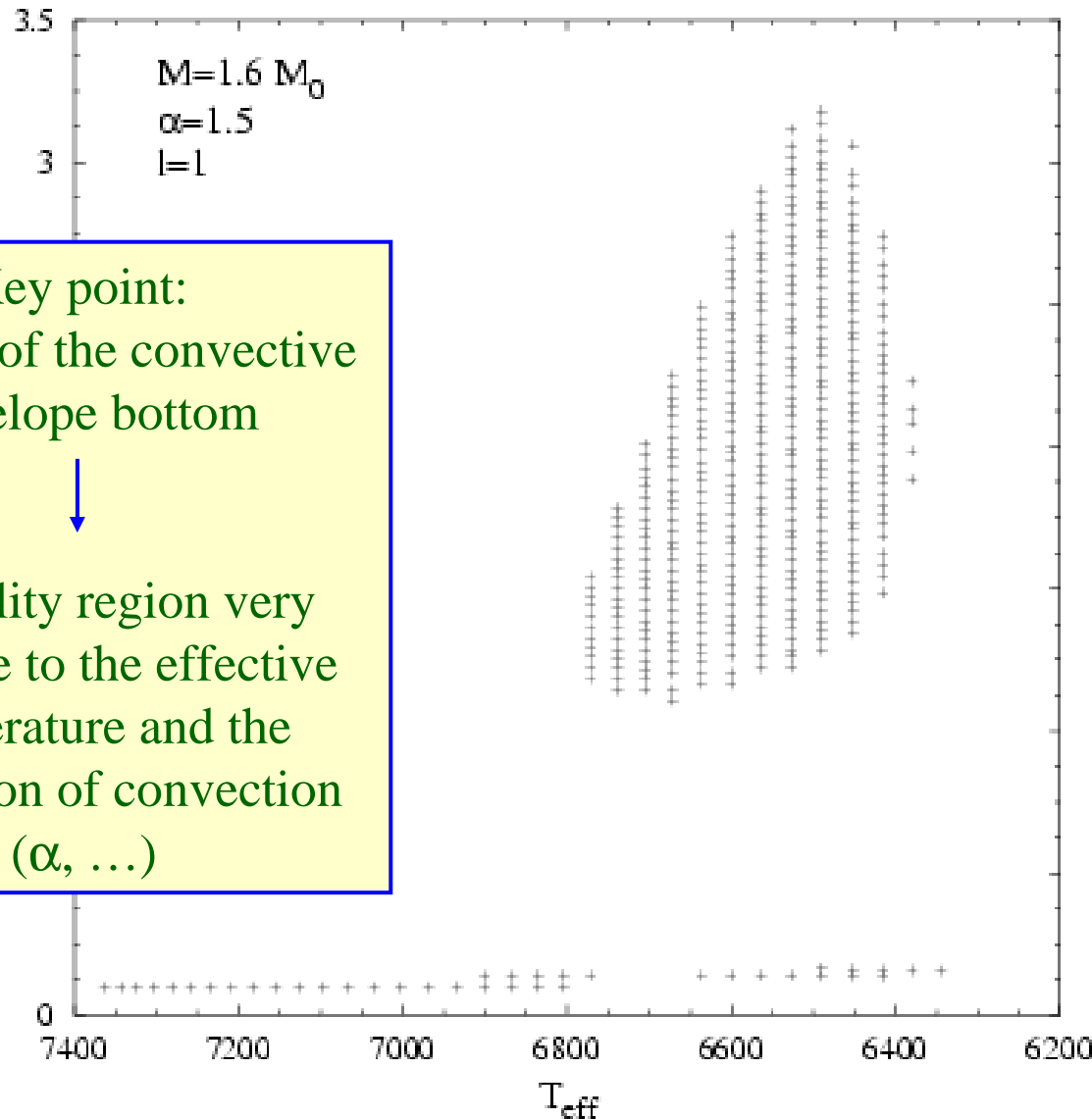
# $\gamma$ Doradus

# Unstable modes



# $\gamma$ Doradus

# Unstable modes



# $\gamma$ Doradus

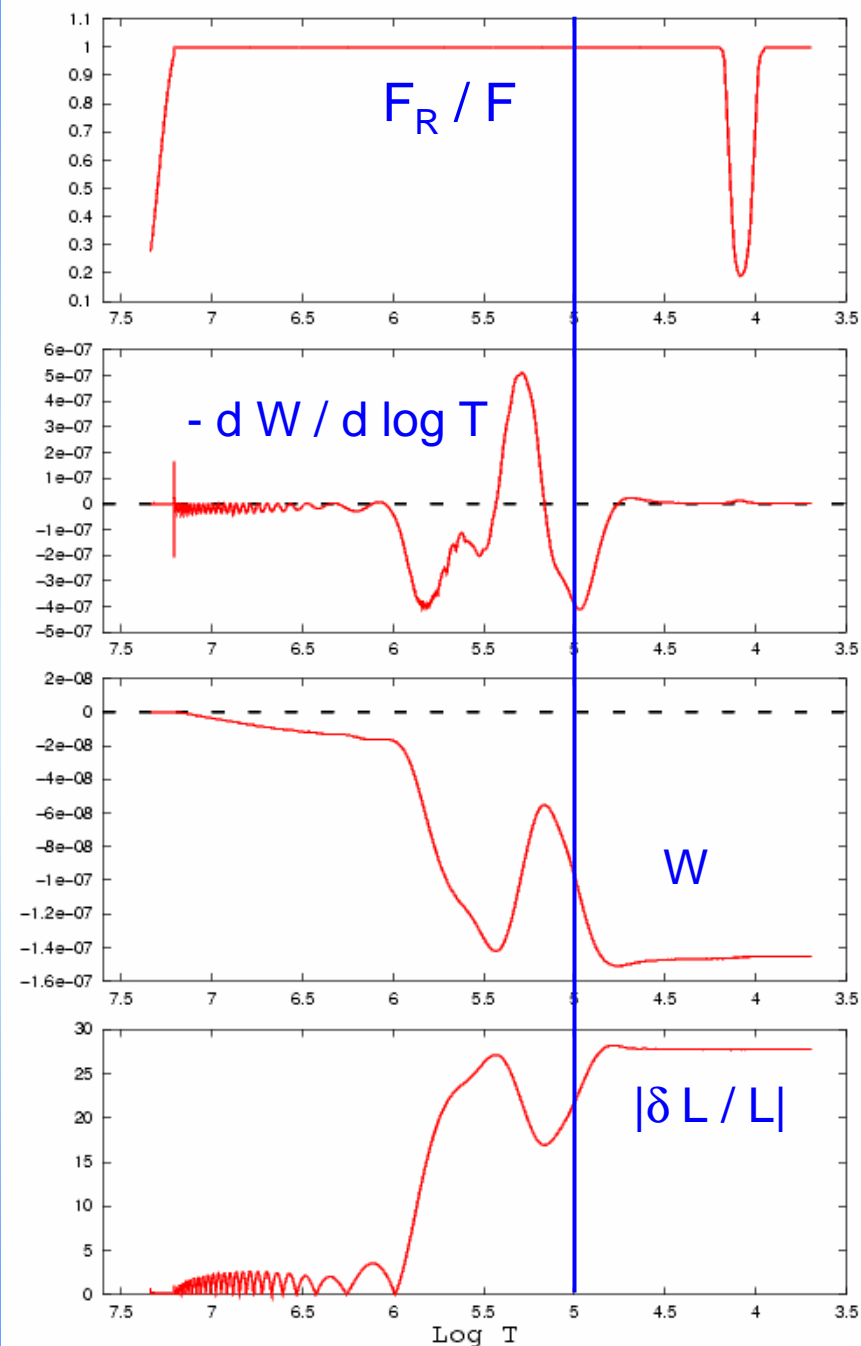
For hot or small  $\alpha$  models:  
very thin convective envelope



the transition region is in the  
radiative zone

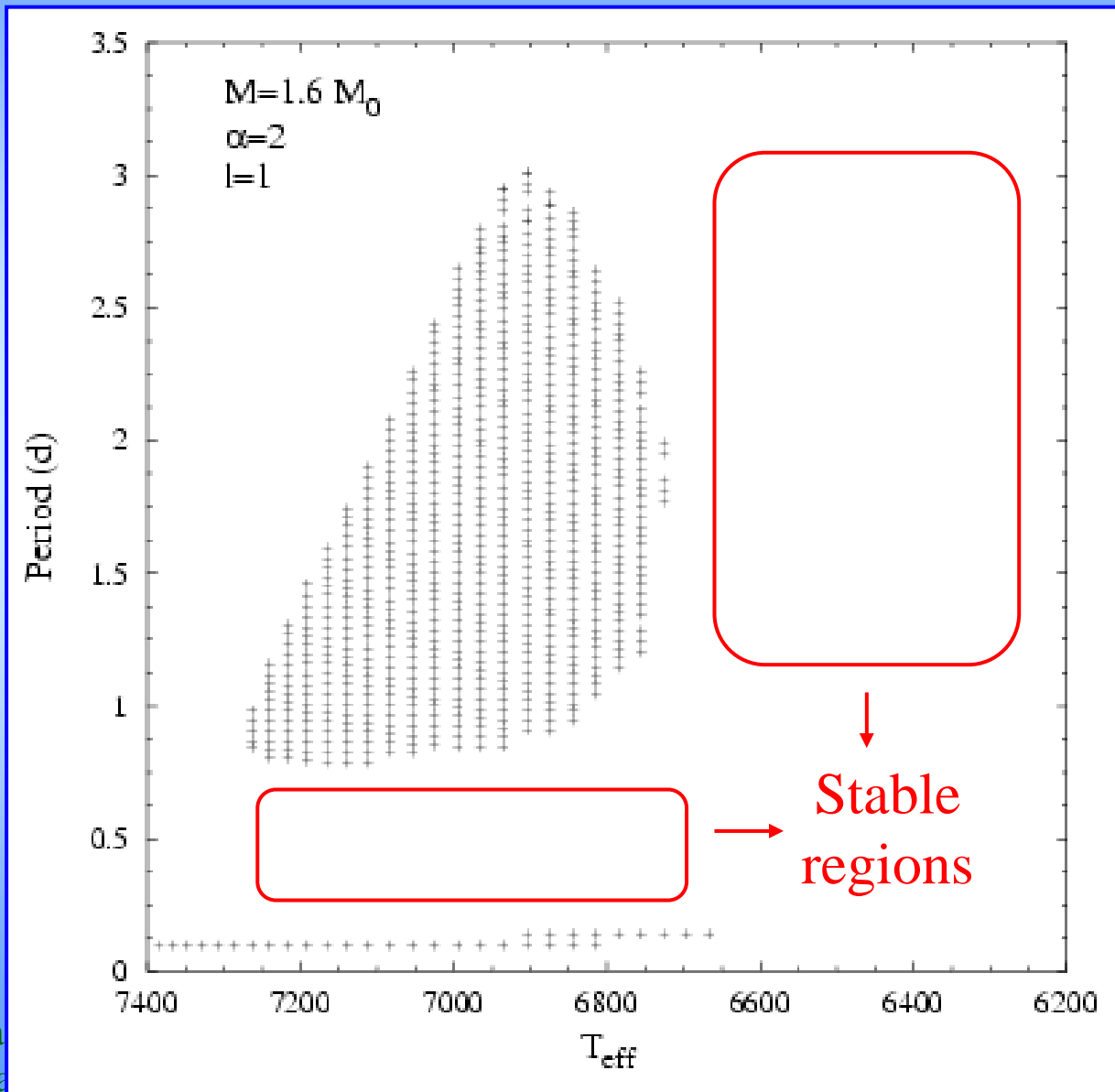


Small  $\kappa$ -driving (Fe,  $\sim$  SPBs)  
compensated by  
large radiative damping  
below and above



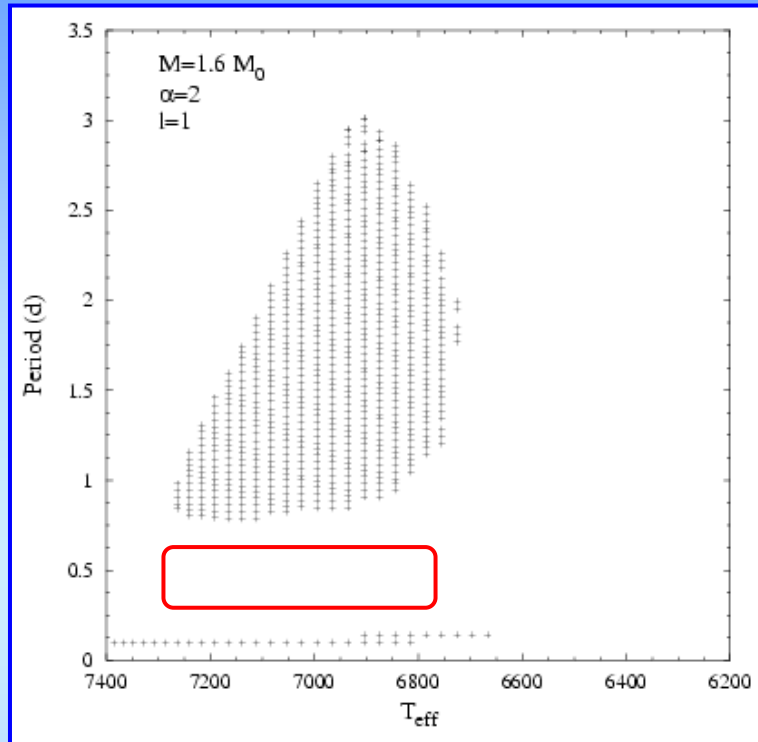
# $\gamma$ Doradus

## Stabilization mechanism

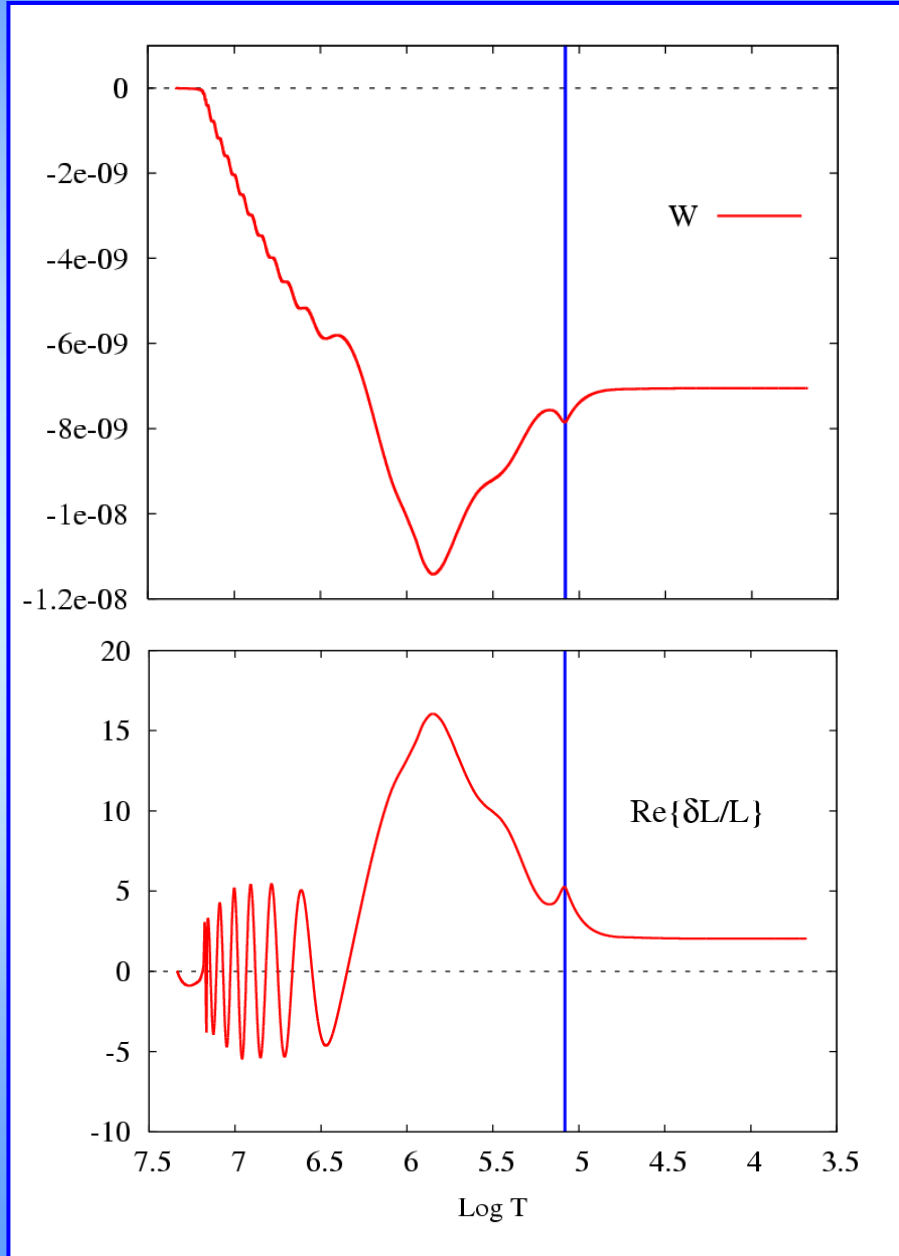


# $\gamma$ Doradus

# Stabilization mechanism



Radiative damping  
in the g-modes cavity

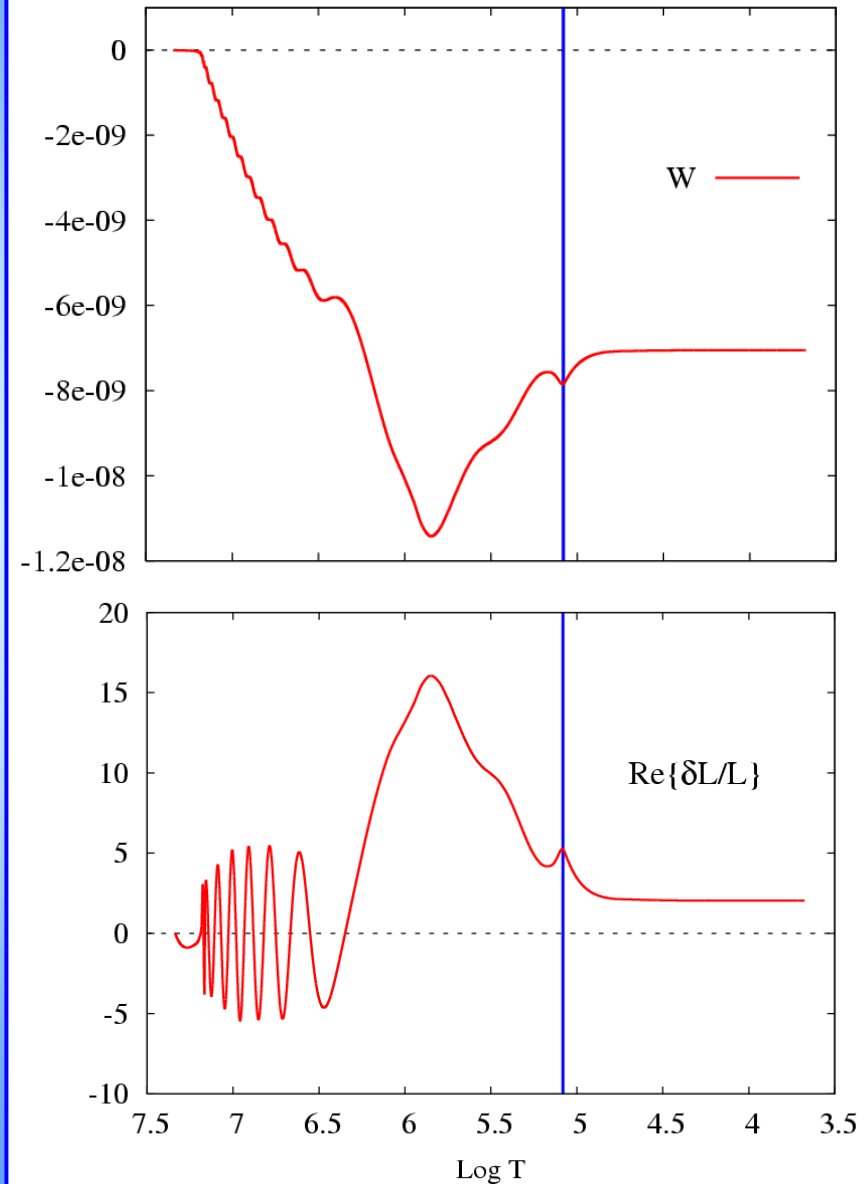


# $\gamma$ Doradus

## Stabilization mechanism

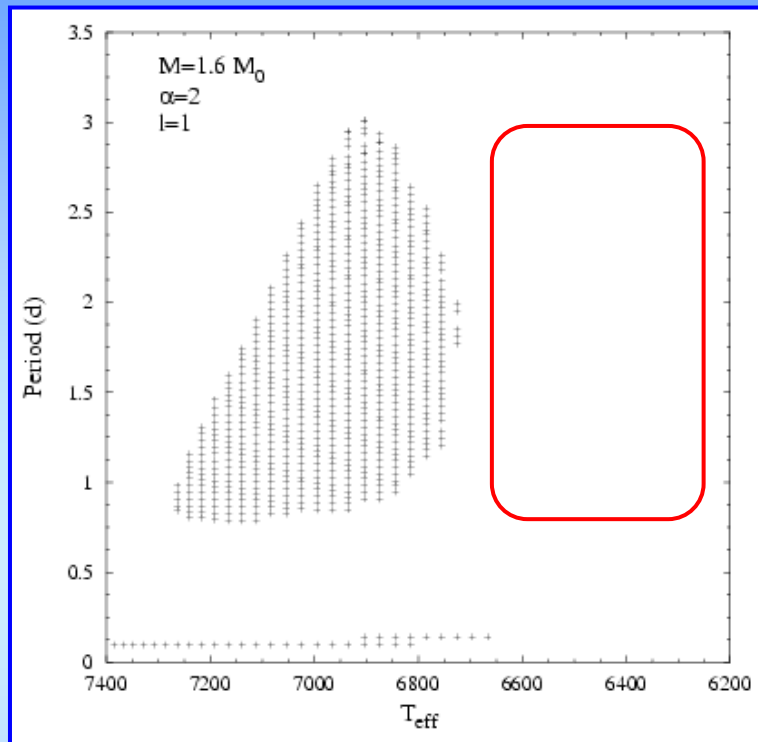
### Radiative damping in the g-modes cavity

$$\sigma_i = \frac{-1}{2\sigma_r^2} \frac{\int_0^M \frac{\delta T}{T} \frac{d\delta L}{dm} dm}{\int_0^M \delta r^2 dm}$$
$$\frac{\delta L}{L} = 2 \frac{\delta r}{r} + 3 \frac{\delta T}{T} - \frac{\delta \kappa}{\kappa} - \frac{\delta \rho}{\rho}$$
$$+ \frac{d\delta T}{dT} - \frac{d\delta r}{dr}$$
$$\sigma_i = \frac{\int_0^R \frac{-L}{d \ln T / dr} \frac{\delta T}{T} \frac{d^2(\delta T / T)}{dr^2} dr}{2\sigma_r^2 \int_0^R \delta r^2 dm}$$

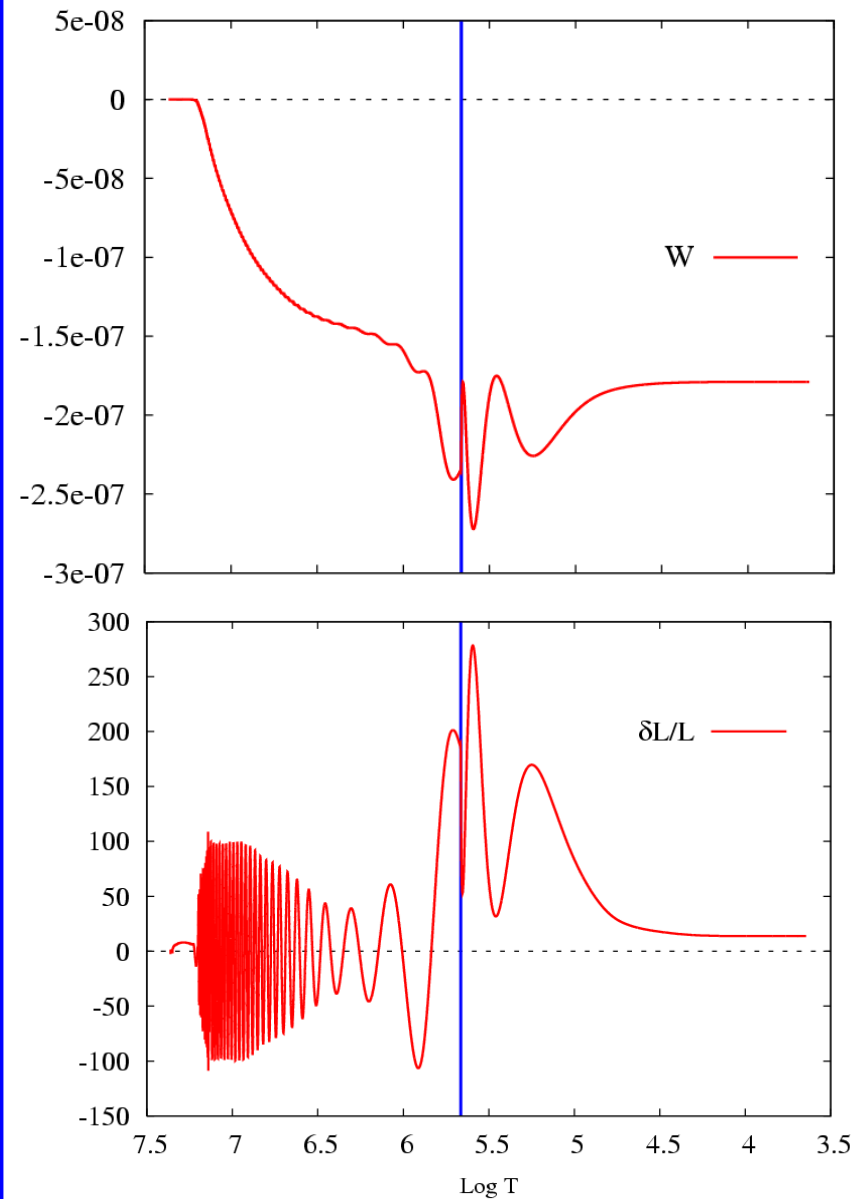


# $\gamma$ Doradus

## Stabilization mechanism

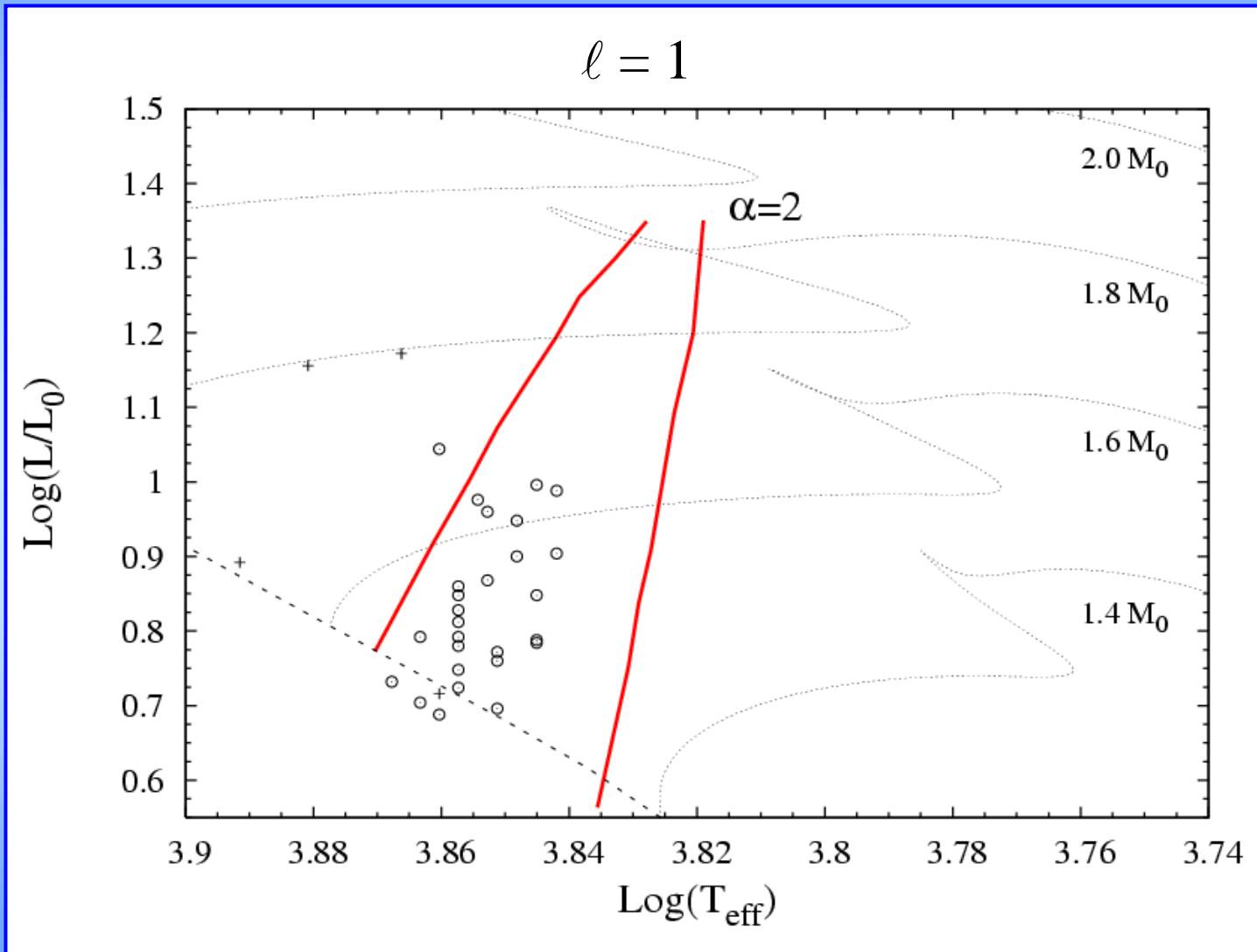


Radiative damping  
in the g-modes cavity



# $\gamma$ Doradus

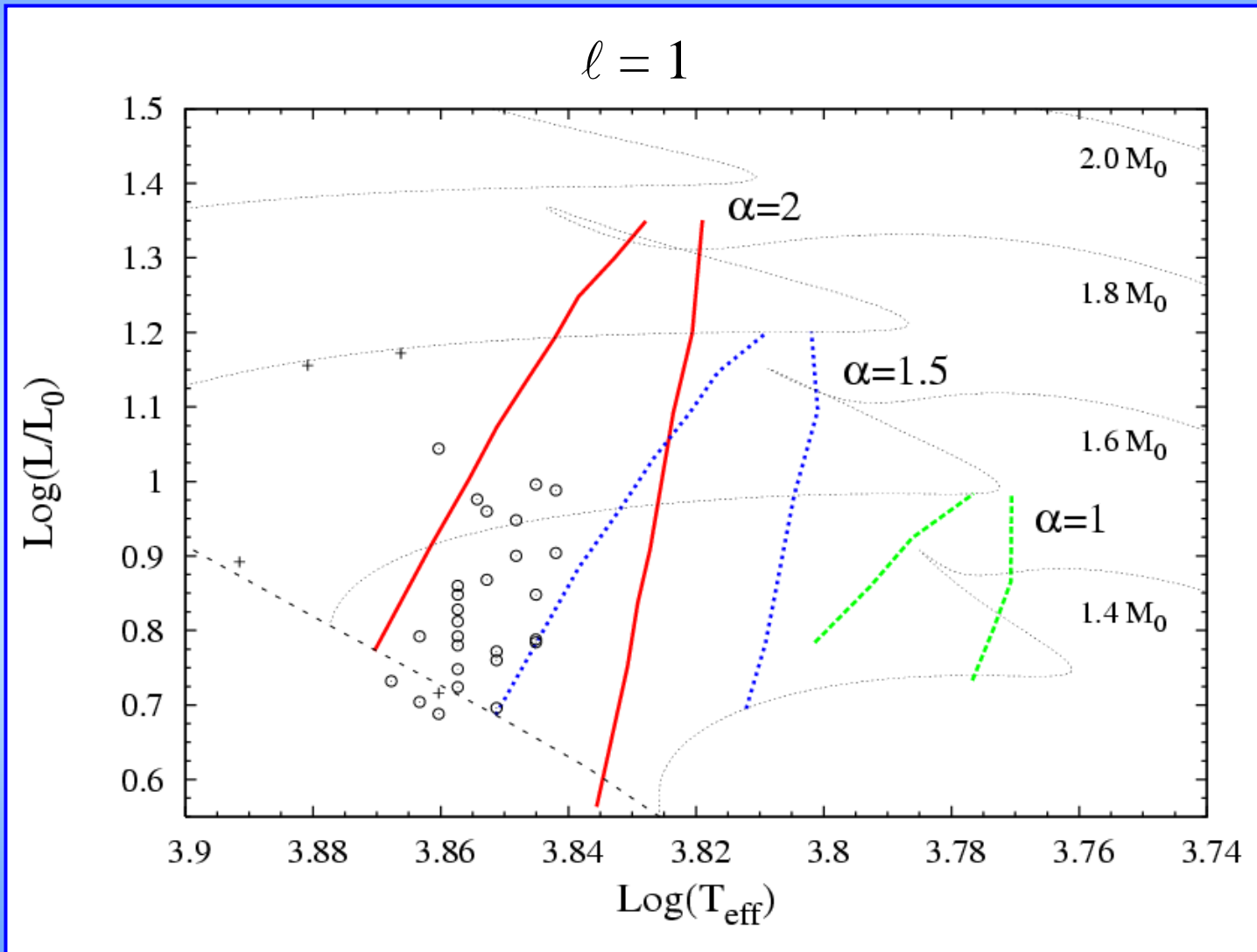
# Instability strips





# $\gamma$ Doradus

# Instability strips

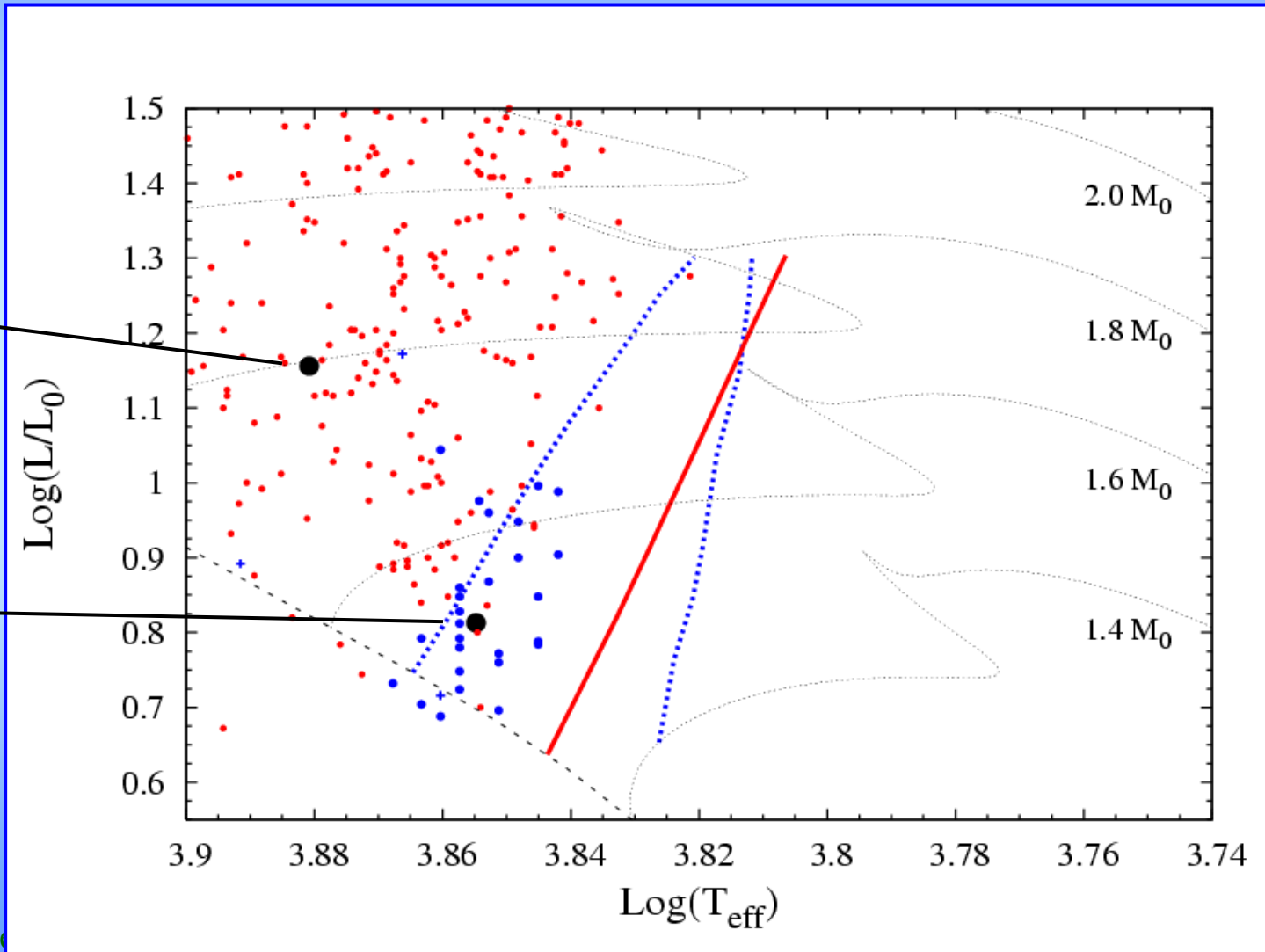


Comparison :  $\delta$  Sct red edge ( $\ell=0, p_1$ )  
 $\gamma$  Dor instability strip ( $\ell=1$ )

Hybrid  
 $\delta$  Sct –  $\gamma$  Dor

HD 209295  
Handler et al.  
(2002)  
Tidally excited ?

HD 8801  
Henry et al.  
(2005)  
Am star



## Influence of turbulent Reynolds stress perturbation

$$\overline{\rho \vec{V} \vec{V}} = \bar{p}_T \mathbf{1} - \bar{\beta}_T$$

$$-\delta(\nabla \cdot \bar{\beta}_T) = \Xi_r Y_1^m(\theta, \phi) + r \Xi_h \nabla_h Y_1^m(\theta, \phi)$$

**Radial component of the equation of momentum conservation**

$$\sigma^2 \delta r = \frac{d\delta\Phi}{dr} - \frac{1}{\rho} \frac{d\delta p_g}{dr} - \frac{1}{\rho} \frac{d\delta p_{\text{turb}}}{dr} + g \frac{\delta\rho}{\rho} + \frac{2A-1}{A} \frac{\bar{p}_T}{r\rho} \frac{\partial \delta r}{\partial r} + \frac{\Xi_r}{\rho}$$

**Transversal component of the equation of momentum conservation**

$$\sigma^2 \delta r_H = \frac{1}{r} \left( \delta\Phi + \frac{\delta p}{\rho} + \frac{r \Xi_h}{\bar{\rho}} + \frac{2A-1}{A} \frac{\bar{p}_T}{\bar{\rho}} \left( \frac{\delta r}{r} - \frac{\delta r_H}{r} \right) \right)$$

$\sigma$  : Angular pulsation frequency

A : Anisotropy parameter

A=1/2 for isotropic turbulence

# Influence of turbulent Reynolds stress perturbation

## Work integral

$$W = \int_0^M dm \Im \left\{ \frac{\delta \rho^* \delta p}{\rho} \right. \\ + \left. (1/\rho) (\xi_r^* \Xi_r + \ell(\ell+1) \xi_h^* \Xi_h) \right. \\ \left. + \frac{2A - 1 p_t}{A \rho} \left[ \frac{\xi_r^*}{r} \frac{d\xi_r}{dr} + \ell(\ell+1) \frac{\xi_h^*}{r} \left( \frac{\xi_r}{r} - \frac{\xi_h}{r} \right) \right] \right\}$$

## Influence of turbulent Reynolds stress perturbation

$$\mathbb{E}_h = (\dots) + \frac{1}{r^3} \frac{d}{dr} \left( r^3 P_{\text{turb}} \frac{\overline{V_r \delta V_h + V_h \delta V_r}}{V_r^2} \right)$$

$$\frac{\overline{V_r \delta V_h + V_h \delta V_r}}{V_r^2} = \delta V V_h = (\dots) + C \frac{d \delta r_h}{dr}$$

$P_{\text{turb}} \longrightarrow 0$  and  $d P_{\text{turb}} / dr$  discontinuous  
at the bottom of the convective envelope

$\longrightarrow$  singularity of the equations

$\longrightarrow$  unphysical discontinuity of the eigenfunctions

## Influence of turbulent Reynolds stress perturbation

$$\Xi_h = (\dots) + \frac{1}{r^3} \frac{d}{dr} \left( r^3 P_{\text{turb}} \frac{\overline{V_r \delta V_h + V_h \delta V_r}}{V_r^2} \right)$$

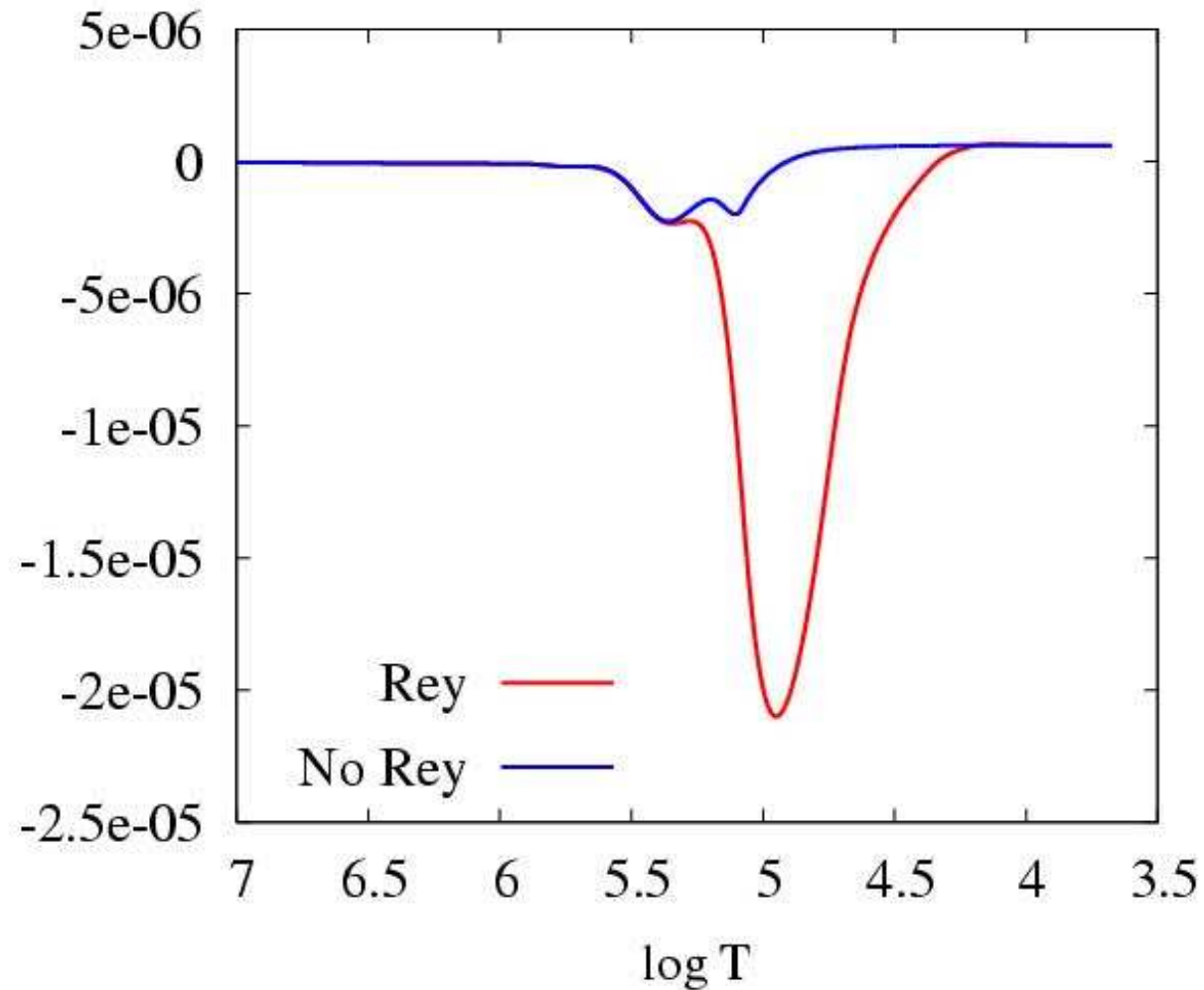
$$\frac{\overline{V_r \delta V_h + V_h \delta V_r}}{V_r^2} = \delta V V_h = (\dots) + C \frac{d \delta r_h}{dr}$$

**Non-local treatments:**

$$\delta V V_{h \text{NL}}(\zeta_0) = \int_{-\infty}^{+\infty} \delta V V_{h \text{L}} e^{-b|\zeta - \zeta_0|} d\zeta \quad ; \quad d\zeta = d \log P$$

**Improves the things (continuity) but problem still present**

## Influence of Reynolds stress: Work integral



# Photometric amplitudes and phases and mode identification

## Hypotheses

- Lagrangian displacement → Distortion of the stellar surface
- Thermal equilibrium in the local atmosphere

→	• Temperature :	$\frac{\delta T}{T} = \frac{\partial \ln T}{\partial \ln T_{\text{eff}}} \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln T}{\partial \ln g} \frac{\delta g_e}{g_e} + \frac{\partial \ln T}{\partial \ln \tau} \frac{\delta \tau}{\tau}$
→	• Flux :	$\frac{\delta F_\lambda}{F_\lambda} = \frac{\partial \ln F_\lambda}{\partial \ln T_{\text{eff}}} \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln F_\lambda}{\partial \ln g} \frac{\delta g_e}{g_e}$
→	• Limb darkening :	$\frac{\delta h_\lambda}{h_\lambda} = \frac{\partial \ln h_\lambda}{\partial \ln T_{\text{eff}}} \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln h_\lambda}{\partial \ln g} \frac{\delta g_e}{g_e} + \frac{\partial \ln h_\lambda}{\partial \ln \mu} \frac{\delta \mu}{\mu}$



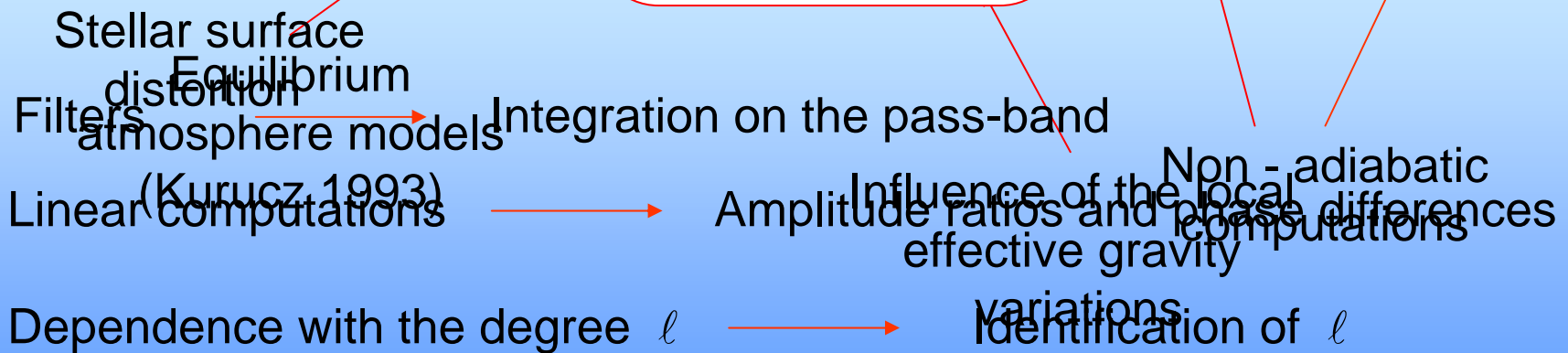
# Monochromatic magnitude variation

$$\delta m_\lambda = - \frac{2.5}{\ln 10} \varepsilon P_\ell^m(\cos i) b_{\ell\lambda}$$

Influence of the local  
effective temperature  
variations

$$\left[ -(\ell - 1)(\ell + 2) \cos(\sigma t) + \left( \frac{\partial \ln F_\lambda^+}{\partial \ln T_{\text{eff}}} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln T_{\text{eff}}} \right) \left| \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \right| \cos(\sigma t + \psi_T) \right]$$

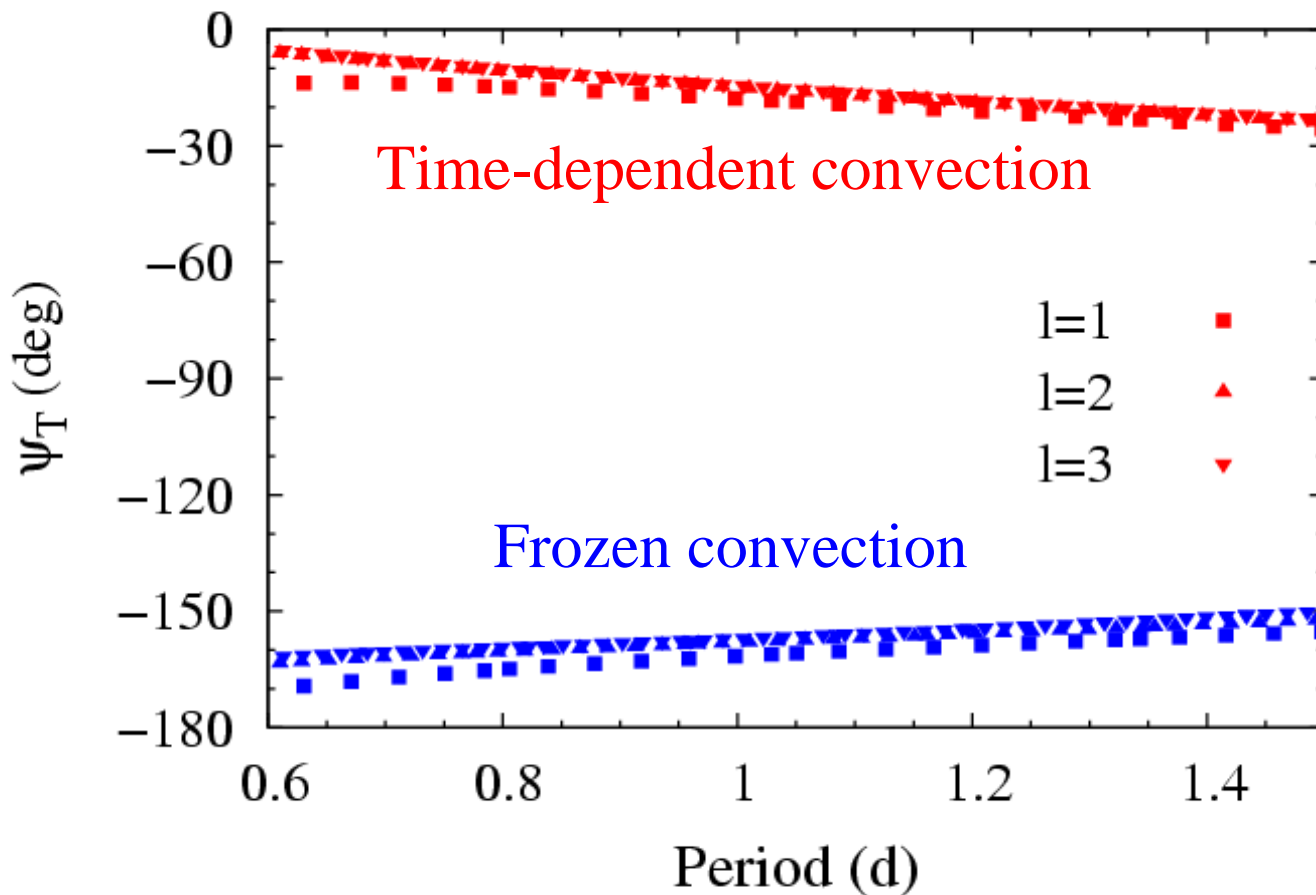
$$- \left( \frac{\partial \ln F_\lambda^+}{\partial \ln g} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln g} \right) \left| \frac{\delta g_e}{g_e} \right| \cos(\sigma t)$$



# Spectro-photometric amplitudes and phases and mode identification

Very sensitive to the non-adiabatic treatment of convection

Phase-lag

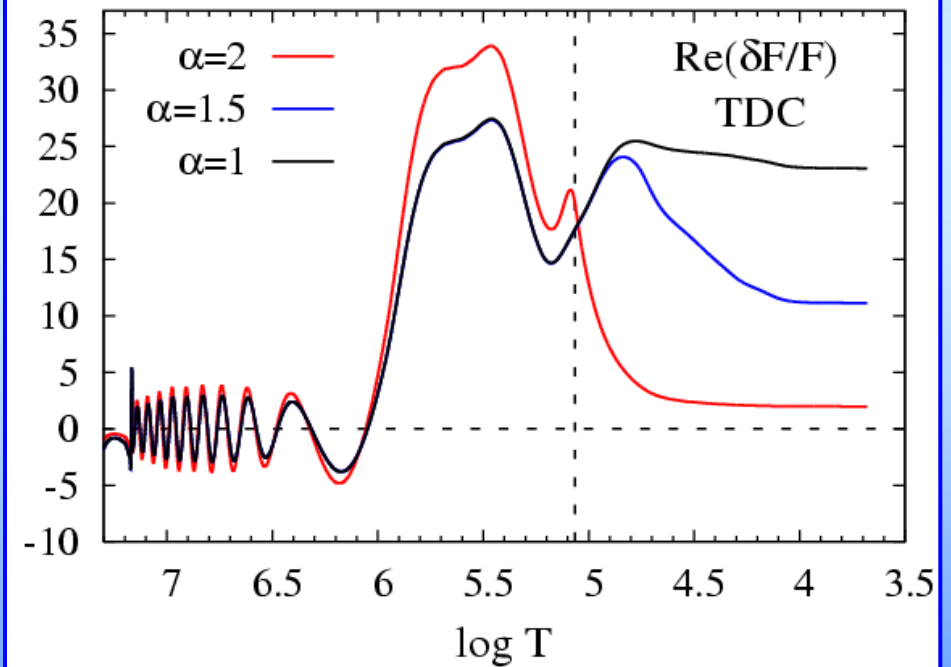
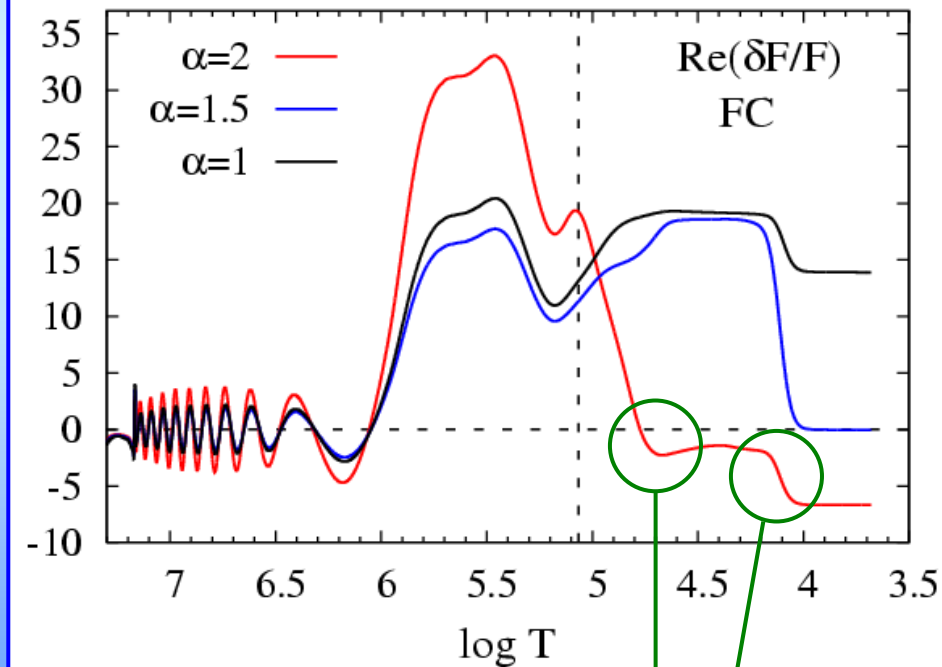


# Spectro-photometric amplitudes and phases and mode identification

Very sensitive to the non-adiabatic treatment of convection

Frozen convection

Time-dependent convection



BAG meeting, asteroseismology  
Liège, 5th of May 2006

$\kappa$ -mechanism  $\rightarrow$

Not allowed by time-dependent convection

# Spectro-photometric amplitudes and phases and mode identification

## $\gamma$ Doradus

3 frequencies:  $f_1=1.32098$  c/d,  $f_2=1.36354$  c/d,  $f_3=1.47447$  c/d

Balona et al. 1994  $\longrightarrow$  Strömngren photometry

Balona et al. 1996  $\longrightarrow$  Simultaneous photometry  
and spectroscopy

$\longrightarrow$  Spectroscopic mode identification:  $(\ell_1, m_1) = (3, 3)$ ,  
 $(\ell_2, m_2) = (1, 1)$ ,  
 $(\ell_3, m_3) = (1, 1)$

# Spectro-photometric phase differences

## $\gamma$ Doradus

Balona et al. 1996  $\longrightarrow$  Simultaneous photometry and spectroscopy

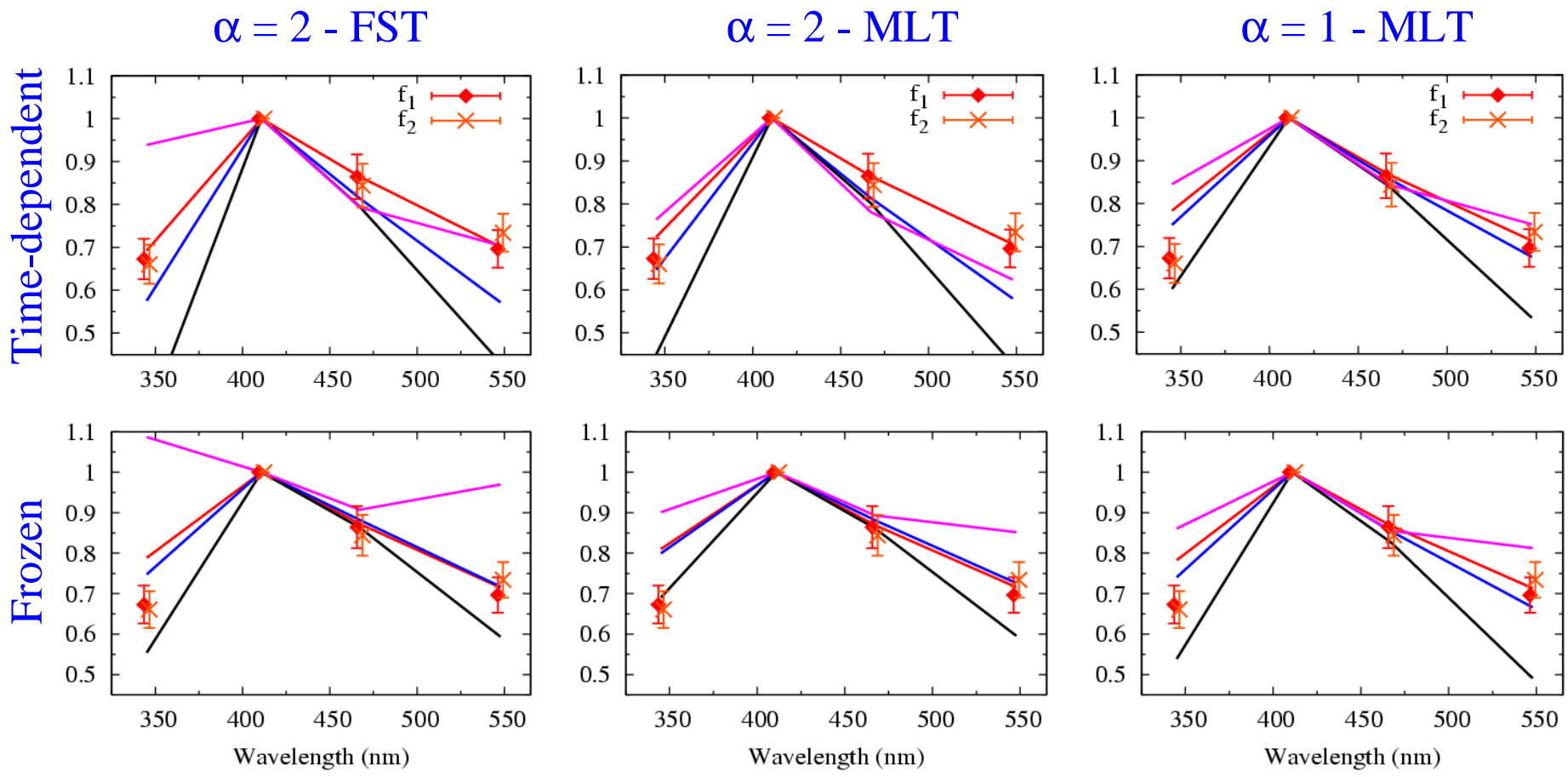
$\longleftarrow$  Phase-lag ( Vmagnitude – displacement ):

**Observations:**  $\Delta\phi_1 = -65^\circ \pm 5^\circ$   $\Delta\phi_3 = -29^\circ \pm 8^\circ$

**Theory:** Time-dependent convection  $\Delta\phi = -30^\circ$  Frozen convection  $\Delta\phi = -165^\circ$   
( $\alpha = 2$ )

# $\gamma$ Doradus

# Photometric mode identification



$\ell = 1$  modes

Best models: Time-dependent convection  
FST atmosphere

# Spectro-photometric amplitudes and phases

## 9 Aurigae

3 frequencies:  $f_1=0.795$  c/d,  $f_2=0.768$  c/d,  $f_3=0.343$  c/d

Zerbi et al. 1994  $\longrightarrow$  Simultaneous photometry  
and spectroscopy

Spectroscopic mode id. :  $(\ell_1, |m_1|) = (3, 1)$ ,  
Aerts & Krisciunas (1996)  $(\ell_3, |m_3|) = (3, 1)$

# Spectro-photometric phase differences

## 9 Aurigae

Balona et al. 1996  $\longrightarrow$  Simultaneous photometry and spectroscopy  
 $\downarrow$  Phase-lag ( Vmagnitude – displacement ):

**Observations:**  $\Delta\phi_1 = -77^\circ \pm 12^\circ$   $\Delta\phi_3 = -41^\circ \pm 10^\circ$

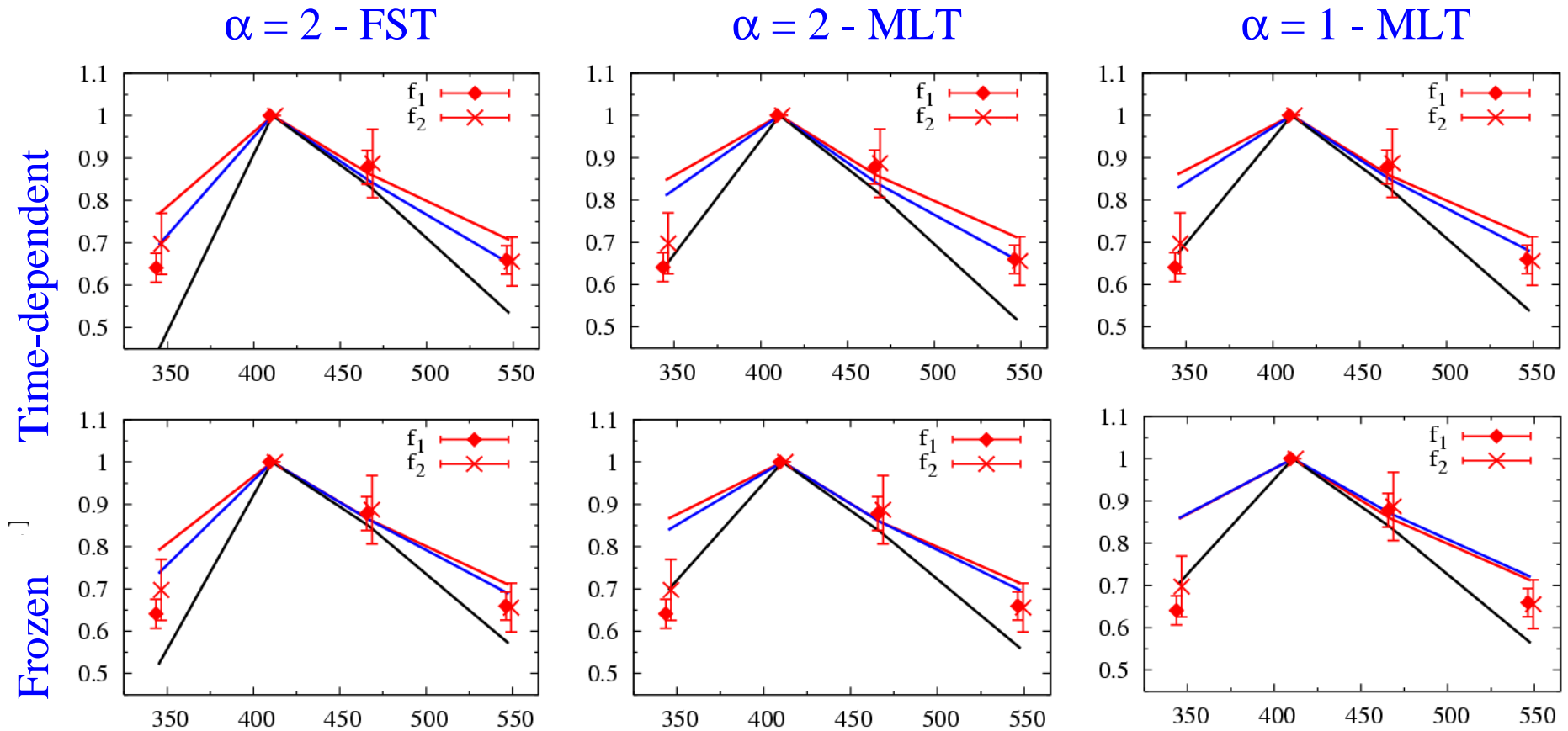
**Theory ( $\alpha = 2$ ):**

<b>TDC</b>	$\Delta\phi_1 = -22^\circ$	$\Delta\phi_3 = -39^\circ$
<b>FC</b>	$\Delta\phi_1 = -156^\circ$	$\Delta\phi_3 = -140^\circ$



# 9 Aurigae

# Photometric mode identification



$\ell = 2$  modes

Best models: Time-dependent convection  
FST atmosphere

# Importance of ultraviolet observations (bracketing the Balmer discontinuity)

Influence of the effective temperature  
variations

$$\delta m_\lambda = -\frac{2.5}{\ln 10} \varepsilon P_\ell^m(\cos i) b_{\ell\lambda}$$

$$\left[ -(\ell - 1)(\ell + 2) \cos(\sigma t) + \left( \frac{\partial \ln F_\lambda^+}{\partial \ln T_{\text{eff}}} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln T_{\text{eff}}} \right) \left| \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \right| \cos(\sigma t + \psi_T) \right]$$

$$- \left( \frac{\partial \ln F_\lambda^+}{\partial \ln g} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln g} \right) \left| \frac{\delta g_e}{g_e} \right| \cos(\sigma t)$$

Gravity derivatives vary quickly in u-v

Changes the weight of  $T_{\text{eff}}$  and  $g_e$  terms

Helps for the mode identification and  
gives constraints on  $|\delta T_{\text{eff}}/T_{\text{eff}}|$

Influence of the effective gravity  
variations

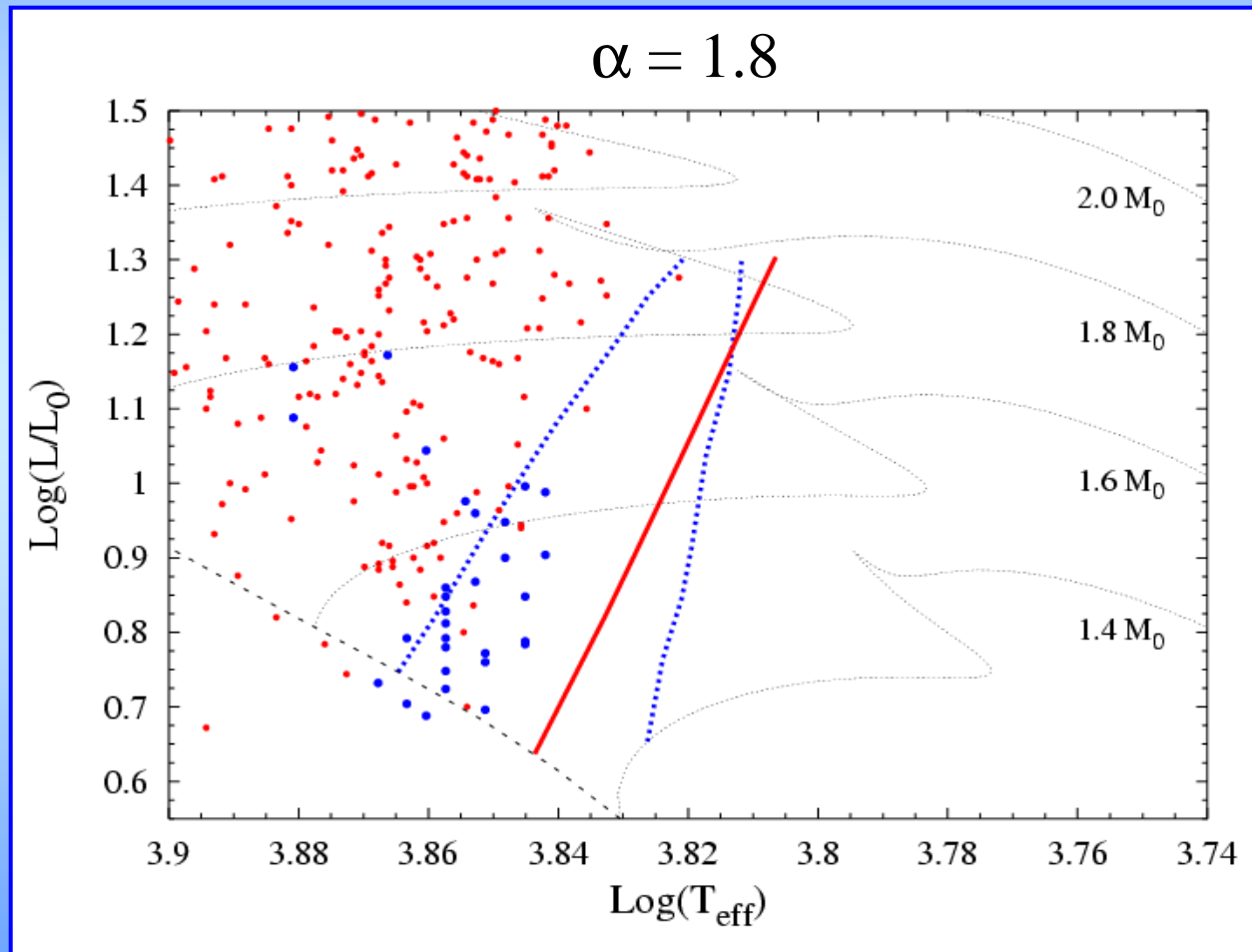
Stromgren, Geneva  
systems are perfects

## Conclusions

# Driving mechanism and energetic aspects in $\gamma$ Doradus stars

- **Excitation mechanism:** Convective blocking  
Time-Dependent convection does not inhibit the mechanism
- **Mode identification, amplitudes and phases:**  
Time-Dependent Convection required

Comparison :  $\delta$  Sct red edge ( $\ell=0, p_1$ )  
 $\gamma$  Dor instability strip ( $\ell=1$ )



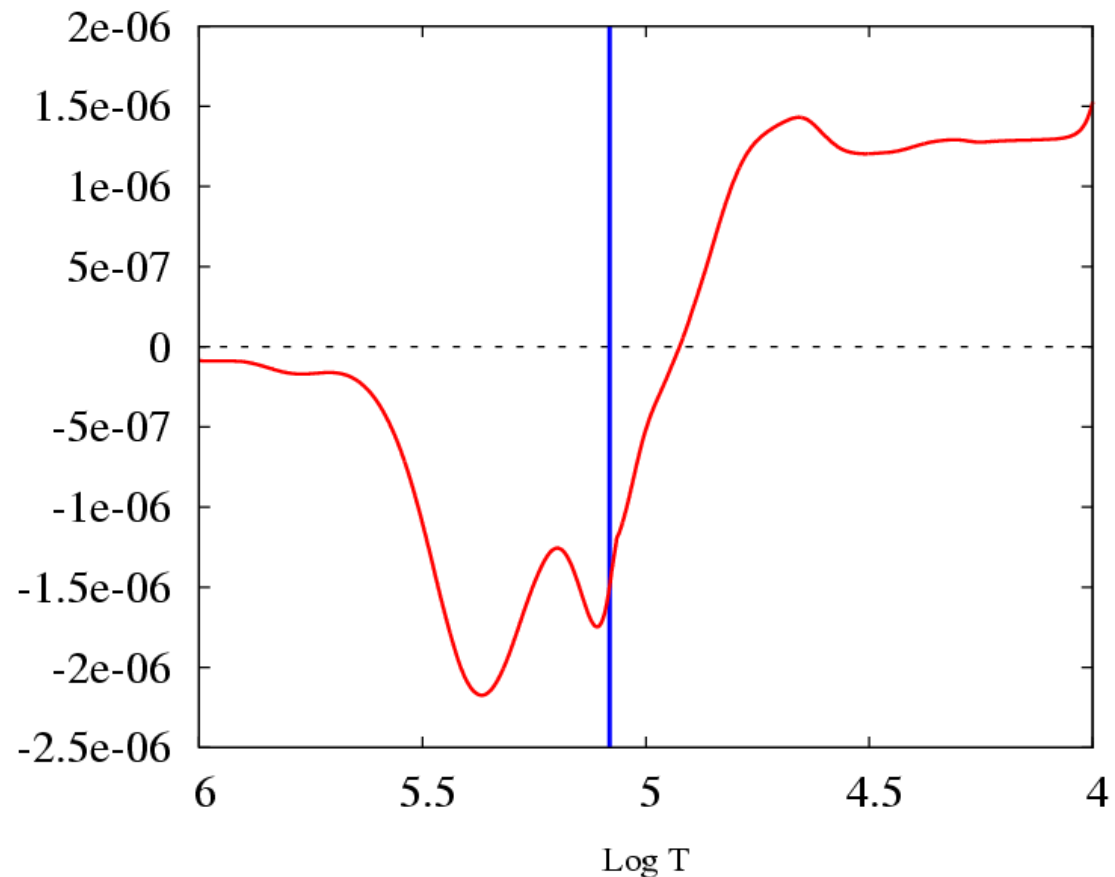
# $\gamma$ Doradus

## Stabilization mechanism

$M = 1.6 M_{\odot}$   
 $T_{\text{eff}} = 7000 \text{ K}$   
 $\alpha = 2$   
Mode  $(\ell=1, g_{50})$

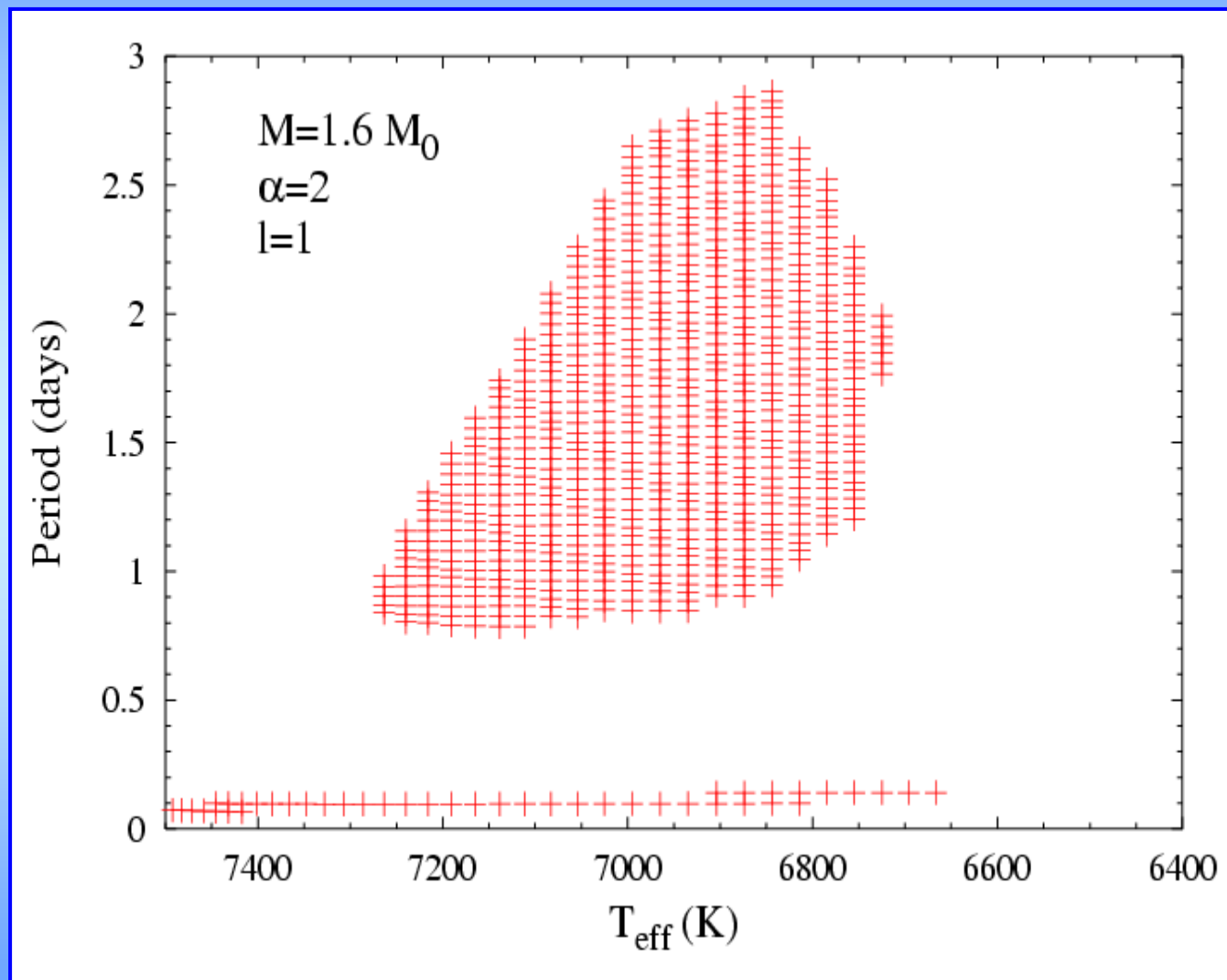
$W_{\text{FRr}}$ : Radial radiative  
flux term

$$\begin{aligned} W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p}{\rho \rho} \right\} dm \\ &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_{\text{R}} + \delta L_{\text{c}})}{dm} \right\} dm \\ &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \\ &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^* \delta p_{\text{t}}}{\rho \rho} \right\} dm \end{aligned}$$



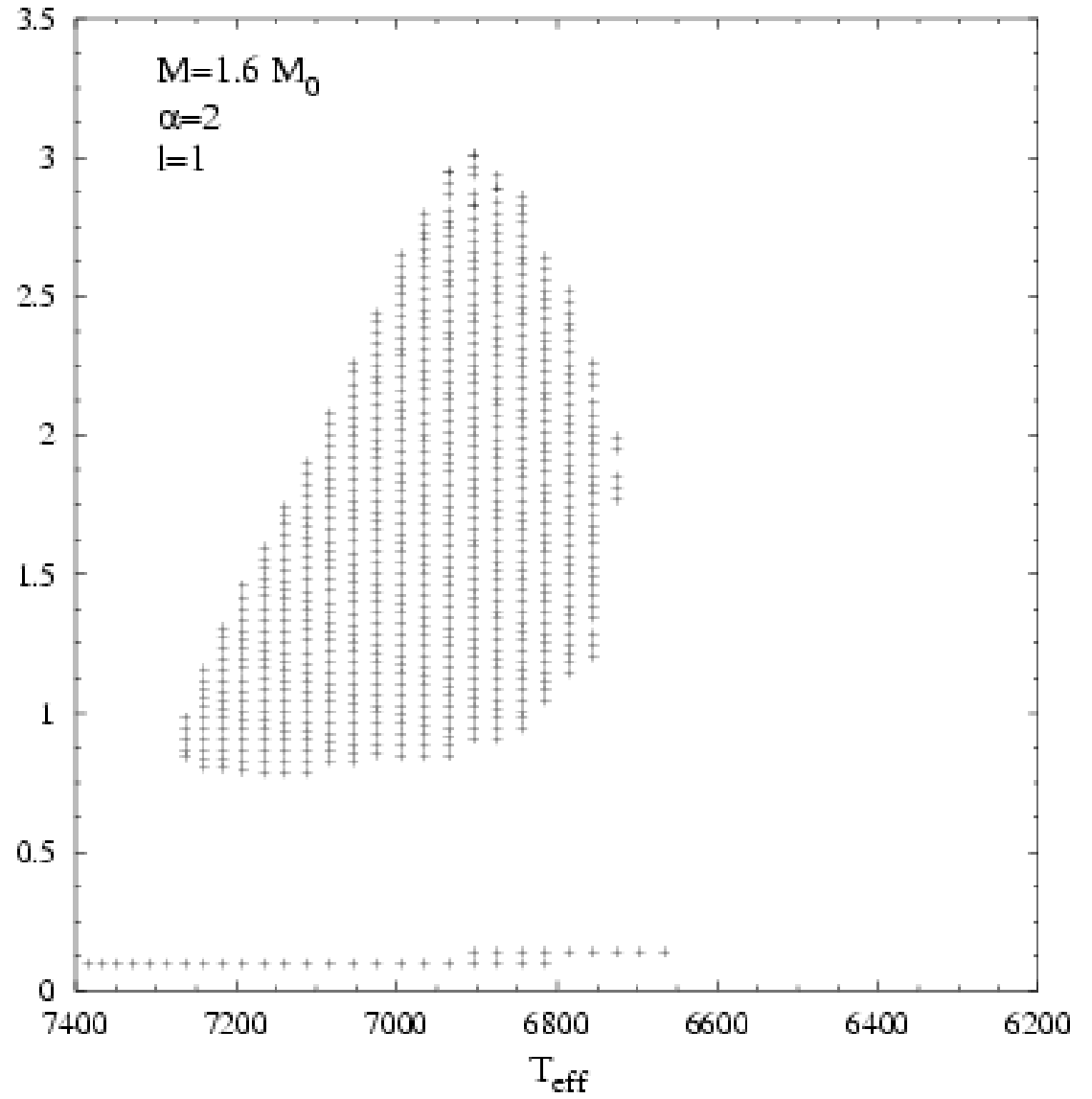
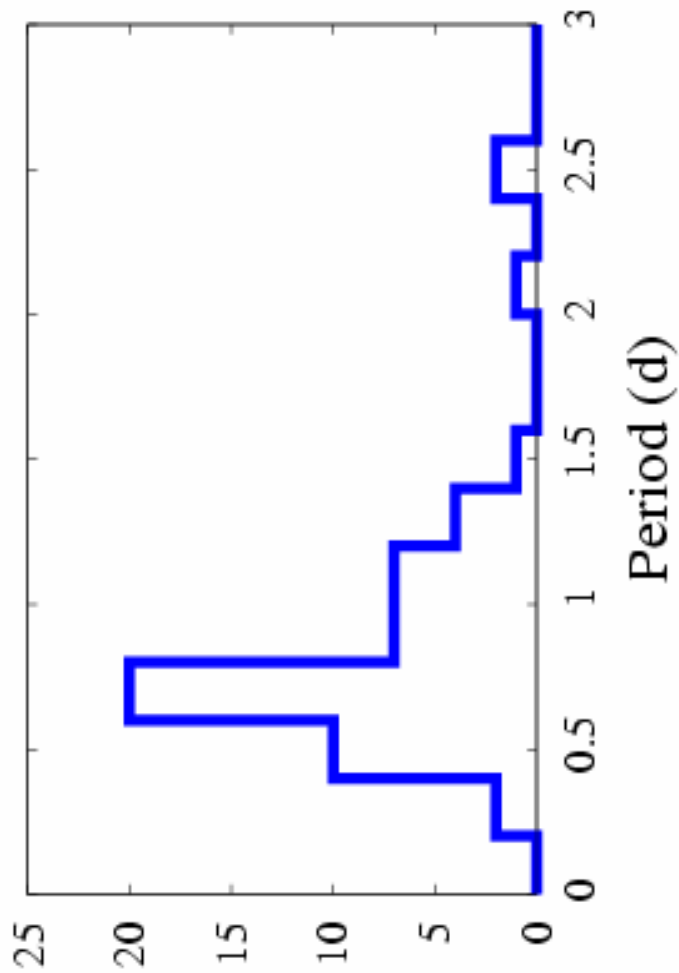
# $\gamma$ Doradus

# Unstable modes



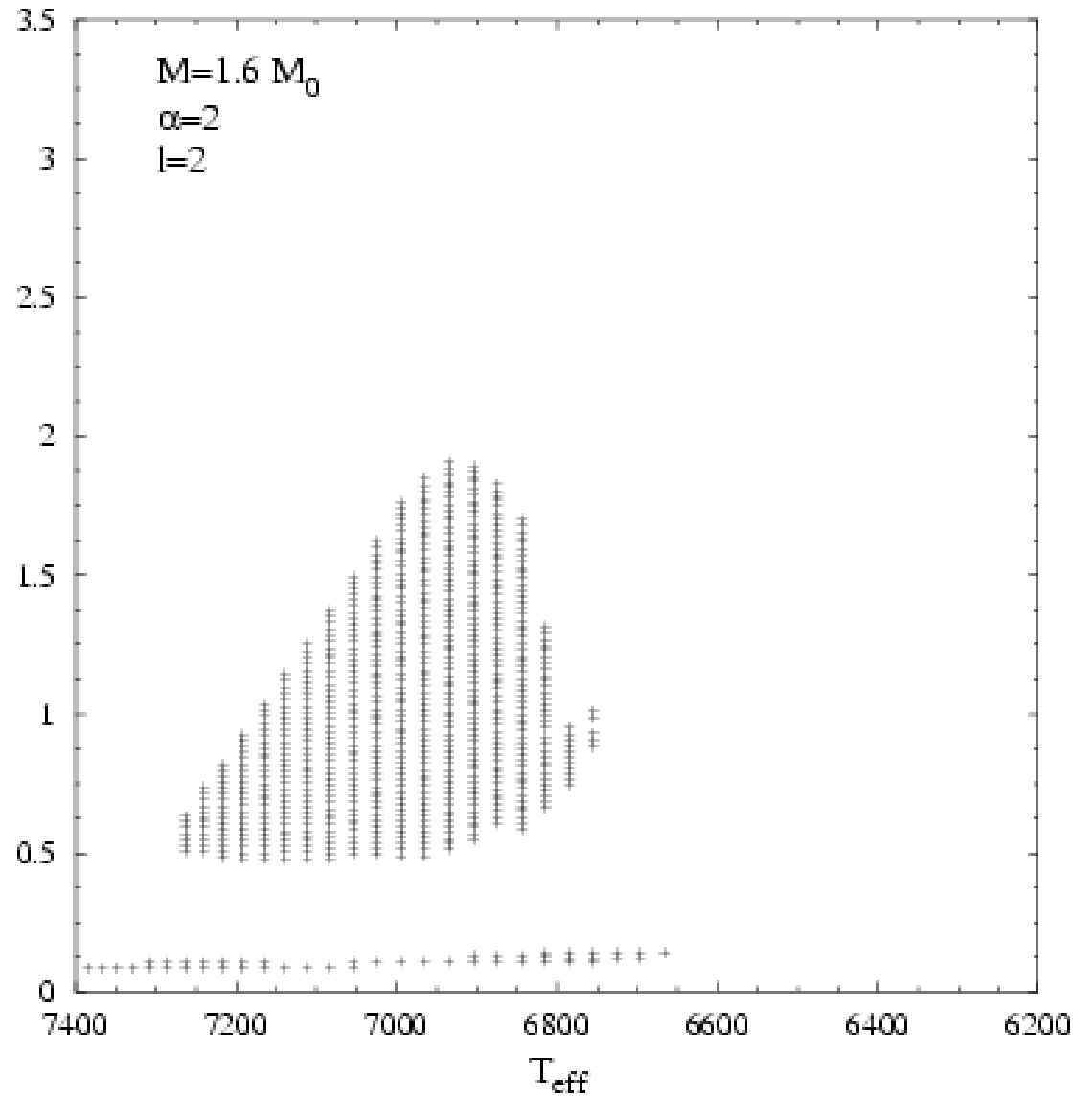
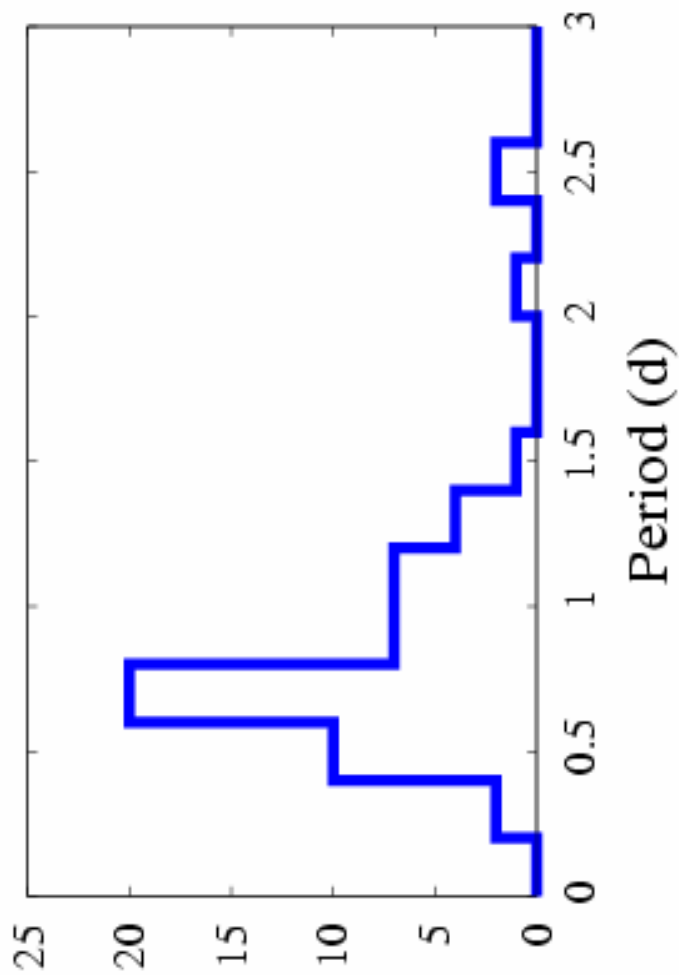
# $\gamma$ Doradus

# Unstable modes



# $\gamma$ Doradus

# Unstable modes





# Non-adiabatic stellar oscillations: utility

Excitation mechanisms

Mode identification

Solar-like oscillations

Unstable modes: growth rates

**Observations**

Stable modes: damping rates

Line-widths in the power spectrum

Stochastic excitation models

Amplitudes

# Convection – pulsation interaction

3-D hydrodynamic simulations

All motions are convective ones

In particular the p-modes  
(present in the solution)

Nordlund & Stein

Perturbative approach

Separation between convection and pulsation  
in the Fourier space of turbulence

Convective motions:  
short wave-lengths

Oscillations:  
long wave-lengths

1. Static solution without oscillations

2. Stability study of this solution

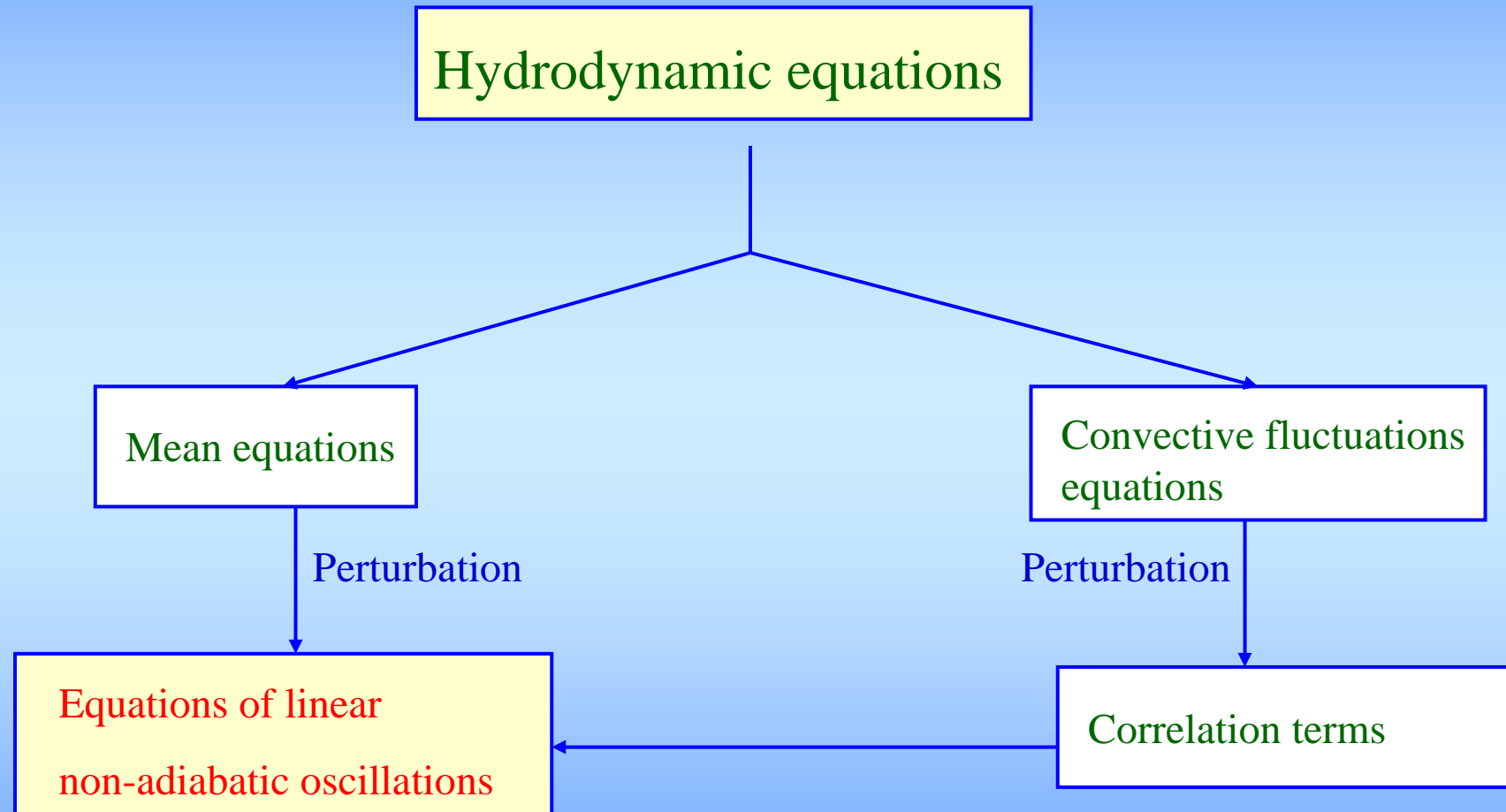
→ Perturbation → Oscillations

MLT

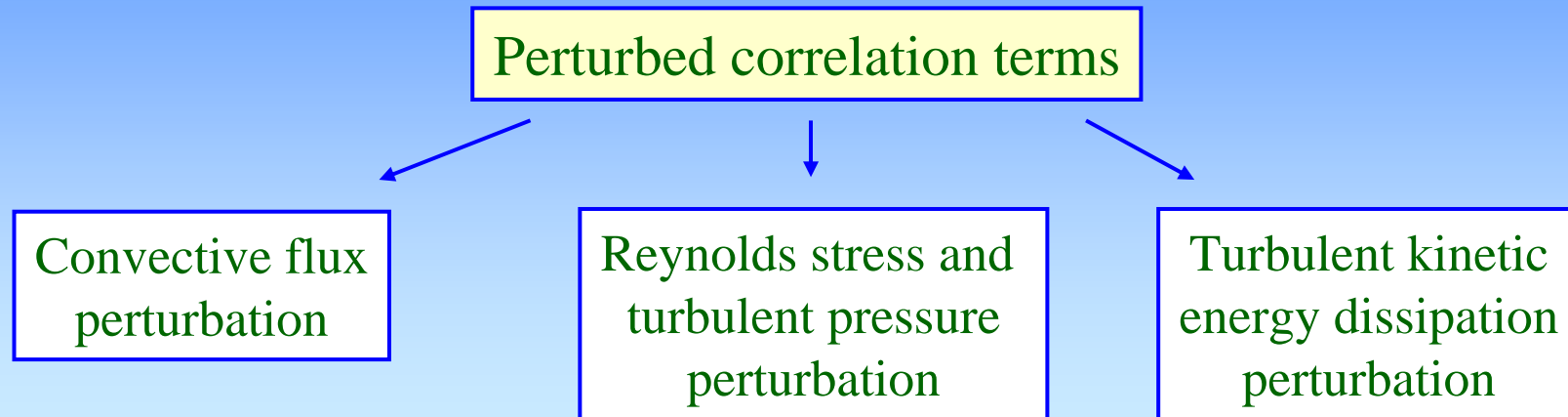
Gough's theory

Gabriel's theory

# Convection – pulsation interaction: Gabriel's theory



# Convection – pulsation interaction: Gabriel's theory



The unknown correlation terms are obtained by perturbing the convective fluctuation equations. The solutions have the form:

$$\delta(\Delta X) = \delta(\Delta X) e^{i\vec{k} \cdot \vec{r}} e^{i\sigma t}$$

Horizontal means

Integration over  $k_\theta, k_\phi$  with  $k_\theta^2 + k_\phi^2 = A k_r^2$

Separation of the variables in term of spherical harmonics

Radial modes

Non-radial modes

# Convection – pulsation interaction: Work integral

Radiative luminosity

Convective luminosity

$$\begin{aligned}
 W_{\text{tot}} &= - \int_0^m \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p}{\rho} \right\} dm \\
 &= - \int_0^m (\Gamma_3 - 1) \Re \left\{ \frac{1}{\sigma} \frac{\delta \rho^*}{\rho} \frac{d(\delta L_R + \delta L_c)}{dm} \right\} dm \\
 &\quad - \int_0^m \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p_t}{\rho} \right\} dm \\
 &\quad + \int_0^m \frac{3}{2} (\Gamma_3 - 1) \Im \left\{ \frac{\delta \rho^*}{\rho} \frac{\delta p_t}{\rho} \right\} dm
 \end{aligned}$$

Turbulent pressure

Turbulent kinetic energy dissipation

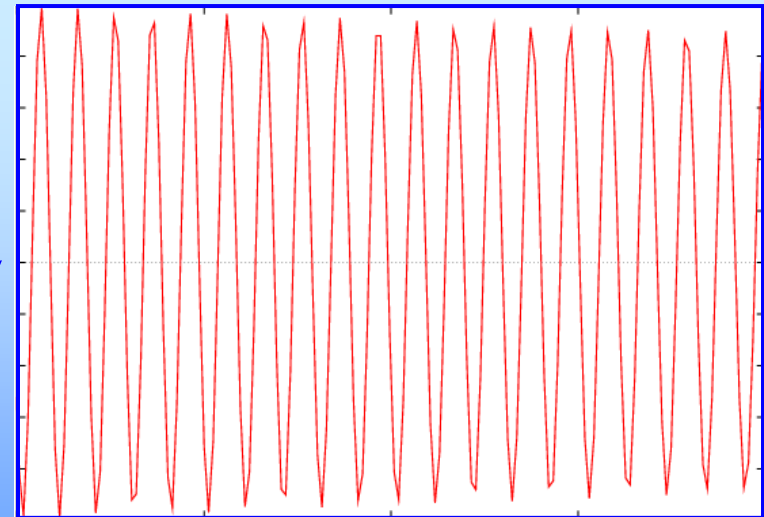
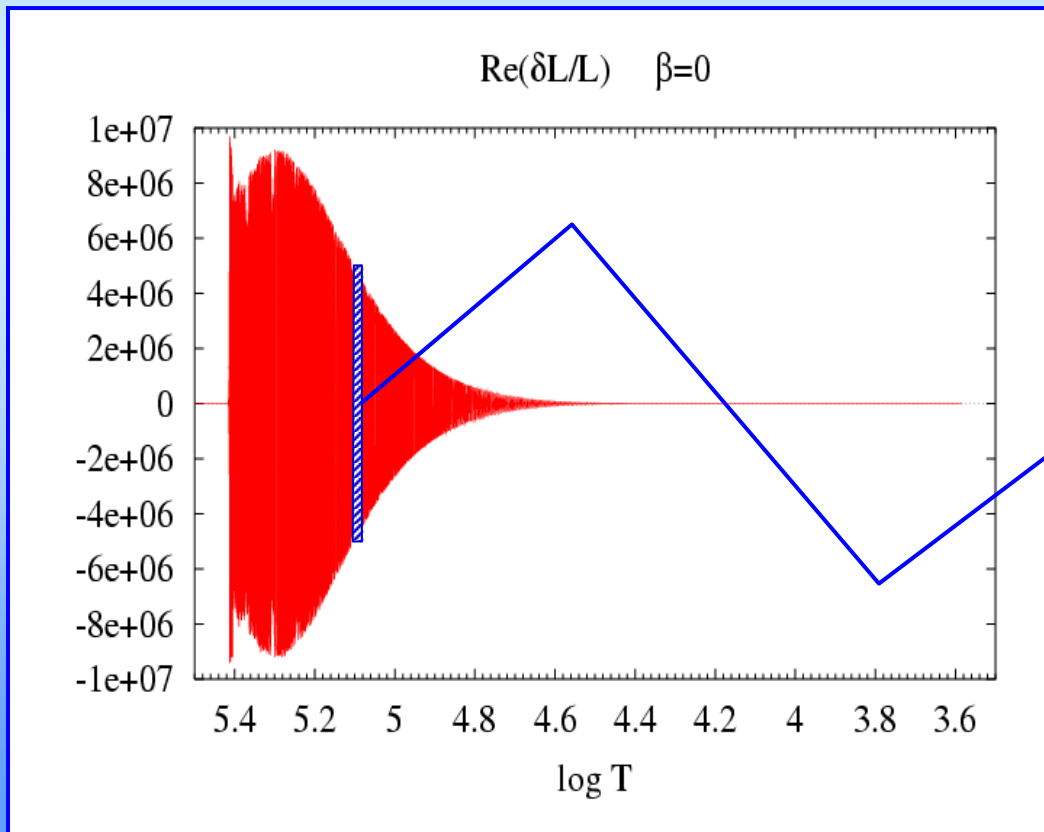
# Solar-type oscillations

Difficulties :

1. Treatment in the efficient part of convection

Local treatment

Very short wave-length oscillations  
of the eigenfunctions



# Solar-type oscillations

Difficulties:

1. Treatment in the efficient part of convection

Origin of the problem:

$$i\sigma T \delta s = -\frac{d\delta L_C}{dm}$$

$$\delta L_C \cong \delta L_1 + \frac{L_C}{i\sigma\tau_C} \frac{d\delta s}{ds}$$

$$\frac{l^2}{i\sigma\tau_C} \frac{d^2 \delta s}{dr^2} + i\sigma\tau_C \delta s + (\dots) = 0$$

$$\delta s = \dots + c_1 \exp(-i\sigma\tau_c r/l) + c_2 \exp(i\sigma\tau_c r/l)$$

Wavelength much shorter than the mixing-length !

# Solar-type oscillations

Difficulties:

1. Treatment in the efficient part of convection

Solutions

Non-local (Balmforth 1992)

Local (Gabriel 2003)

Introduction of new free parameters



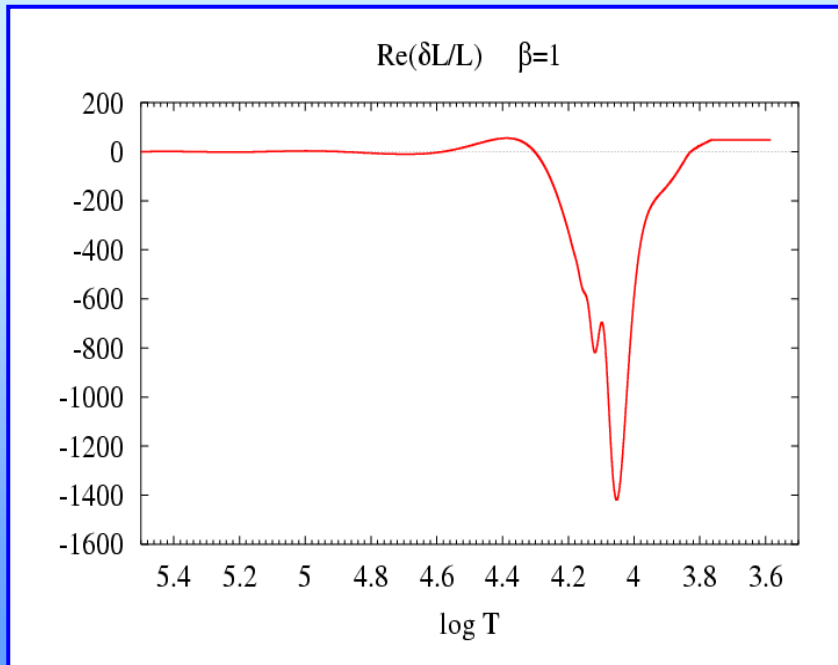
# Solar-type oscillations

Difficulties:

1. Treatment in the efficient part of convection

Solutions

Local (Gabriel 2003)



$$\frac{\Delta s}{\tau_c} = \frac{1}{\overline{\rho T}} \left[ \overline{\rho \varepsilon_2} - \overline{\rho \varepsilon_2} + \overline{\rho T \nabla s \cdot \vec{V}} - \overline{\rho T \nabla s \cdot \vec{V}} \right] - \left( \nabla \cdot \vec{F}_R - \overline{\nabla \cdot \vec{F}_R} \right)$$

$$\delta \left( \frac{\Delta s}{\tau_c} \right) = \frac{\Delta s}{\tau_c} \left( (1 + \beta \sigma \tau_c) \frac{\delta \Delta s}{\Delta s} - \frac{\delta \tau_c}{\tau_c} \right)$$

$$\frac{l^2}{(i + \beta) \sigma \tau_c} \frac{d^2 \delta s}{dr^2} + i \sigma \tau_c \delta s + (\dots) = 0$$

# Solar-type oscillations

Difficulties:

2. Treatment of turbulent pressure perturbation

**Increases the order  
of the system**

**Very stiff problem  
at the boundaries**



**Numerical instabilities**

# Solar-type oscillations: confrontation to observations

## Models of stochastic excitation

- The Sun is a vibrationally stable oscillator.
- Excitation of the mode is due to stochastic forcing coming from turbulent convective motions.

Non-adiabatic models

Stochastic models

Damping rate:  $\eta$

Acoustical noise generation rate:  $P$

Theory

$$V_s = \sqrt{\frac{P}{2\eta I}}$$

Velocity

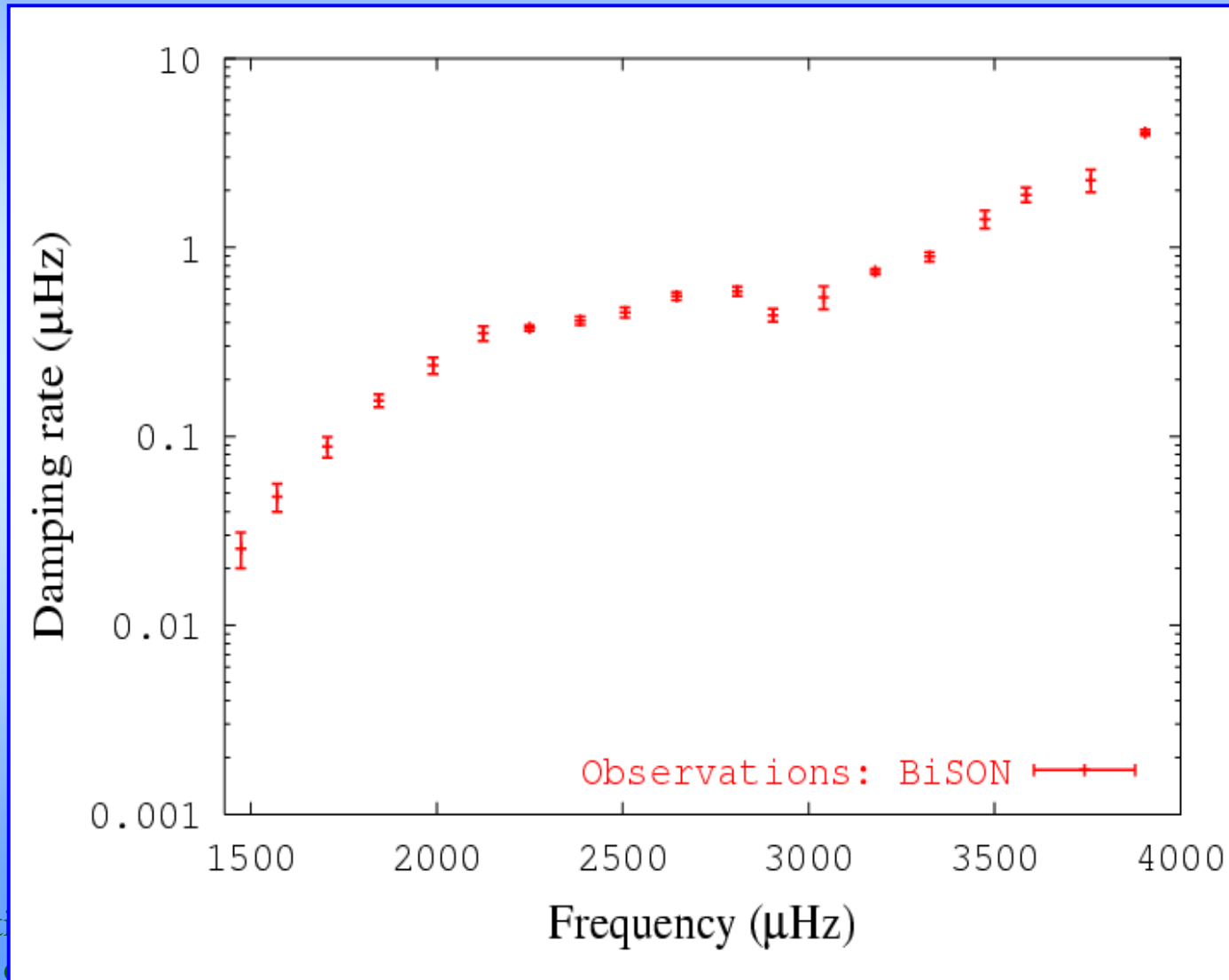
Line-widths

Observations

Observed amplitudes

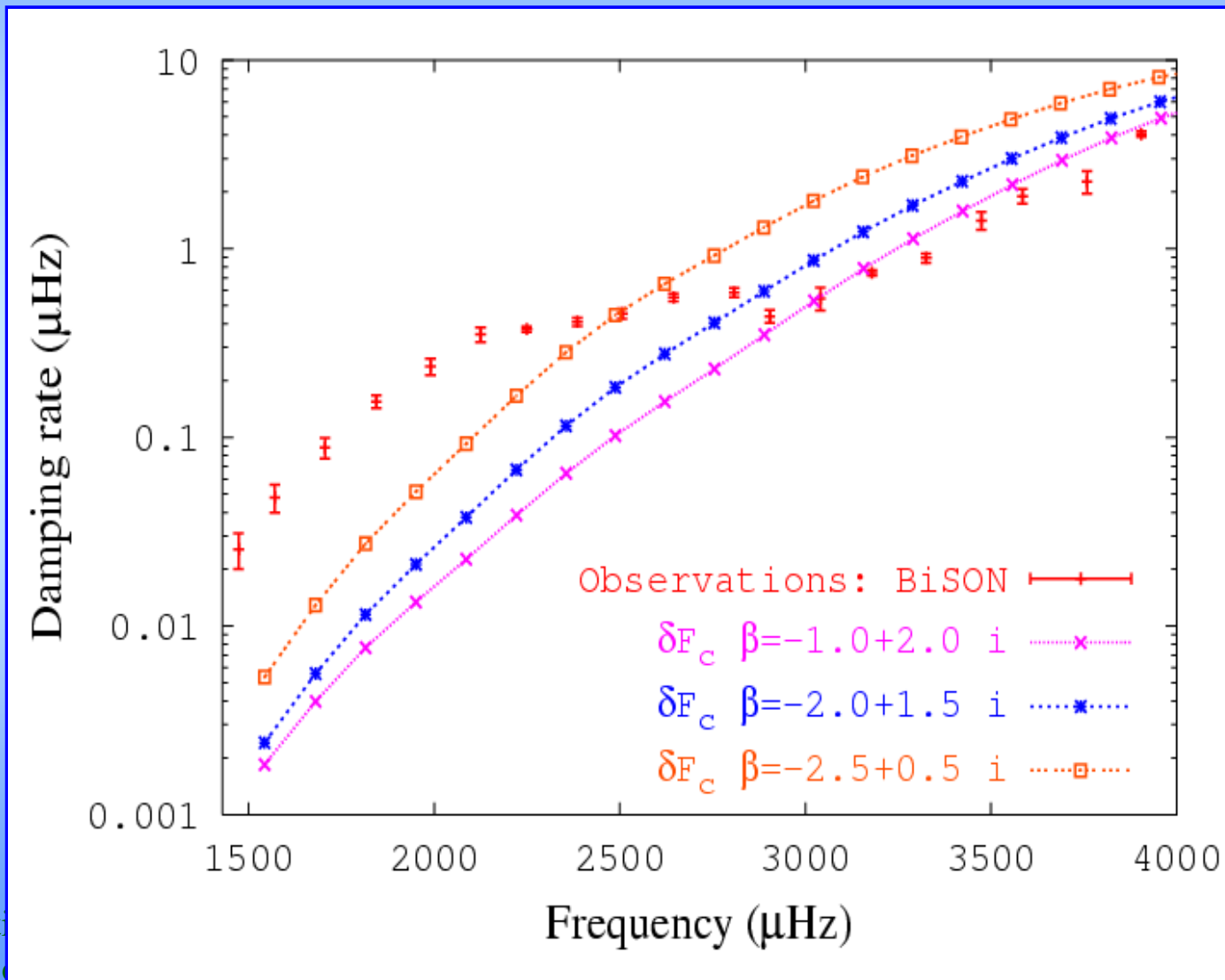
## Solar-type oscillations: confrontation to observations

Theoretical damping rates  $\leftrightarrow$  line-widths observed by BiSON (Chaplin et al. 1997)



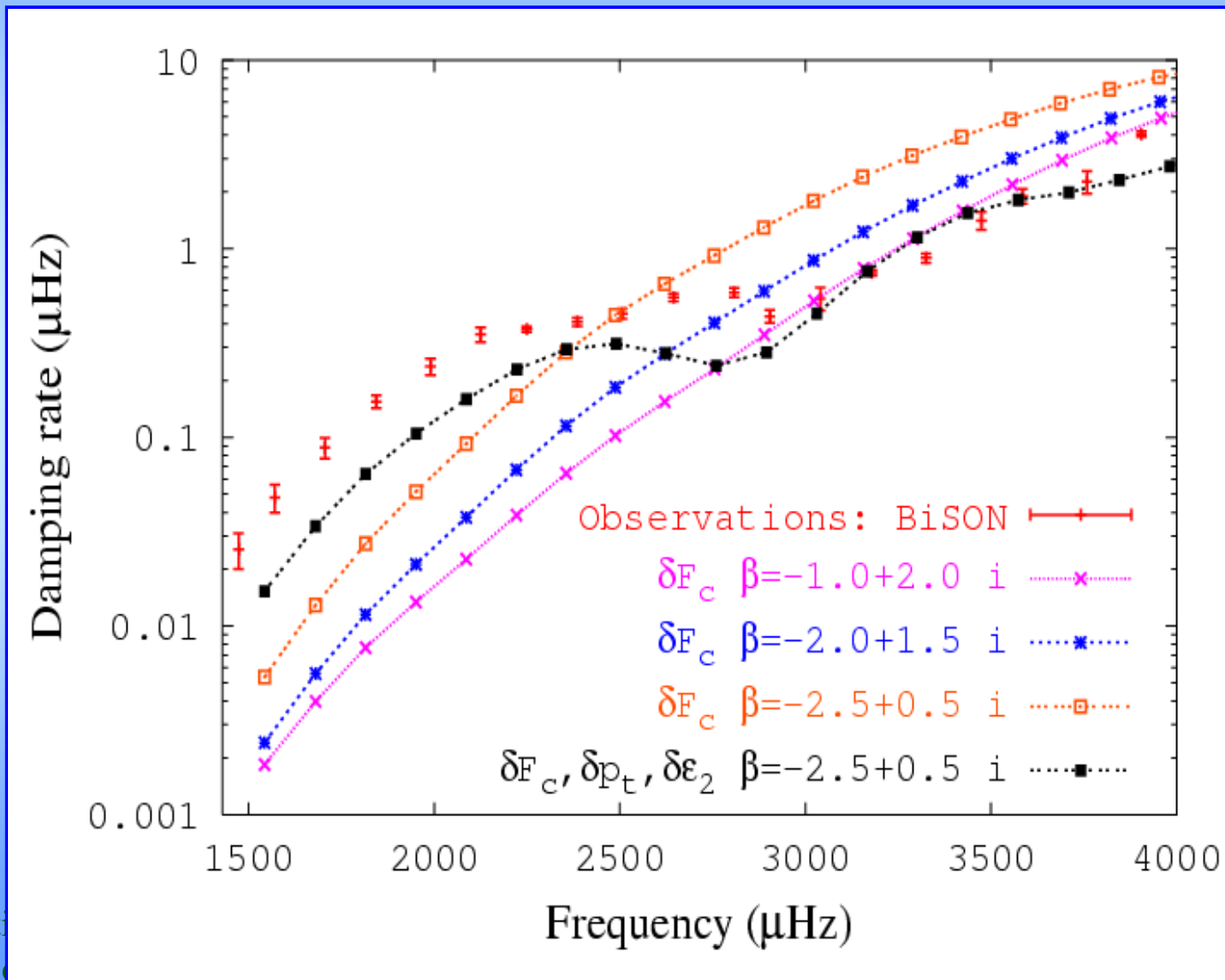
# Solar-type oscillations: confrontation to observations

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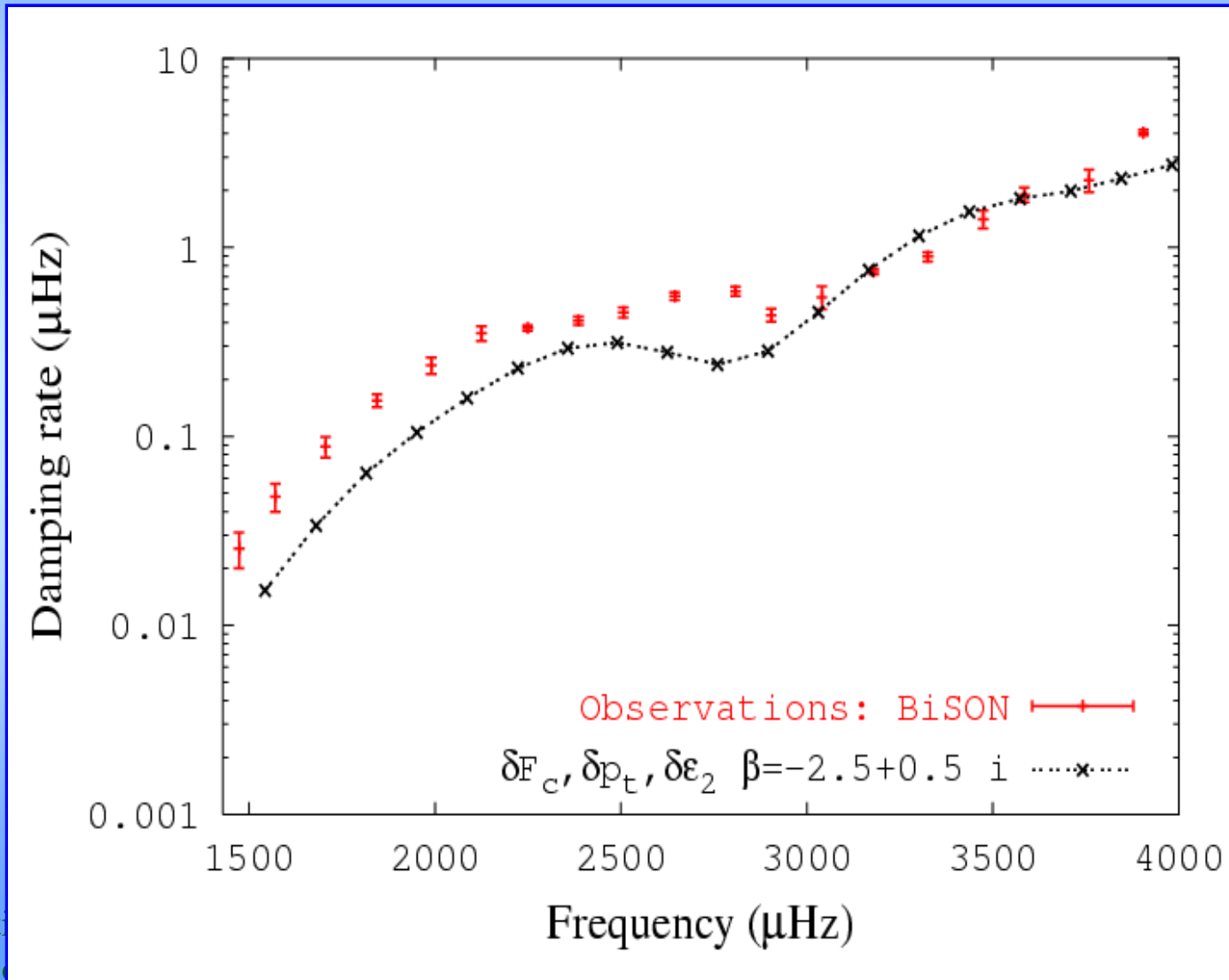
# Solar-type oscillations: confrontation to observations

Theoretical damping rates  $\leftrightarrow$  line-widths observed by BiSON (Chaplin et al. 1997)



# Solar-type oscillations: confrontation to observations

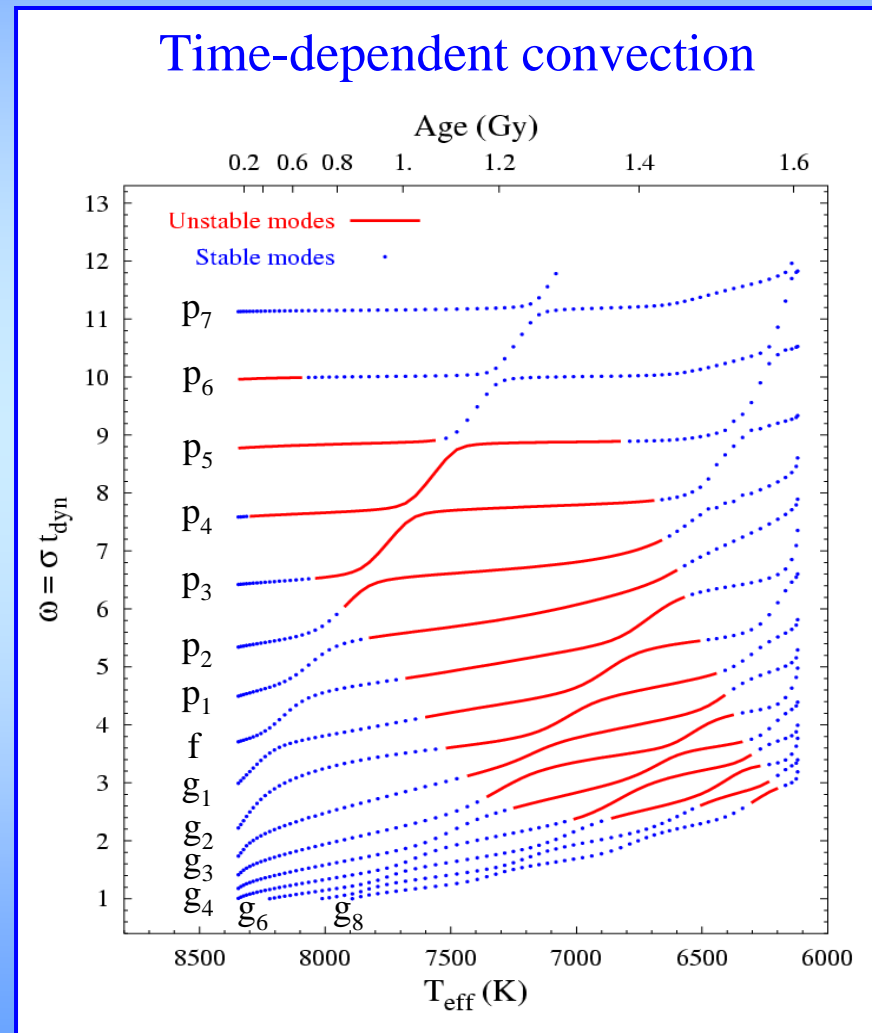
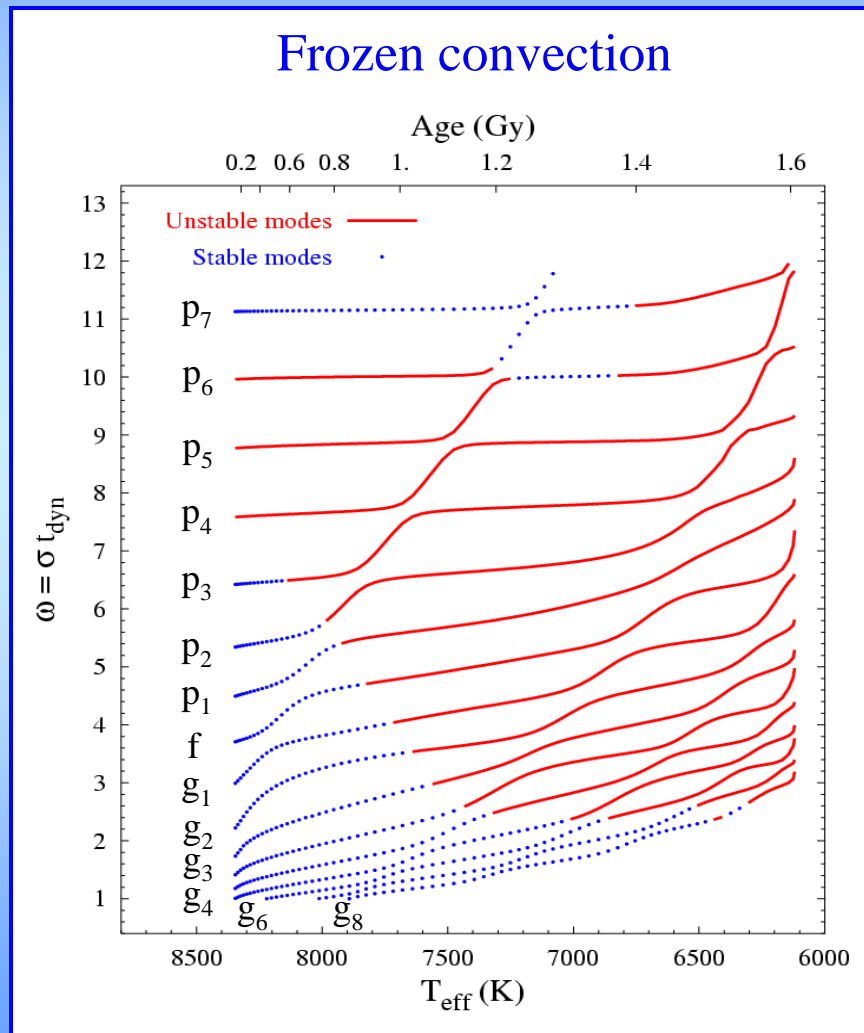
Theoretical damping rates  $\leftrightarrow$  line-widths observed by BiSON (Chaplin et al. 1997)



# $\delta$ Scuti

## Stables and unstable modes

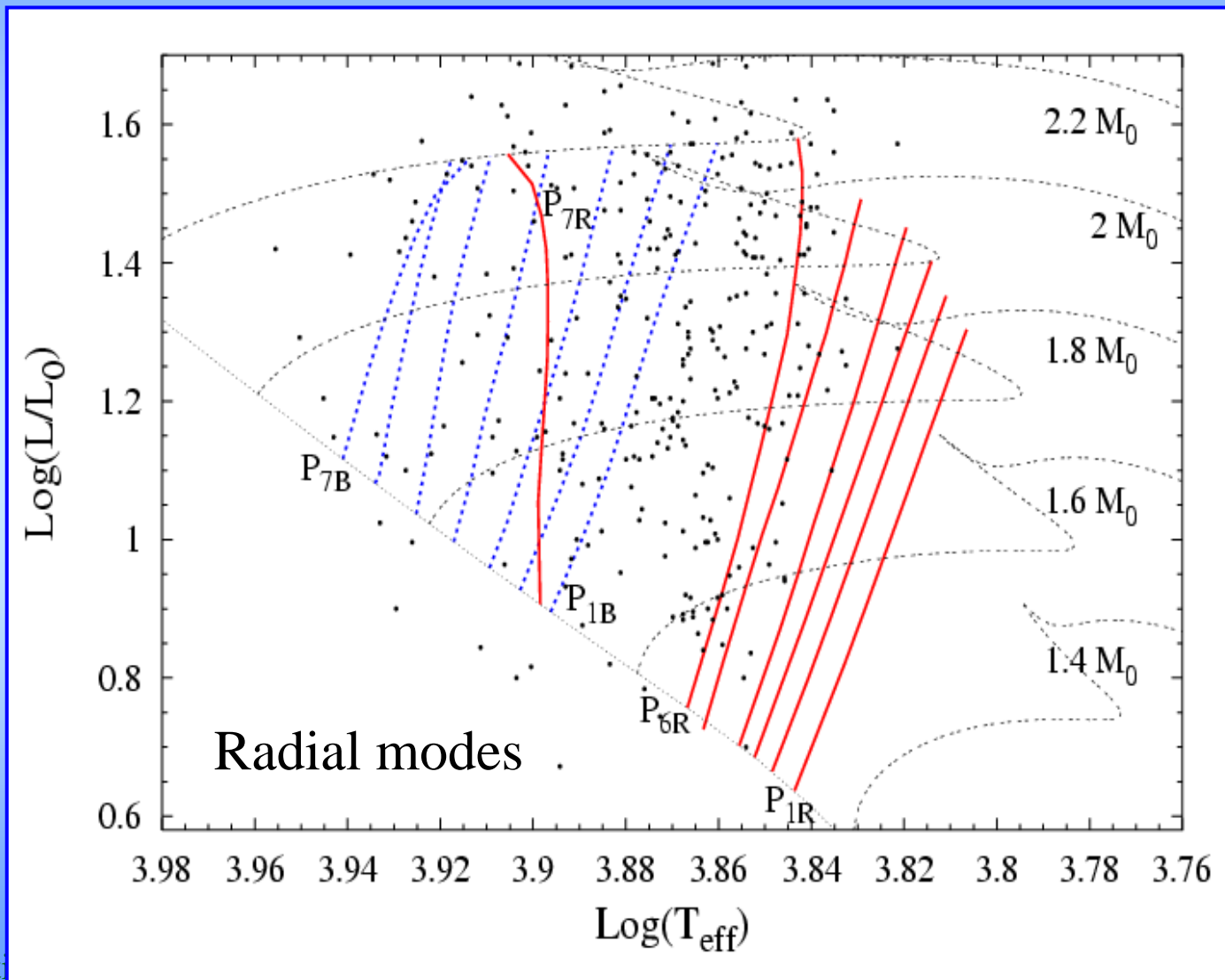
$$\ell = 2 - 1.8 M_0 - \alpha = 1.5$$





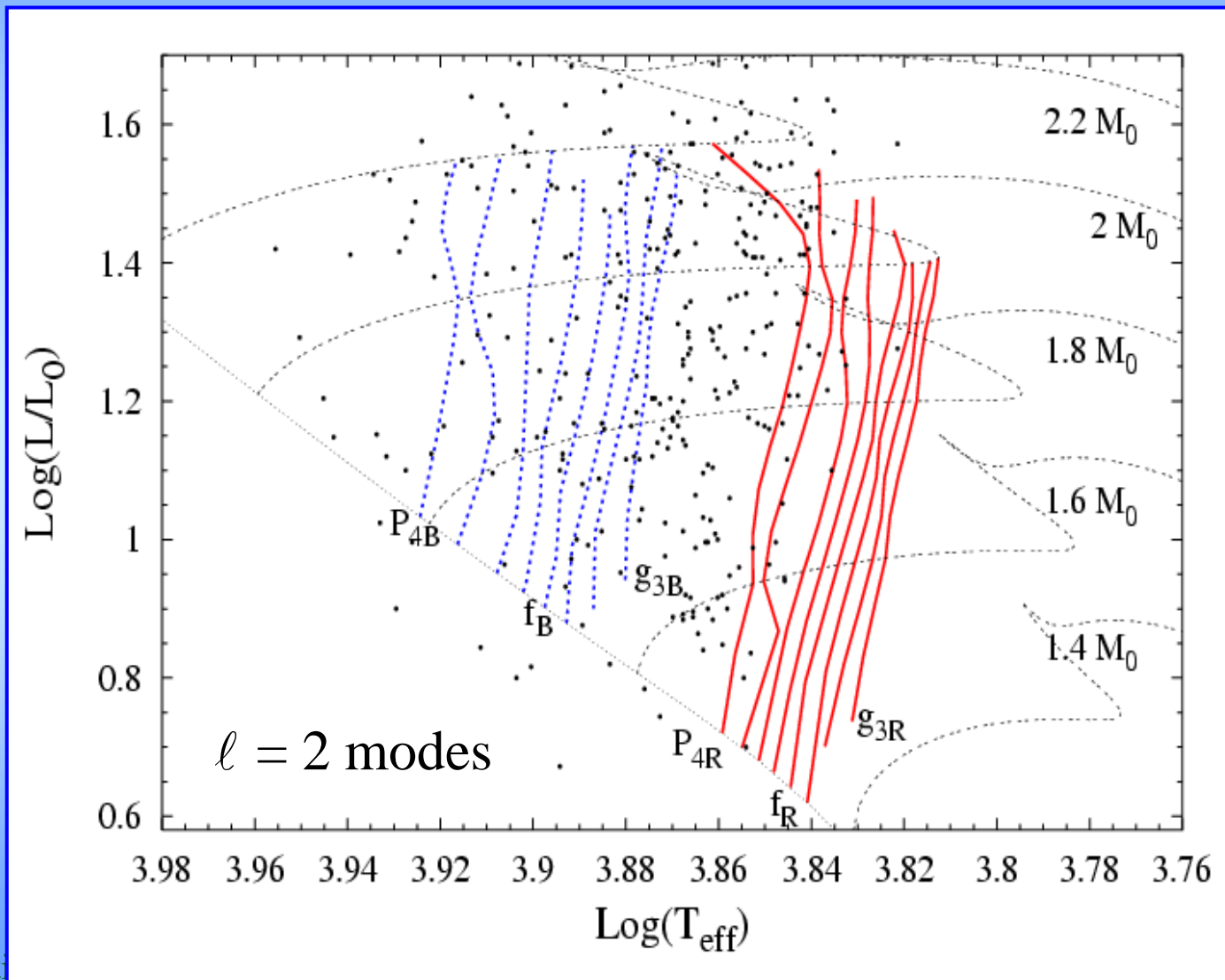
# $\delta$ Scuti

# Instability strips

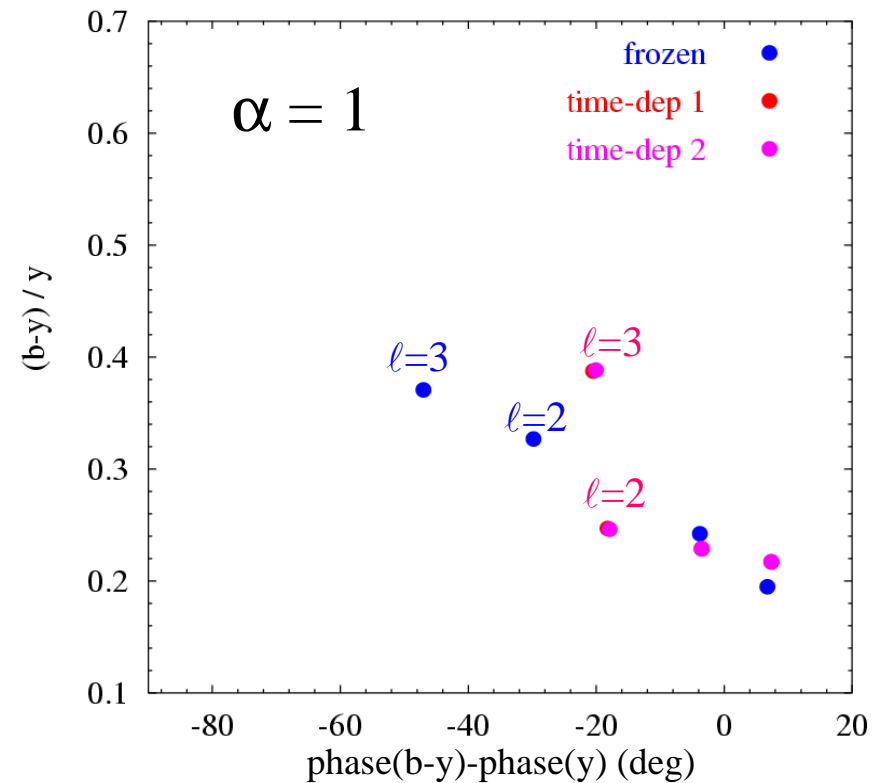
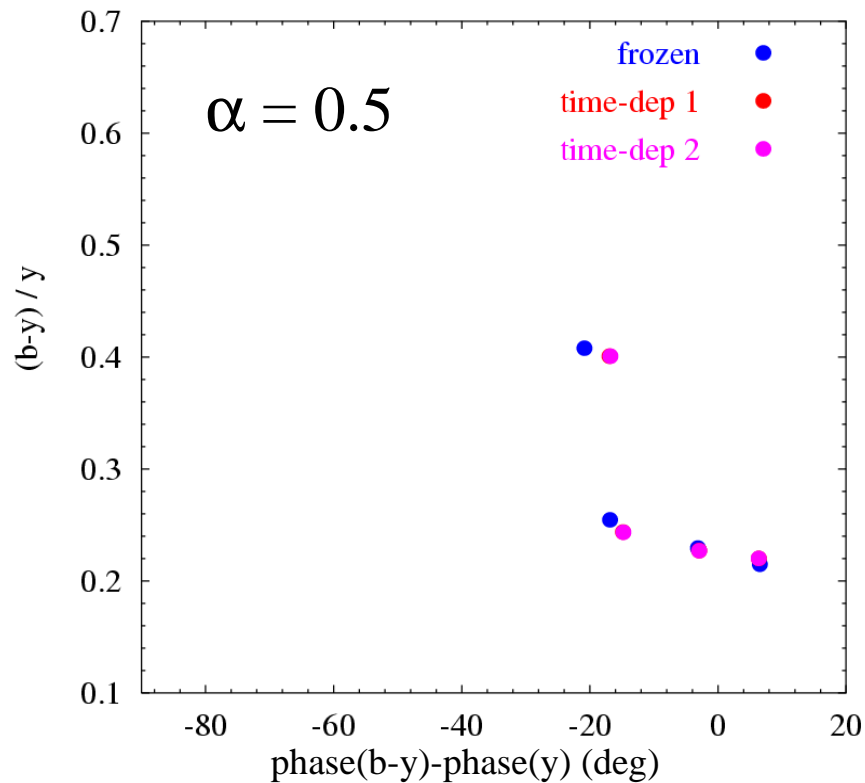


$\delta$  Scuti

Instability strips



Amplitude ratios vs. phase difference - Stroemgren photometry



## Conclusions

# Convection-pulsation interaction in solar-like stars

## Theoretical difficulties

## Confrontation to observations

### Efficient part of convective envelope

Local treatment  
→ Spatial oscillations  
of the eigenfunctions  
→ Introduction of a free  
parameter  $\beta$  in the perturbation  
of the closure equations

### Perturbation of turbulent pressure

→ Numerical instabilities

### Damping rates

### Line-widths

### Stochastic excitation

### Amplitudes

We found a model fitting the  
observed damping rates but ...



## Oscillations de type solaire

Mécanisme d'excitation : Excitation stochastique

↳ Oscillateur vibrationnellement stable  
forcé stochastiquement par la convection

Taux de d'amortissement confrontables  
aux observations (largeurs de raie)

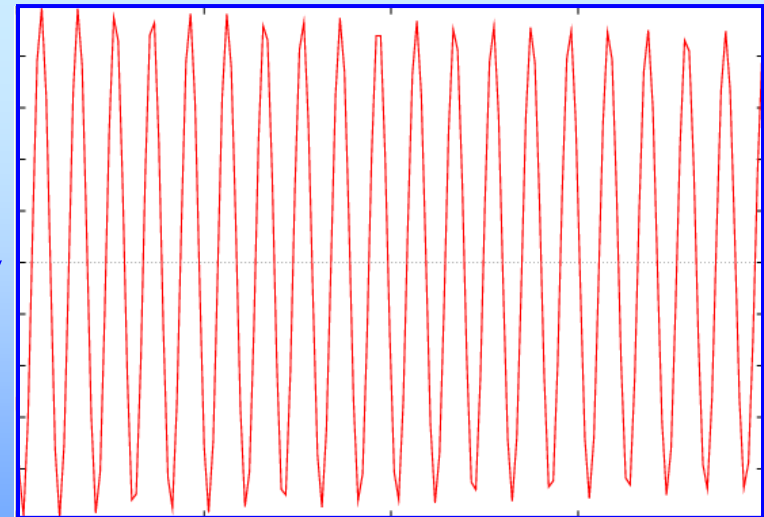
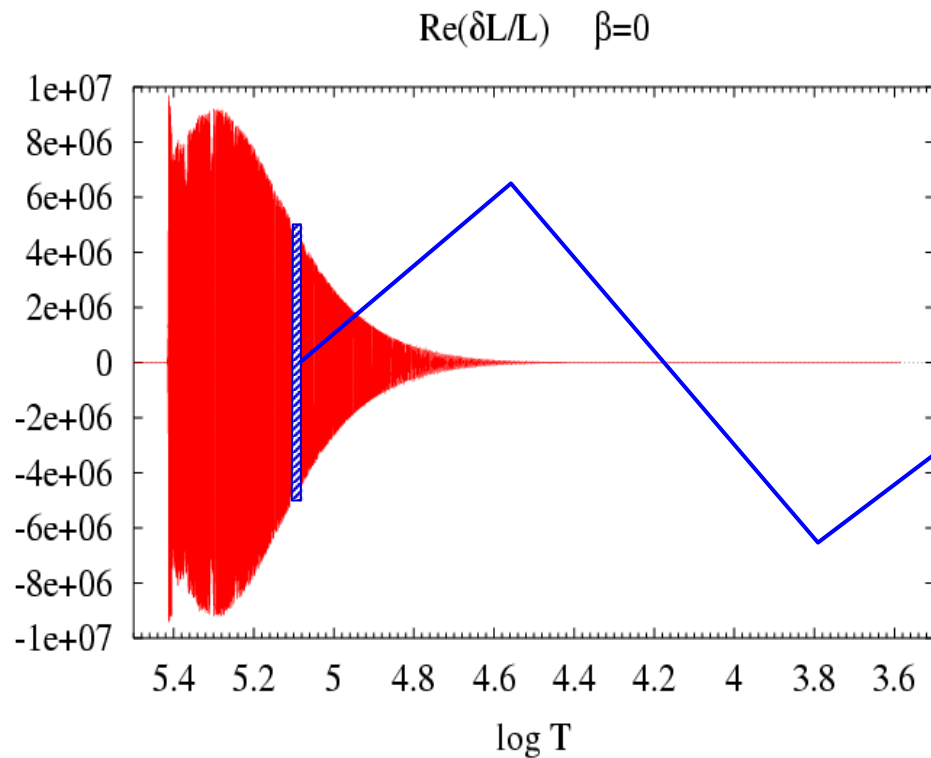
↳ Contrainte sur les modèles non-adiabatiques  
d'interaction convection-oscillations

# Oscillations de type solaire

Difficulté des modèles non-adiabatiques:  
interaction convection-oscillations

Grande enveloppe convective  
Convection efficace ( $t_c \gg t_p$ )

Oscillations spatiales non-physiques  
des fonctions propres



# Oscillations de type solaire

Difficulté des modèles non-adiabatiques:  
interaction convection-oscillations

Grande enveloppe convective  
Convection efficace ( $t_c \gg t_p$ )

Oscillations spatiales non-physiques  
des fonctions propres

## Solutions

Non-locales (Balmforth 1992)

Locales (Gabriel 2003)

Introduction de paramètres  
libres supplémentaires



# Oscillations de type solaire

Difficulté des modèles non-adiabatiques:  
interaction convection-oscillations

Grande enveloppe convective  
Convection efficace ( $t_c \gg t_p$ )

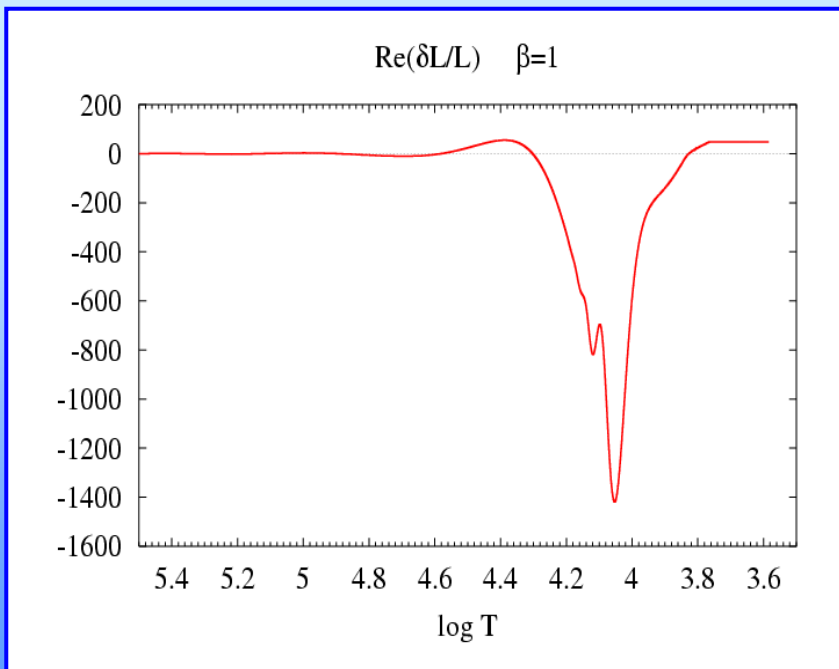
Oscillations spatiales non-physiques  
des fonctions propres

## Solutions

Locales (Gabriel 2003)

$$\frac{\Delta s}{\tau_c} = \frac{1}{\rho T} \left[ \rho \varepsilon_2 - \overline{\rho \varepsilon_2} + \rho T \nabla s \cdot \vec{V} - \overline{\rho T \nabla s \cdot \vec{V}} \right] - \left( \nabla \cdot \vec{F}_R - \overline{\nabla \cdot \vec{F}_R} \right)$$

$$\delta \left( \frac{\Delta s}{\tau_c} \right) = \frac{\Delta s}{\tau_c} \left( (1 + \beta \sigma \tau_c) \frac{\delta \Delta s}{\Delta s} - \frac{\delta \tau_c}{\tau_c} \right)$$



# Conclusions

Mécanismes d'excitation

Amplitudes et phases  
photométriques

Bandes d'instabilité

Astérosismologie  
non-adiabatique

Identification  
des modes

Astérosismologie  
"classique" (fréquences)

# Conclusions

Mécanismes d'excitation

Amplitudes et phases  
photométriques

Bandes d'instabilité

Astérosismologie  
non-adiabatique

Identification  
des modes

$\beta$  Cephei

Mécanisme  $\kappa$  (Fe)

Contraintes sur la  
métallicité

Marche  
très bien

Modes p

SPB

Idem

Idem

OK mais effet  
de la rotation ?

Modes g

# Conclusions

## Mécanismes d'excitation

## Amplitudes et phases photométriques

Bandes d'instabilité

Astérosismologie non-adiabatique

Identification des modes

$\delta$  Scuti

Frontière bleue:  
mécanisme  $\kappa$  (HeII)  
Frontière rouge:  
convection

Modes p

Contraintes sur les  
modèles de convection  
et d'interaction  
convection - pulsation

OK mais reste  
difficile

Effet de la  
rotation ?

$\gamma$  Dor

Bon accord  
avec  $\alpha$  solaire

Modes g

Idem

Nettement mieux  
avec l'interaction  
convection-pulsation

## Conclusions

### Mécanismes d'excitation

### Amplitudes et phases photométriques

### Type solaire

Modes p

**Excitation stochastique,  
modélisation difficile  
de l'interaction  
convection – oscillations**

**Amplitudes données par  
les modèles d'excitation  
stochastique.**

**Grands espoirs futurs :**



## Oscillations de type solaire

Difficulté des modèles non-adiabatiques:  
interaction convection-oscillations:

Solutions

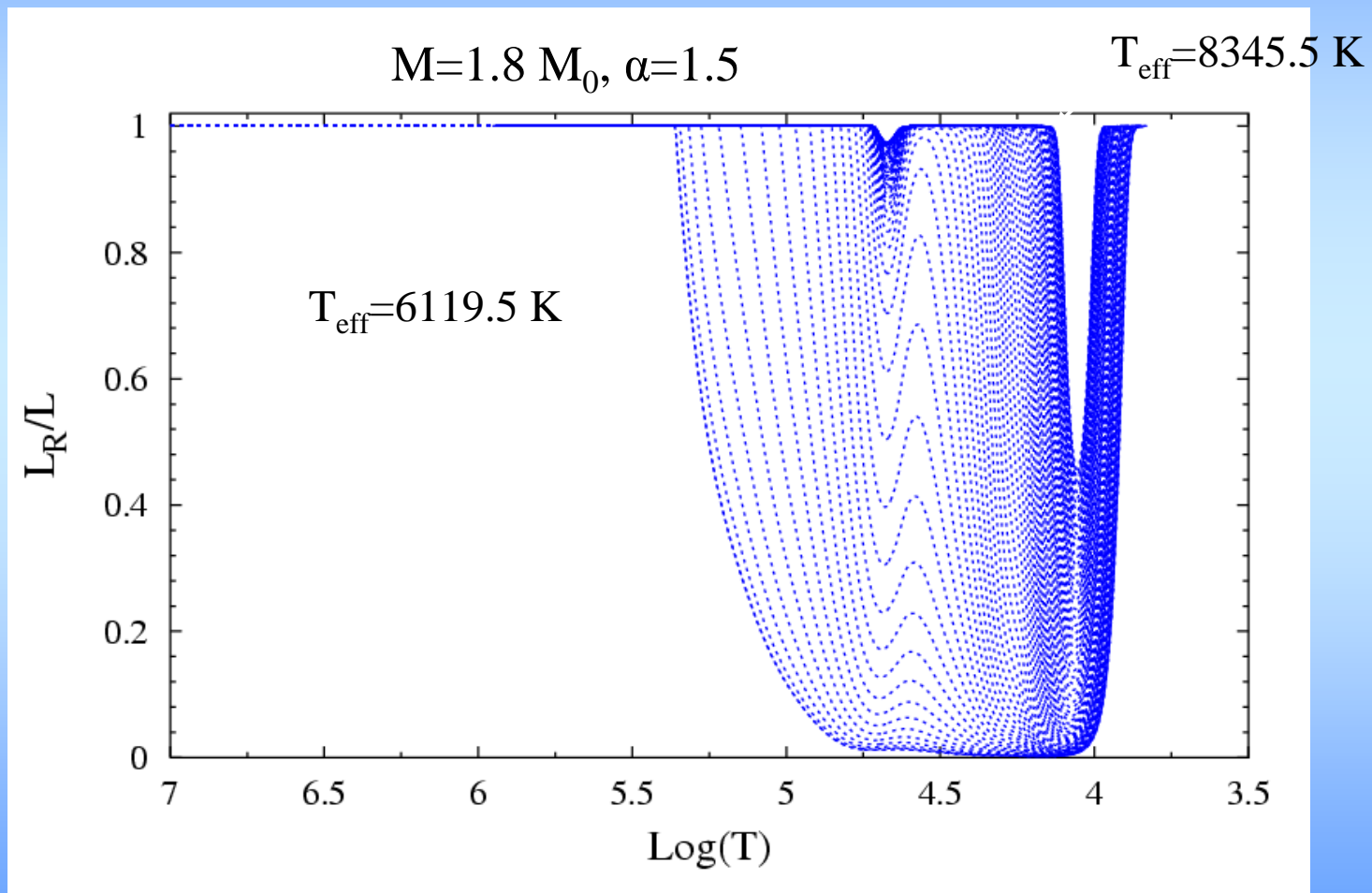
```
graph TD; A[Solutions] --> B[Non-locales (Balmforth 1992)]; A --> C[Locales (Gabriel 2003)];
```

Non-locales (Balmforth 1992)

Locales (Gabriel 2003)

$\delta$  Scuti

Taille de l'enveloppe convective pour différentes températures effectives

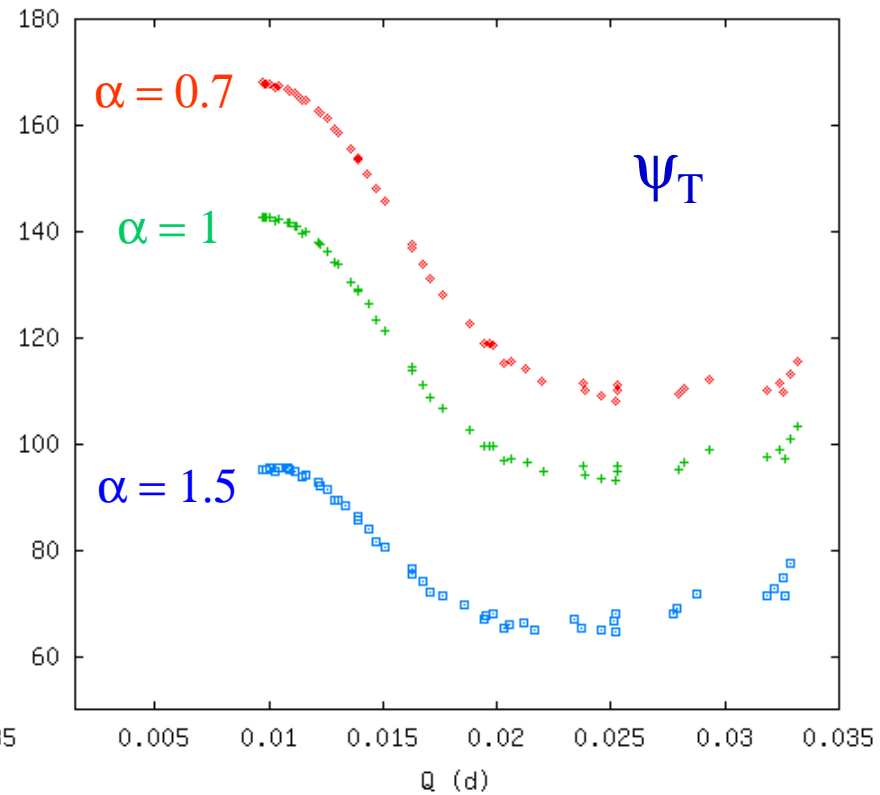
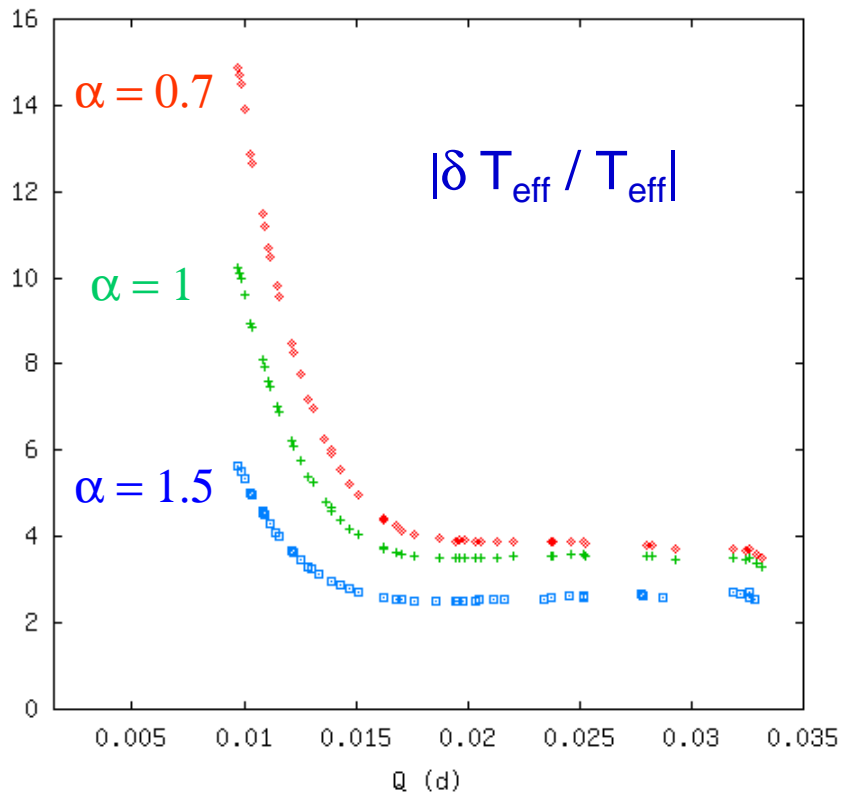


$\delta$  Scuti

# Amplitudes et phases photométriques

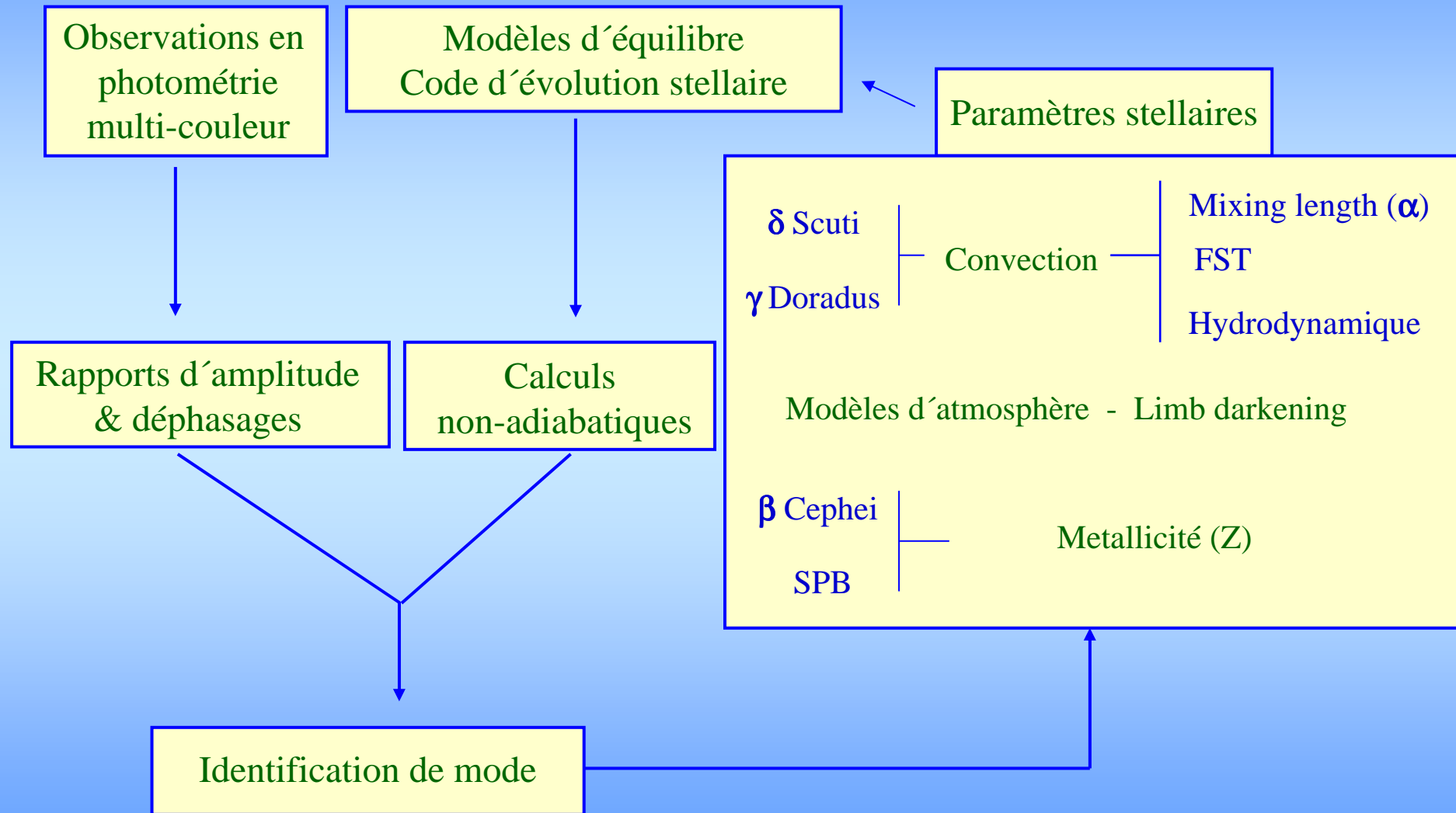
Sensibilité à la structure de l'enveloppe convective

Longueur de mélange - différents  $\alpha$  - convection gelée





# Photométrie multi-couleur et astérosismologie non - adiabatique



$\delta$  Scuti

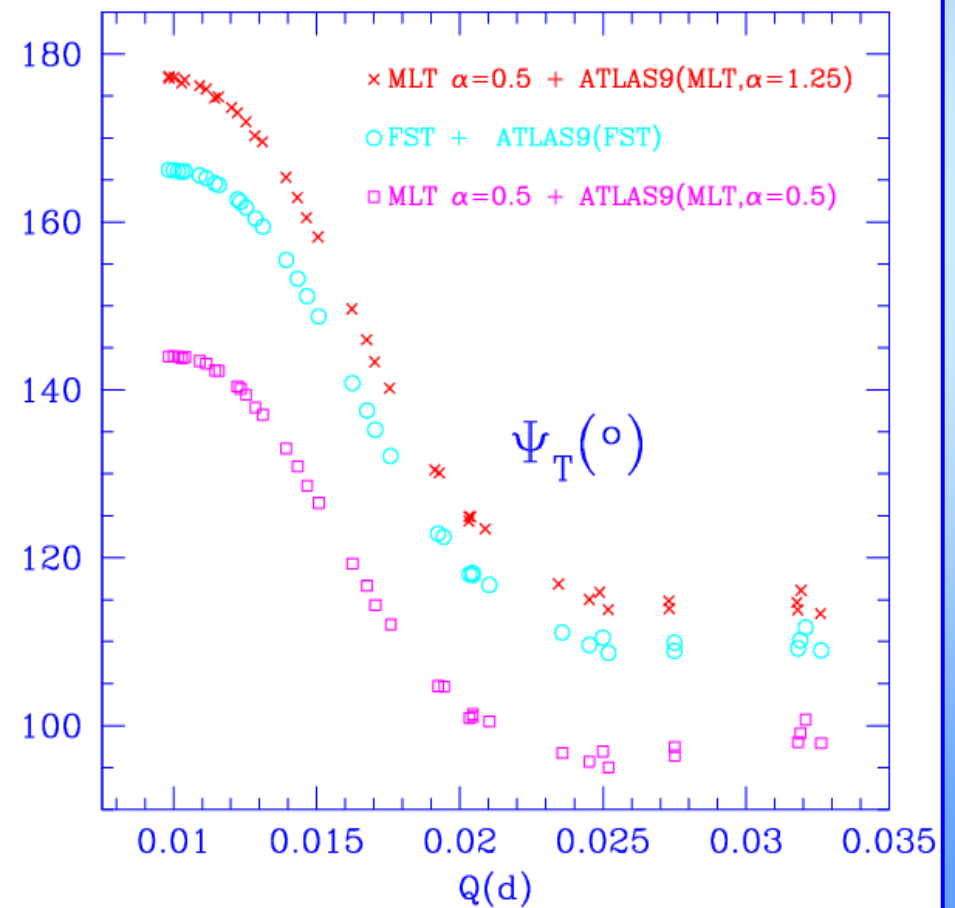
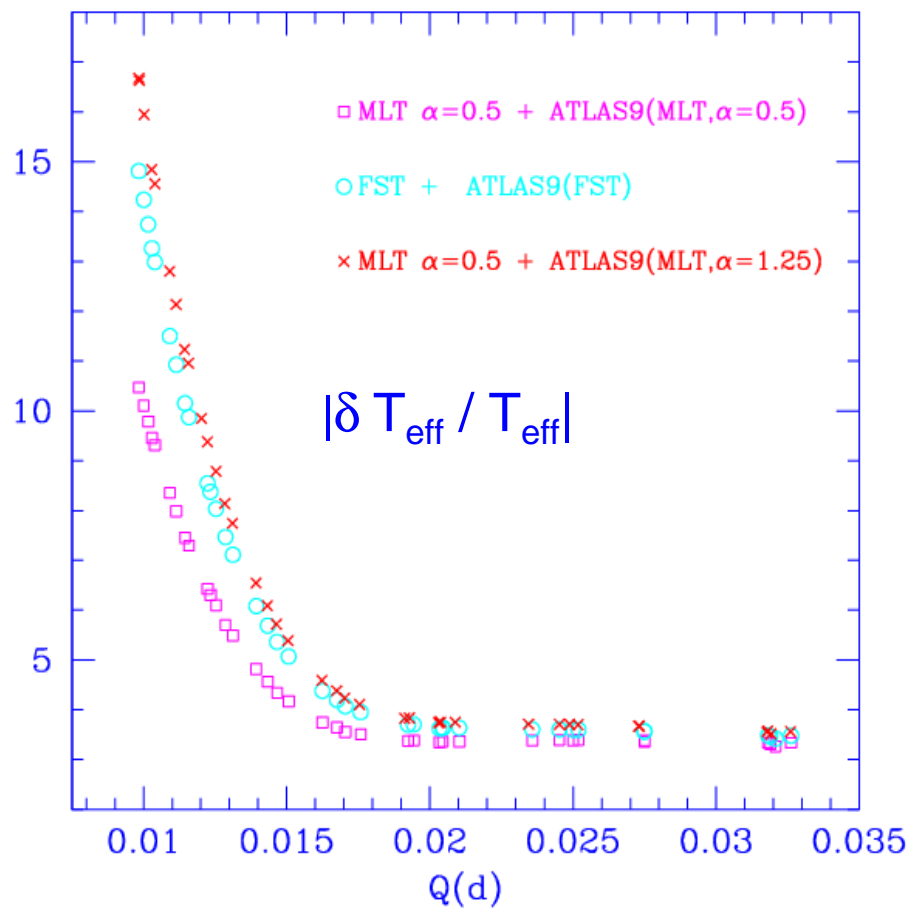
# Amplitudes et phases photométriques

Sensibilité à la structure de l'enveloppe convective

Full Spectrum of Turbulence



Longueur de mélange

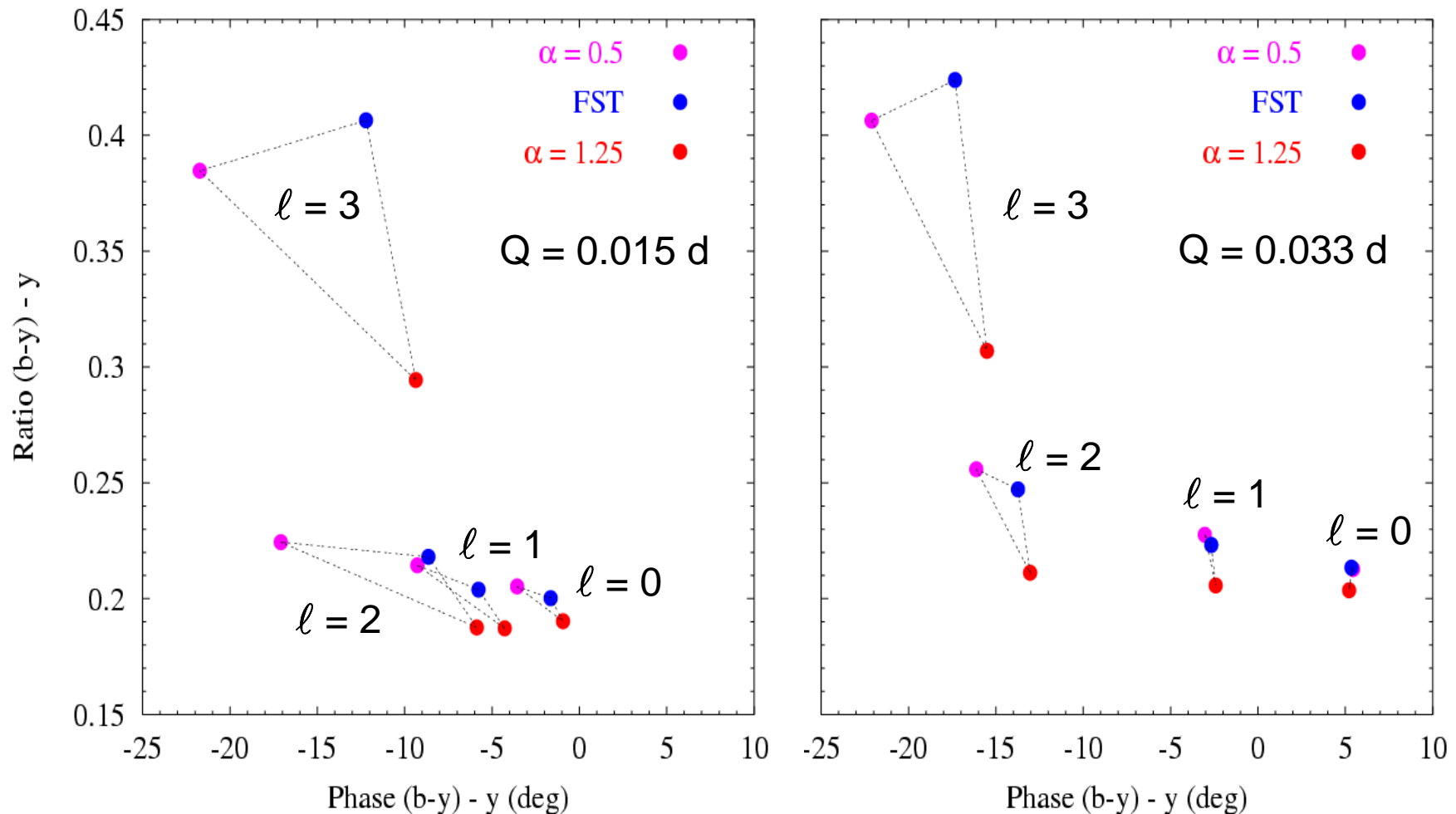


$\delta$  Scuti

# Amplitudes et phases photométriques

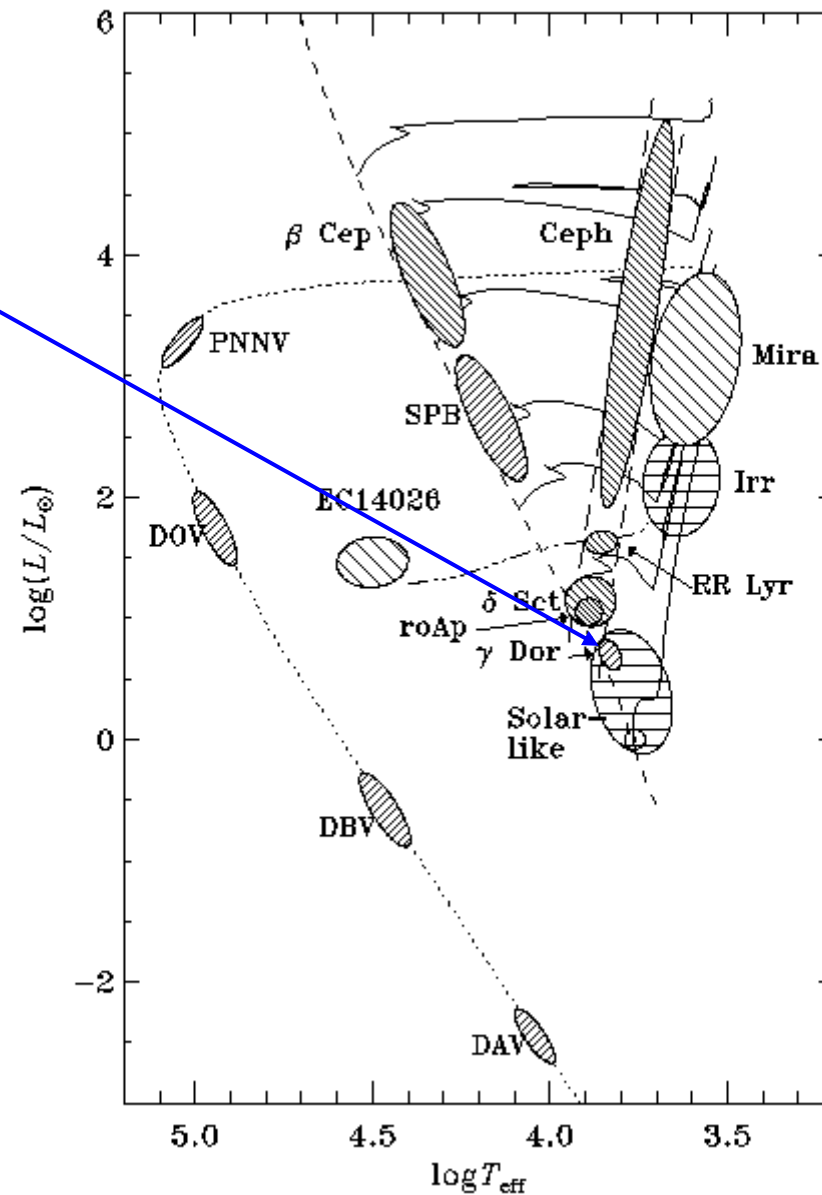
Full Spectrum of Turbulence  $\longleftrightarrow$  Longueur de mélange

Rapport d'amplitude vs. Différences de phase - photométrie Stroemgren



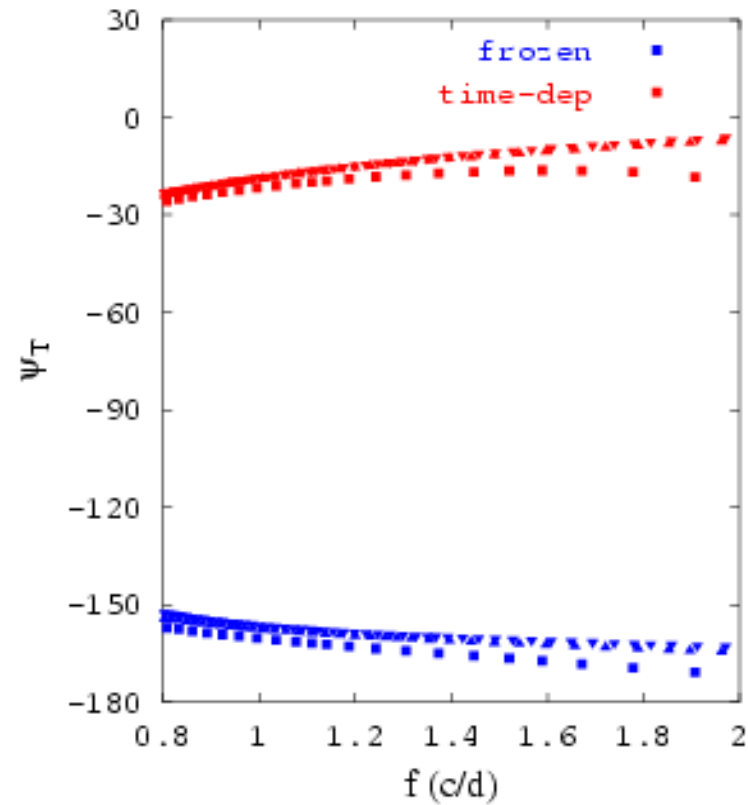
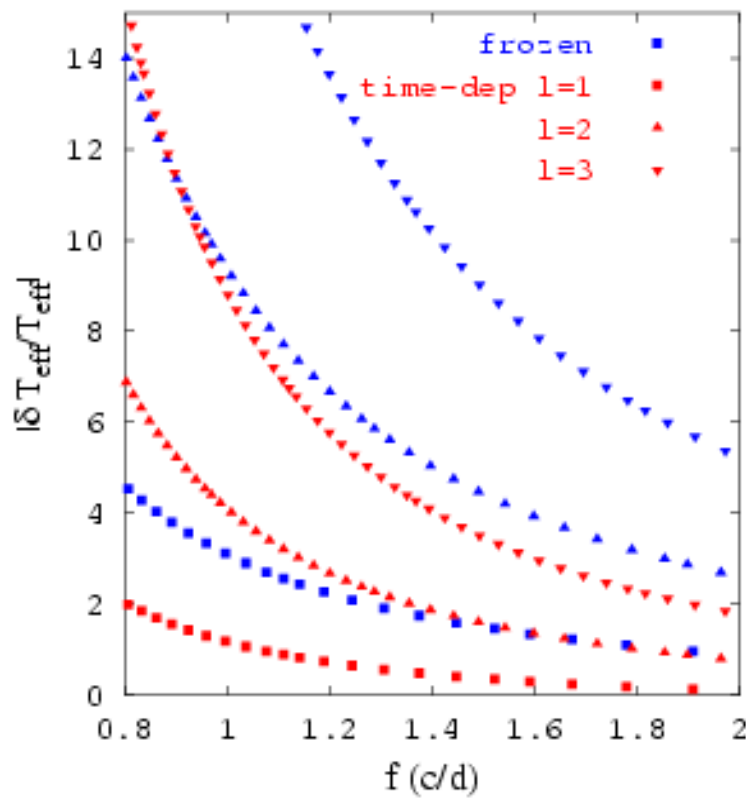
## $\gamma$ Doradus

- Types spectraux  
F
- Masses  
+/-  $1.5 M_{\odot}$
- Périodes  
0.3 à 3 jours  
modes g



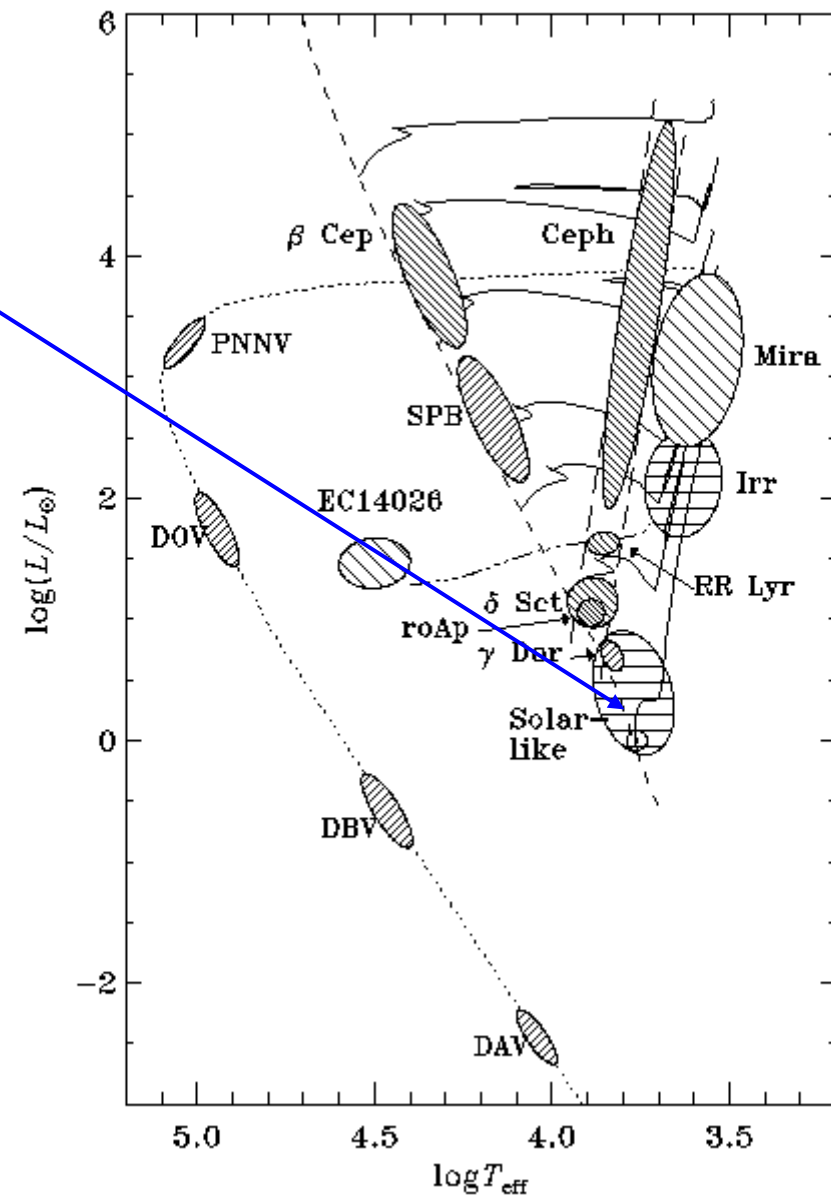
Comparaison : convection gelée  
convection dépendant du temps

$M = 1.5 M_0$  -  $T_{\text{eff}} = 7000 \text{ K}$  -  $\alpha = 1.8$



## Type solaire

- Types spectraux  
F et G
- Masses  
 $1 M_0$  à  $1.5 M_0$
- Périodes  
Quelques minutes  
Modes p élevés  
Faibles amplitudes



# Utility of our non - adiabatic code

- Excitation mechanisms
- Multi-colour photometry

Photometric amplitudes  
and phases in different filters

Identification  
of the degree  $\ell$

Non - adiabatic  
asteroseismology

Non adiabatic oscillations  
at the photosphere

Need of a non - adiabatic code  
for the confrontation between theory  
and observations

# Introduction

## Stellar pulsations

- Pressure modes

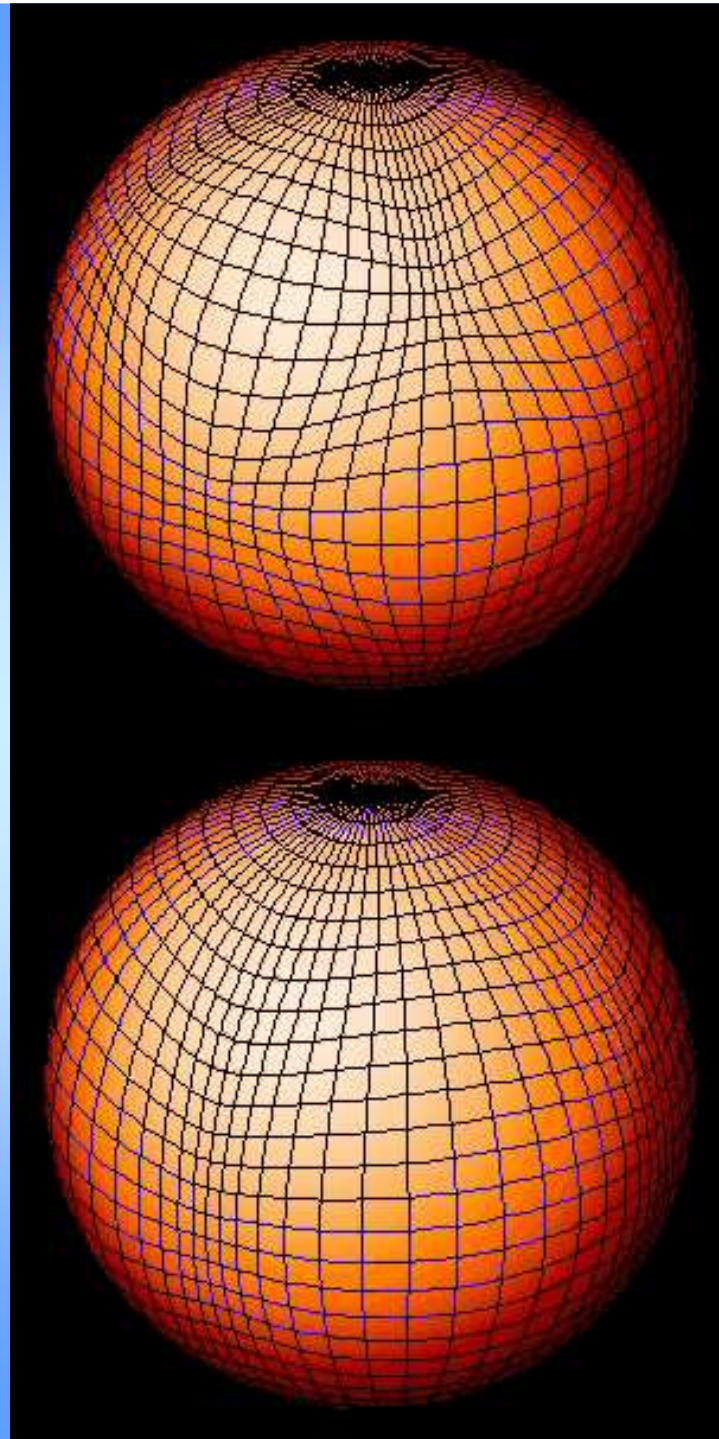
Acoustic waves

- Gravity modes

Buoyancy force

## Asteroseismology

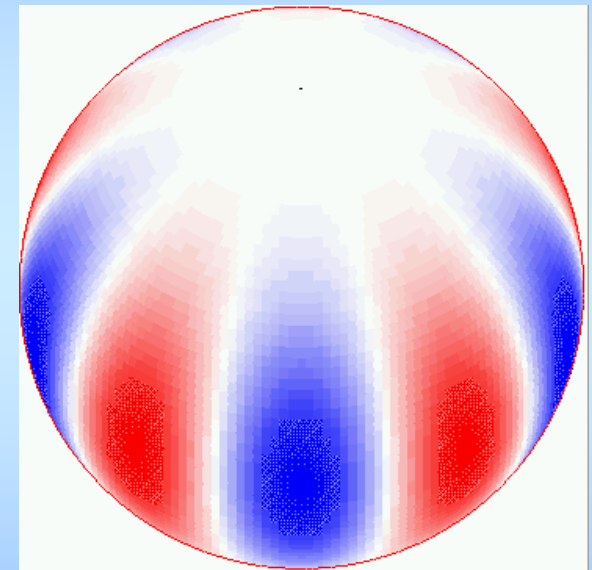
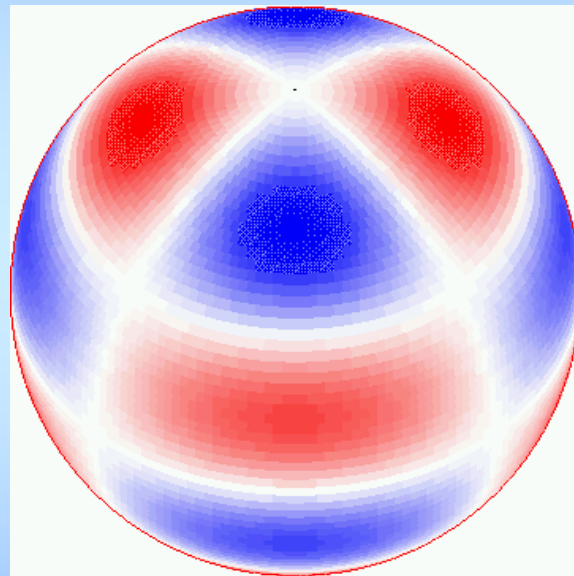
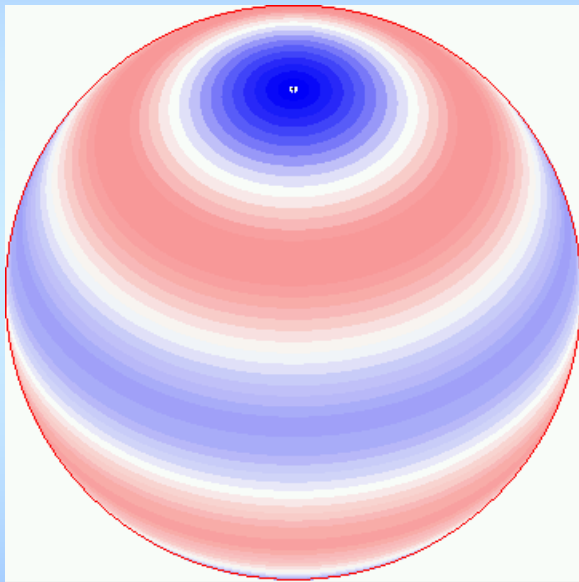
BAG meeting, asteroseismology of  $\gamma$  Dor stars  
Liège, 5th of May 2006





# 1) **Non - radial** non - adiabatic stellar oscillations

Splitting in spherical harmonics



p - modes

Acoustic waves

g - modes

Buoyancy force

# 1) Non - radial **non - adiabatic** stellar oscillations

$$\delta S \neq 0$$

Coupling between the dynamical and thermal equations

- Equation of momentum conservation
- Equation of mass conservation
- Poisson equation
- Equation of energy conservation
- Equations of transfer by radiation and convection

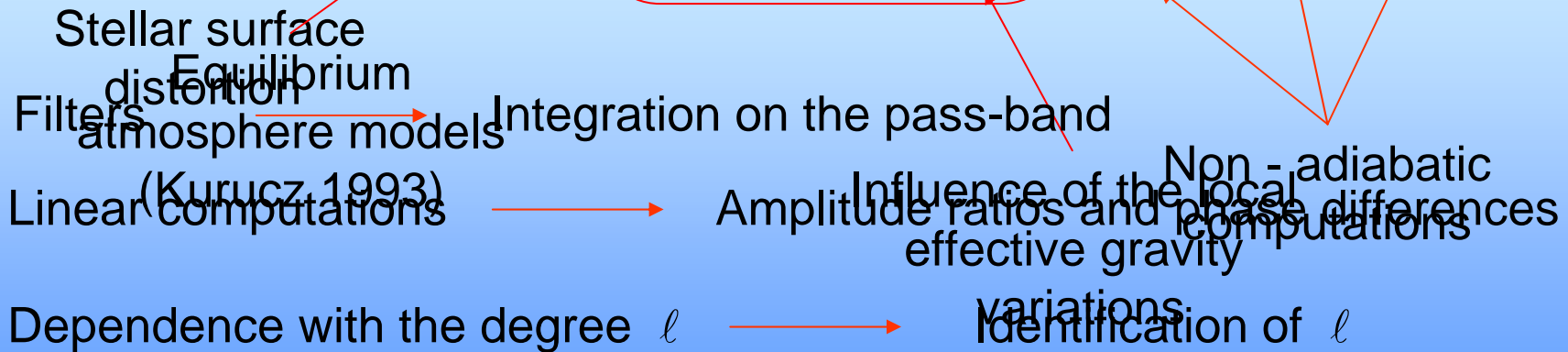
# Monochromatic magnitude variation

$$\delta m_\lambda = -\frac{2.5}{\ln 10} \varepsilon P_\ell^m(\cos i) b_{\ell\lambda}$$

Influence of the local effective temperature variations

$$\left[ -(\ell-1)(\ell+2) \cos(\sigma t) + \left( \frac{\partial \ln F_\lambda^+}{\partial \ln T_{\text{eff}}} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln T_{\text{eff}}} \right) \left| \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \right| \cos(\sigma t + \psi_T) \right]$$

$$- \left( \frac{\partial \ln F_\lambda^+}{\partial \ln g} + \frac{\partial \ln b_{\ell\lambda}}{\partial \ln g} \right) \left| \frac{\delta g_e}{g_e} \right| \cos(\sigma t)$$



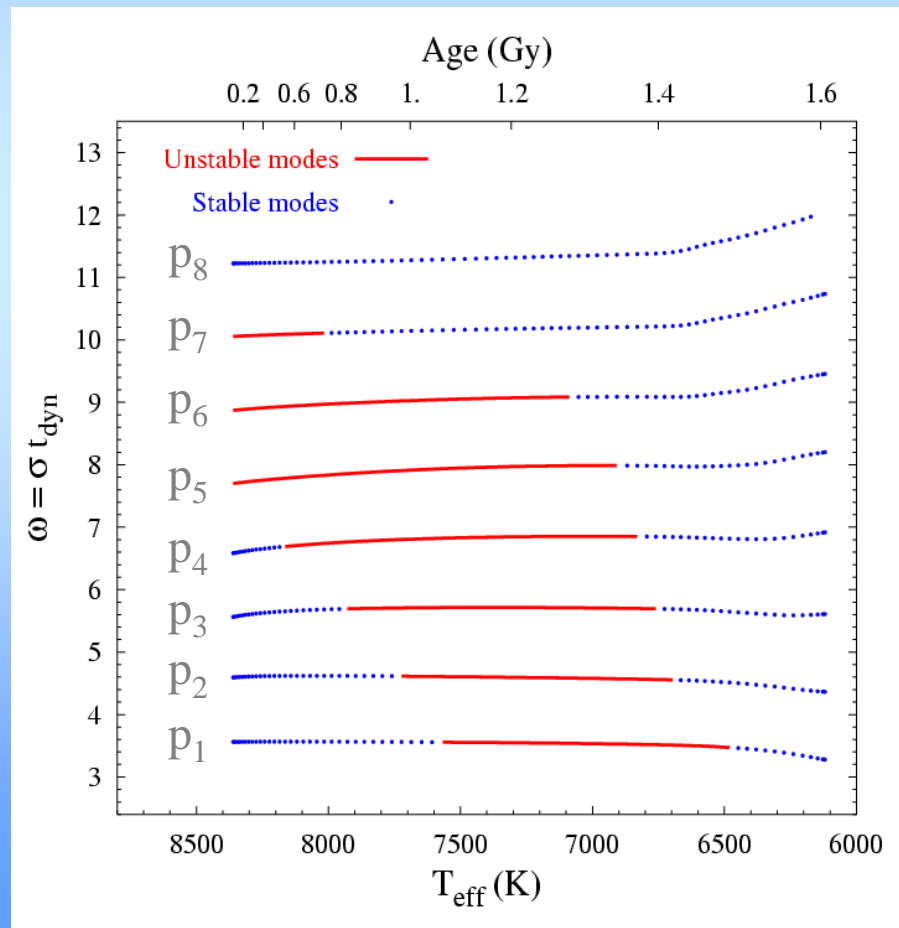
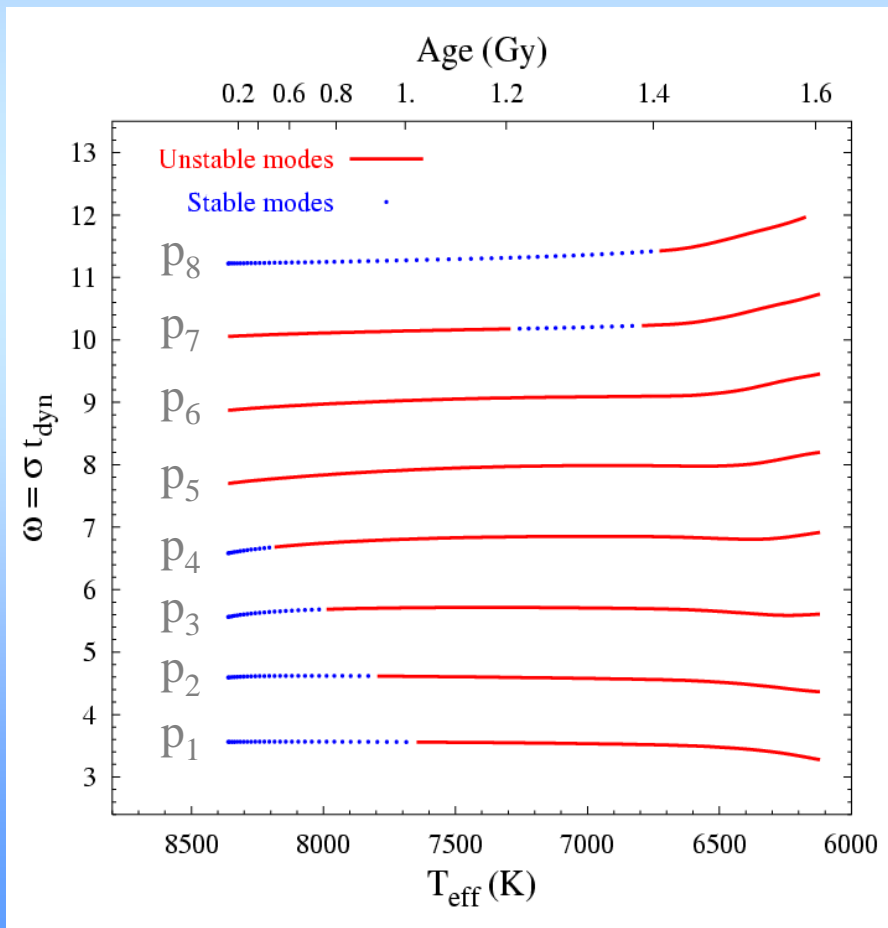
# 4.2 $\delta$ Scuti Time dependent convection - MLT Theory of M. Gabriel

## Red edge of the instability strip

Radial modes –  $1.8 M_{\odot}$ ,  $\alpha = 1.5$

Frozen convection

Time-dependent convection

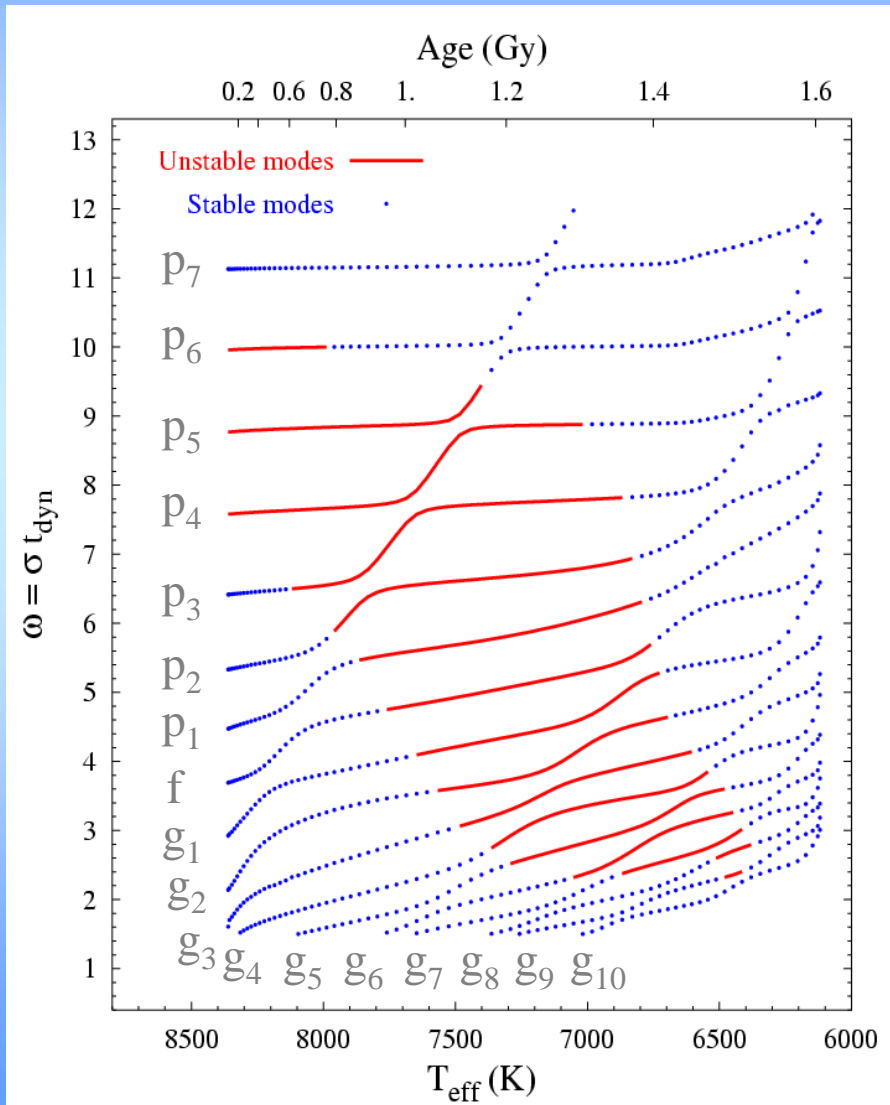
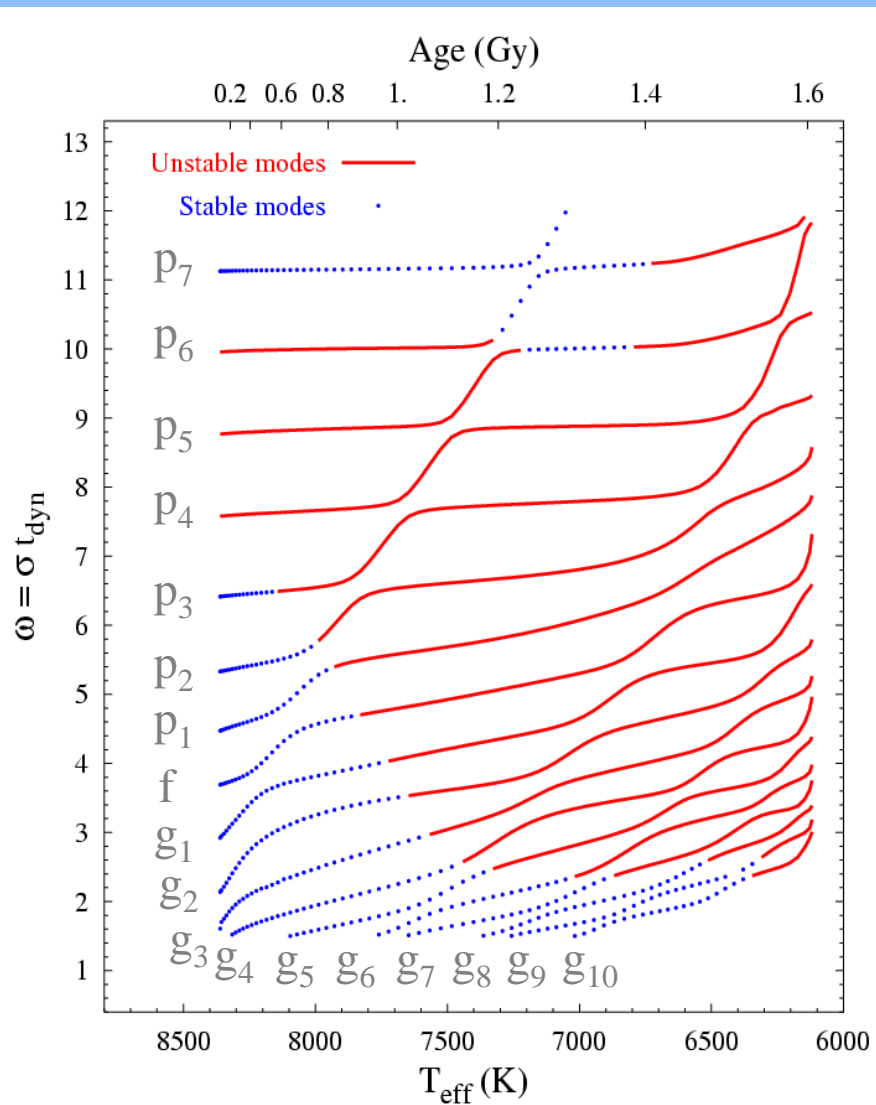


## 4.2 $\delta$ Scuti      Red edge of the instability strip

$\ell = 2$  modes –  $1.8 M_{\odot}$ ,  $\alpha = 1.5$

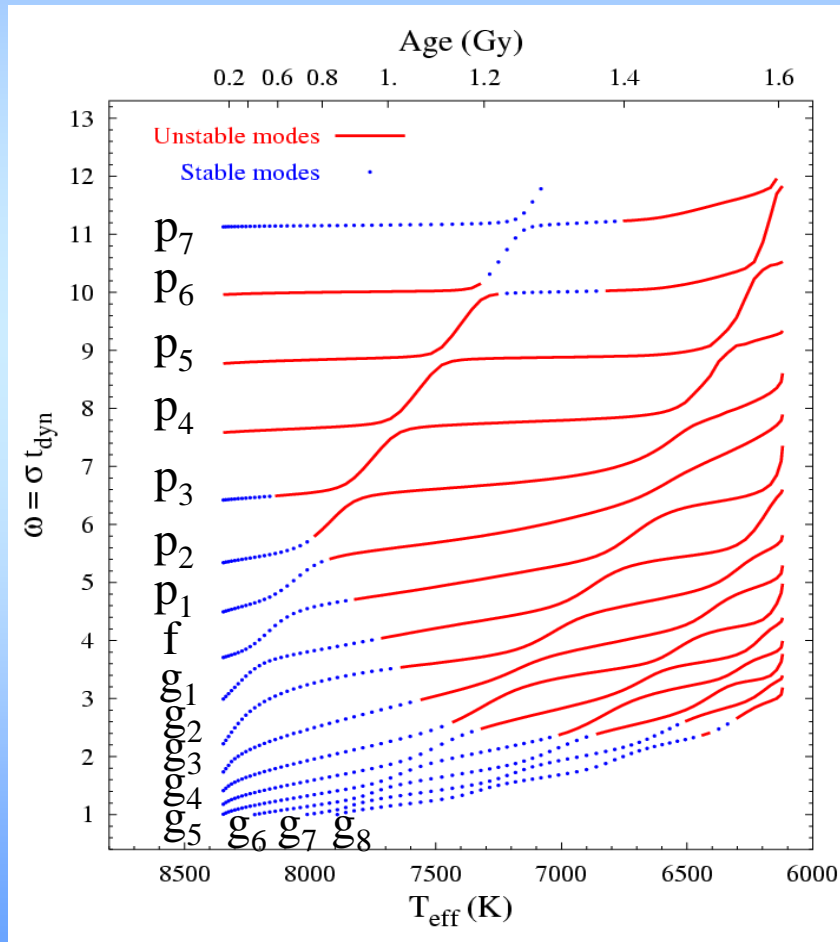
Frozen convection

Time-dependent convection

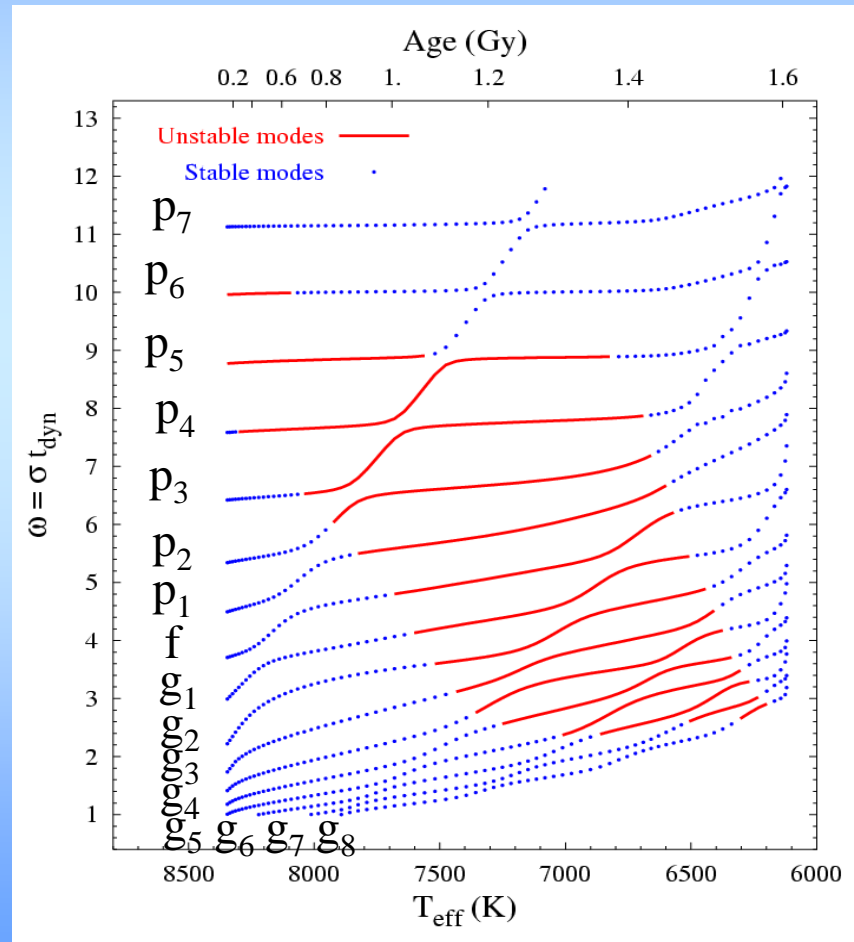


III. Instability Strip  
 III. 1.  $\delta$  Scuti stars

## Frozen Convection



## Time-dependent convection



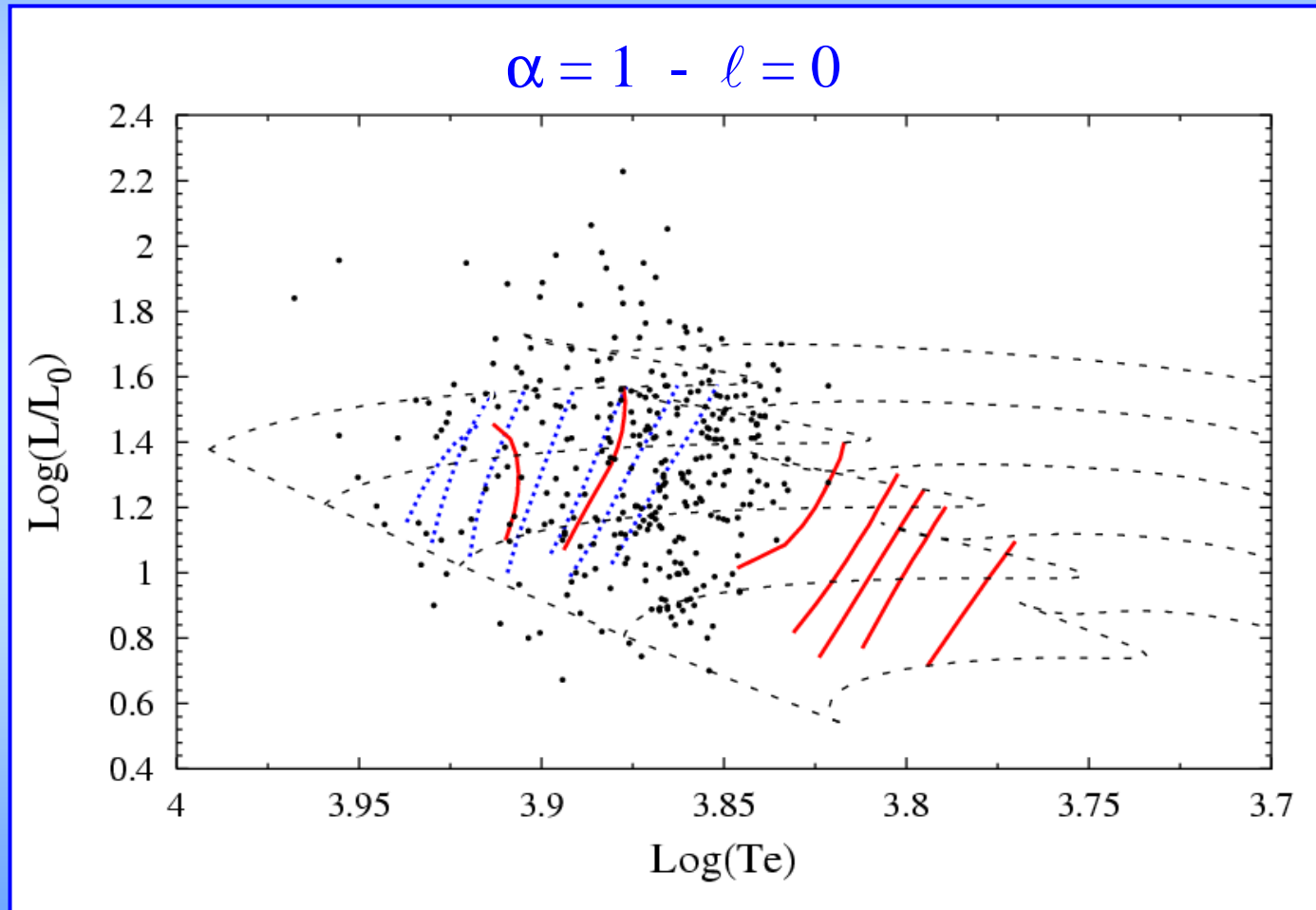
BAG meeting, asteroseismology of  $\gamma$  Dor stars  
 Liège, 5th of May 2006

Figure 5

Figure 6

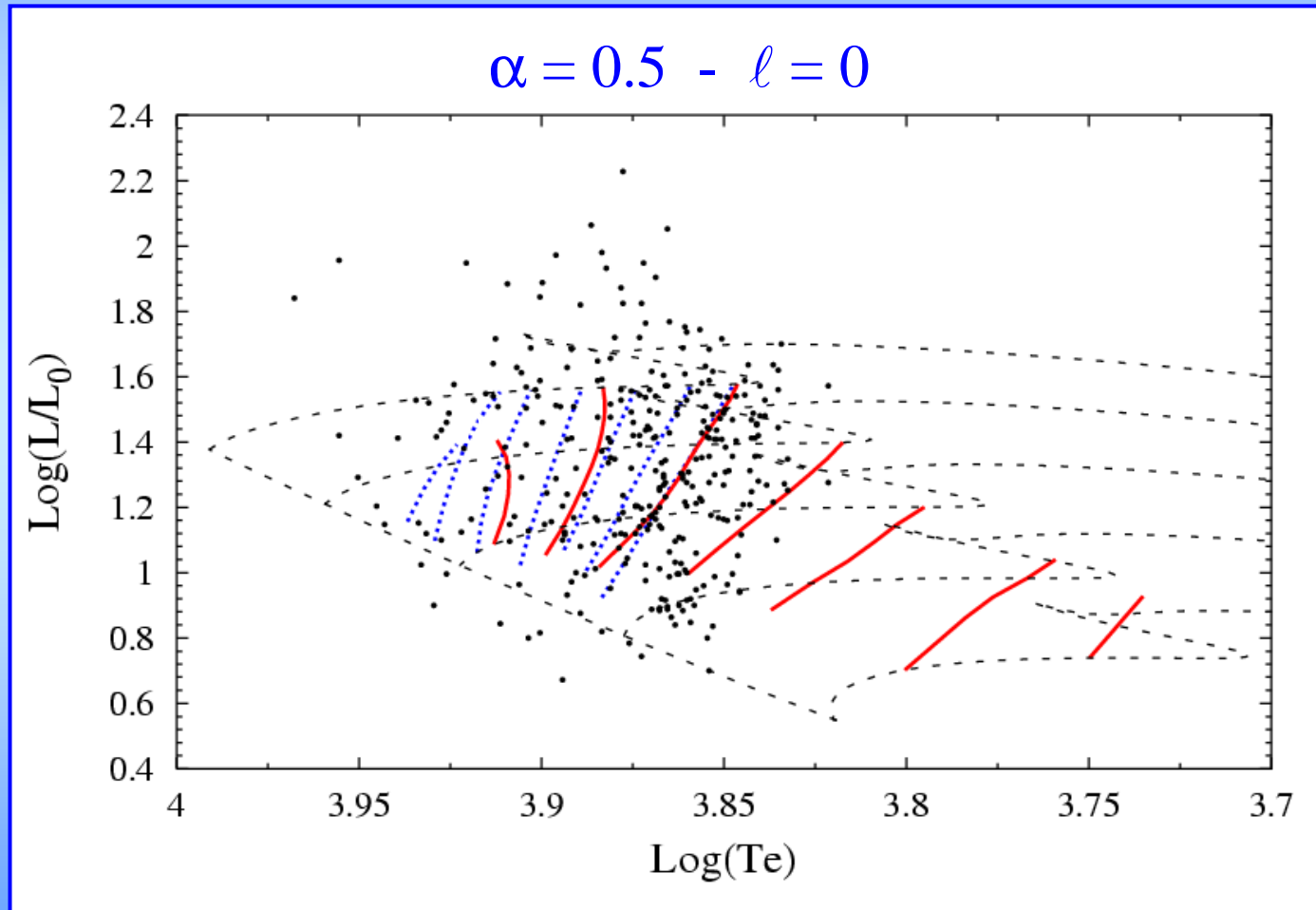
$\delta$  Scuti

Bandes d'instabilité



$\delta$  Scuti

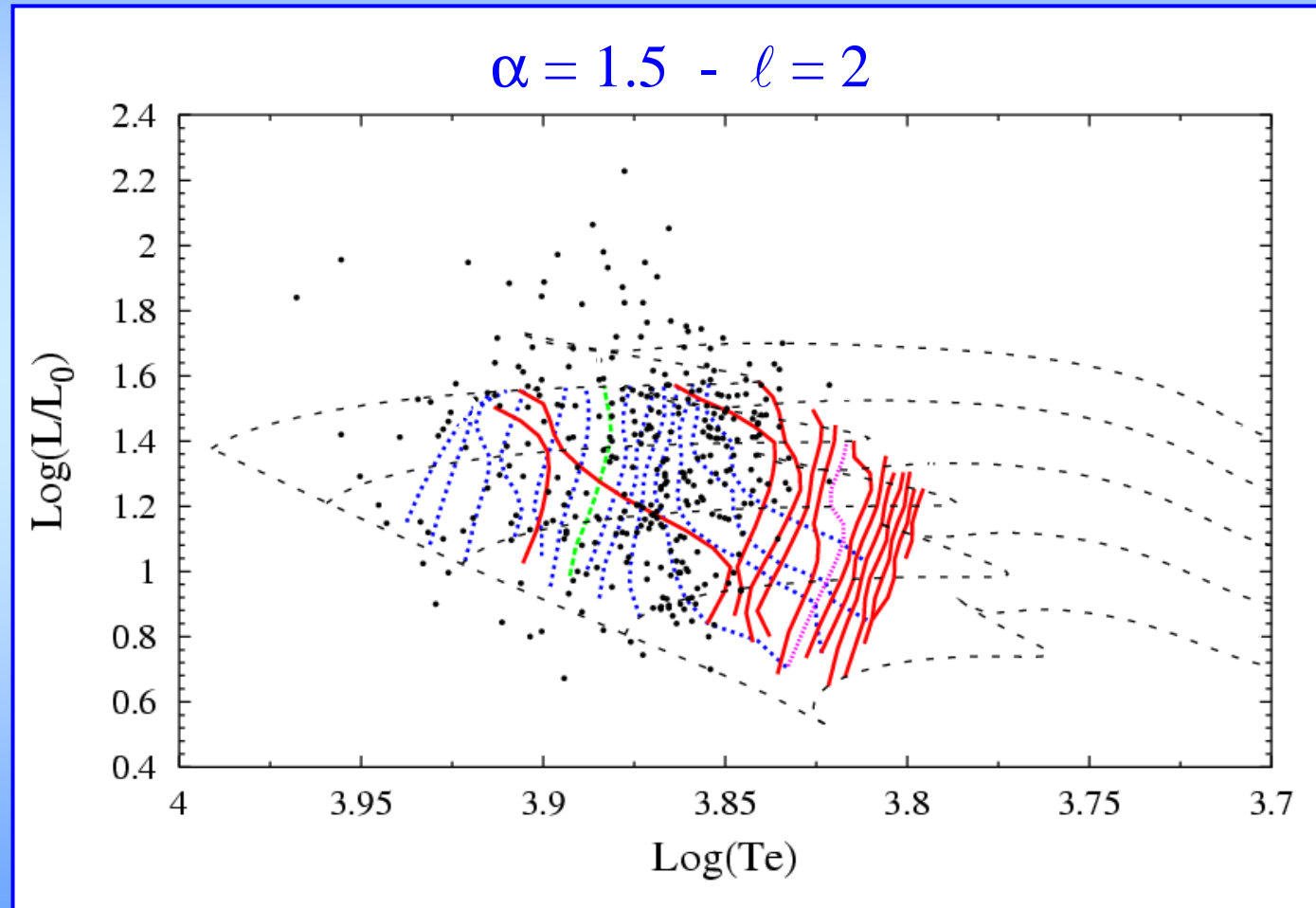
Bandes d'instabilité





$\delta$  Scuti

Bandes d'instabilité



# Plan de l'exposé

1. Introduction

2. Oscillations stellaires non-adiabatiques : utilité

3. Modélisation du problème

- Atmosphère
- Convection

4. Applications

- $\beta$  Cephei
- Slowly Pulsating B
- $\delta$  Scuti
- $\gamma$  Doradus
- Type solaire

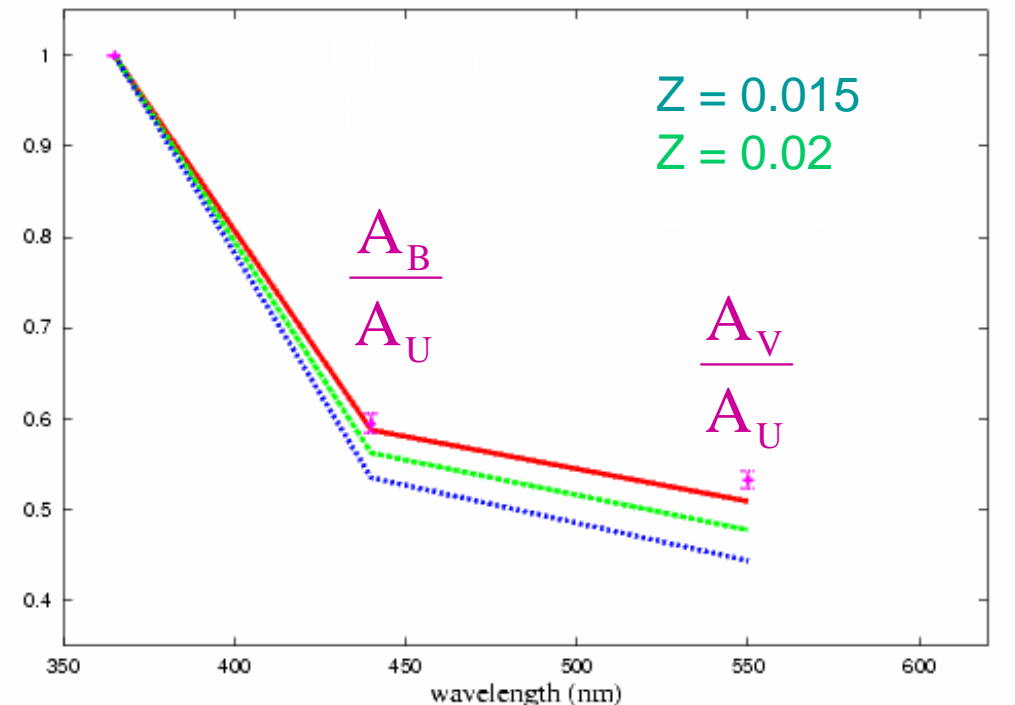
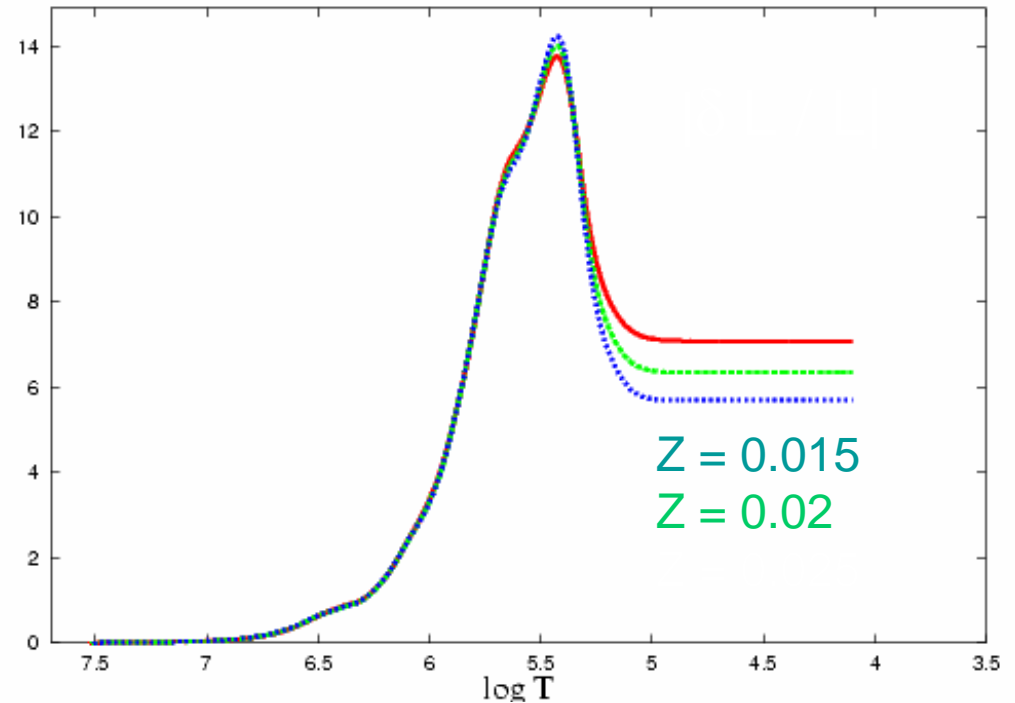
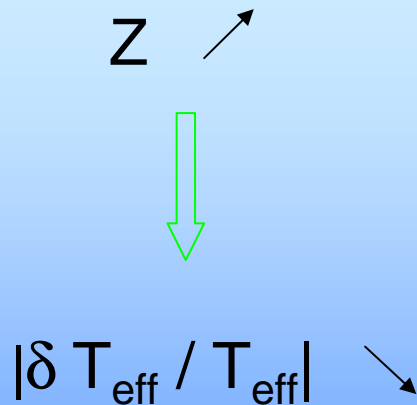
5. Conclusions

# $\beta$ Cephei

## Multi-colour photometry

16 Lacertae

High sensitivity of the non - adiabatic results to the metallicity



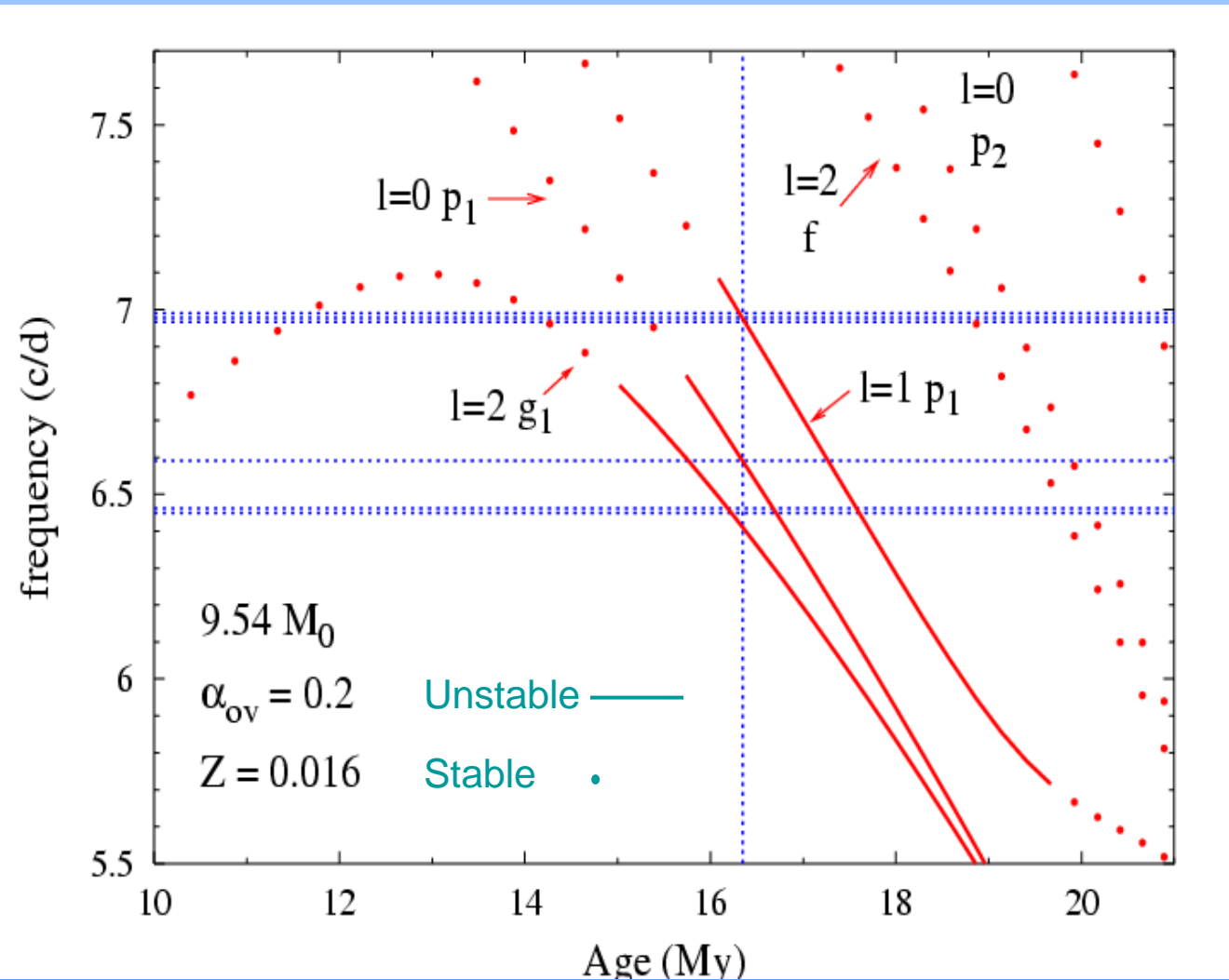
# 4.1 $\beta$ Cephei : HD 129929

## Non-adiabatic constraints on the metallicity

Modes excitation



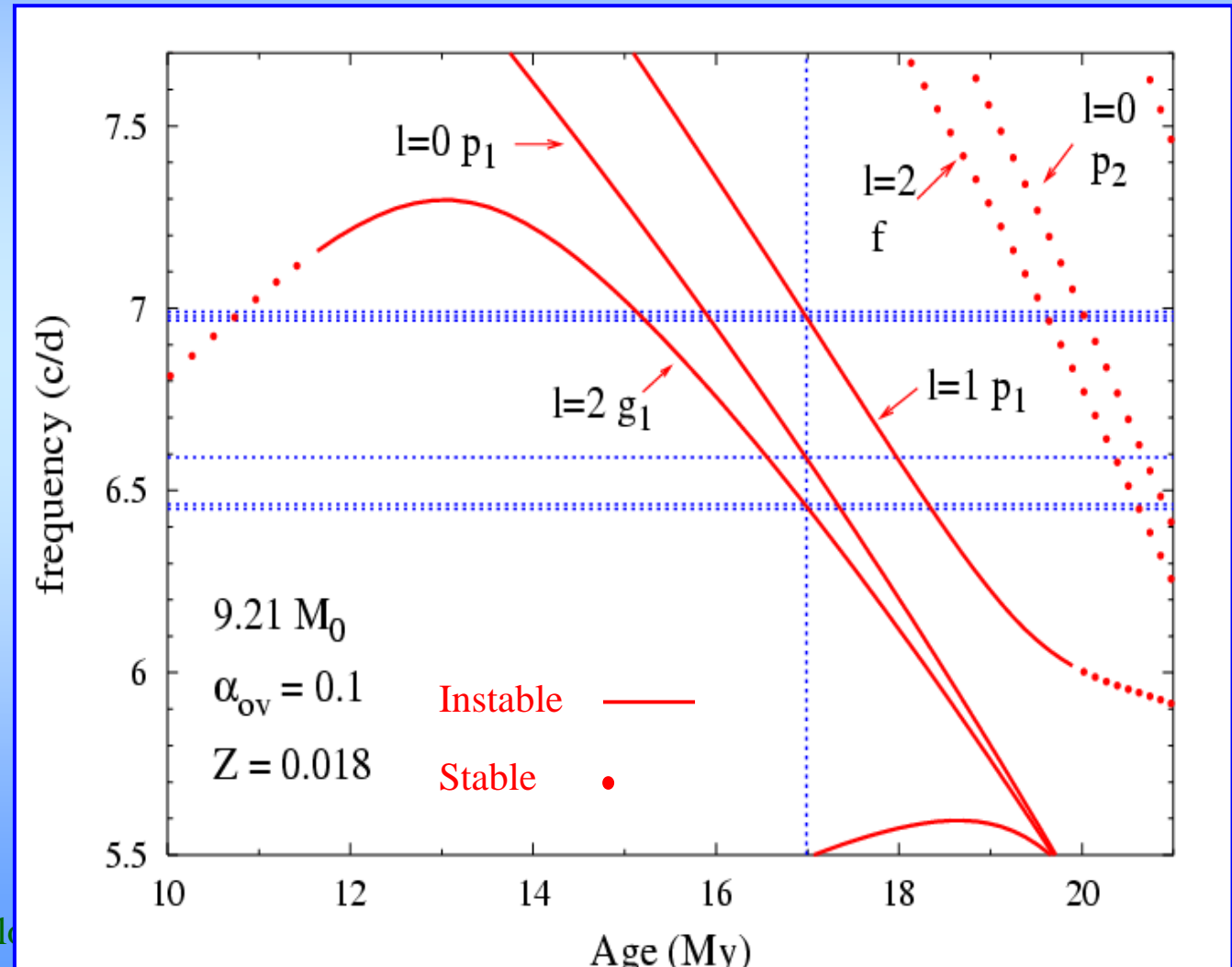
$Z > 0.016$



$\beta$  Cephei : HD 129929

Contraintes sismiques sur l'overshooting

$$\alpha_{\text{ov}} = 0.1 \pm 0.05$$



# Slowly Pulsating B stars

## Excitation mechanism

Work integral and luminosity variation from the center to the surface of the star

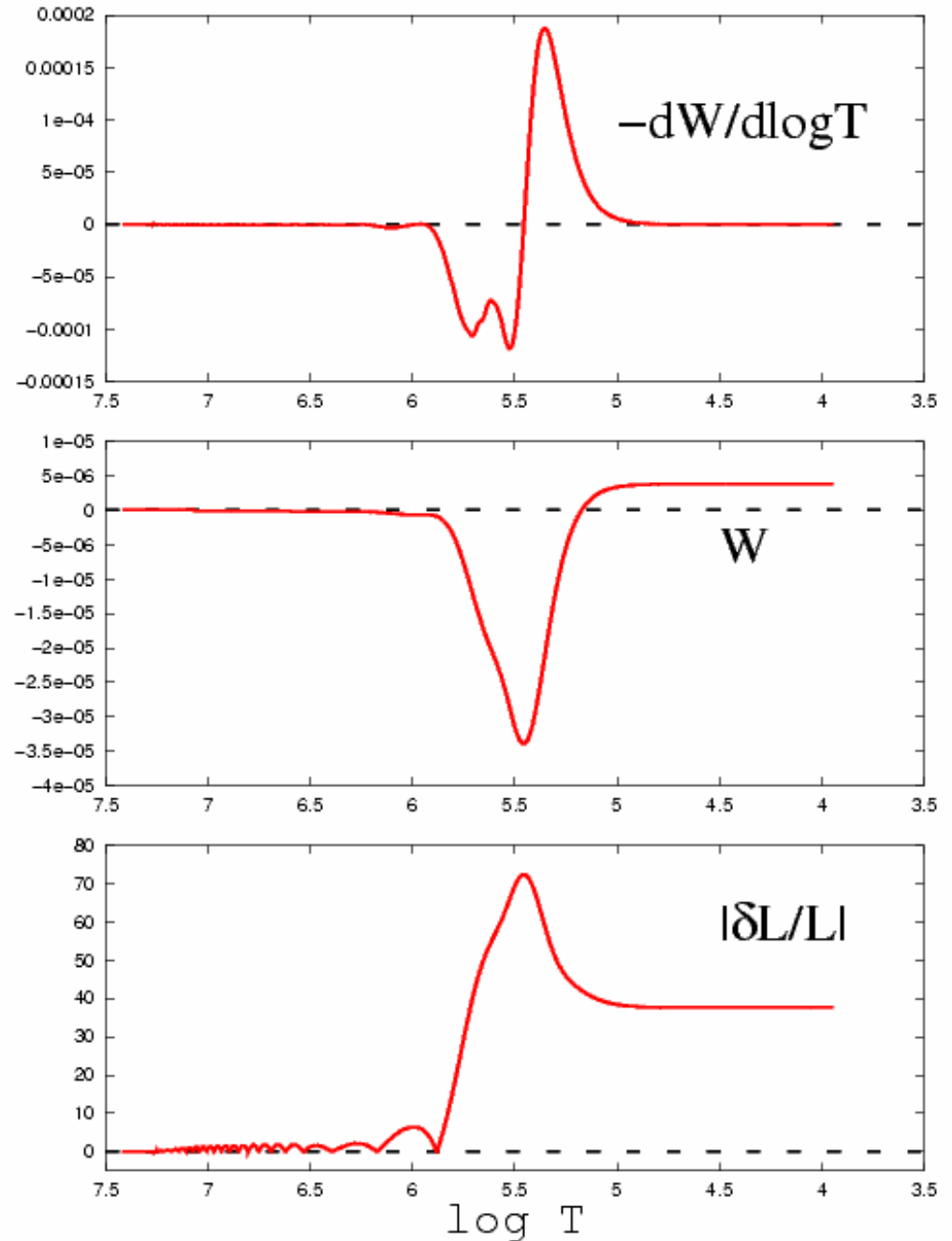
$$M = 4 M_0$$

$$T_{\text{eff}} = 13\,955 \text{ K}$$

$$Z = 0.02$$

$$\text{Mode } \ell = 1 \quad g_{22}$$

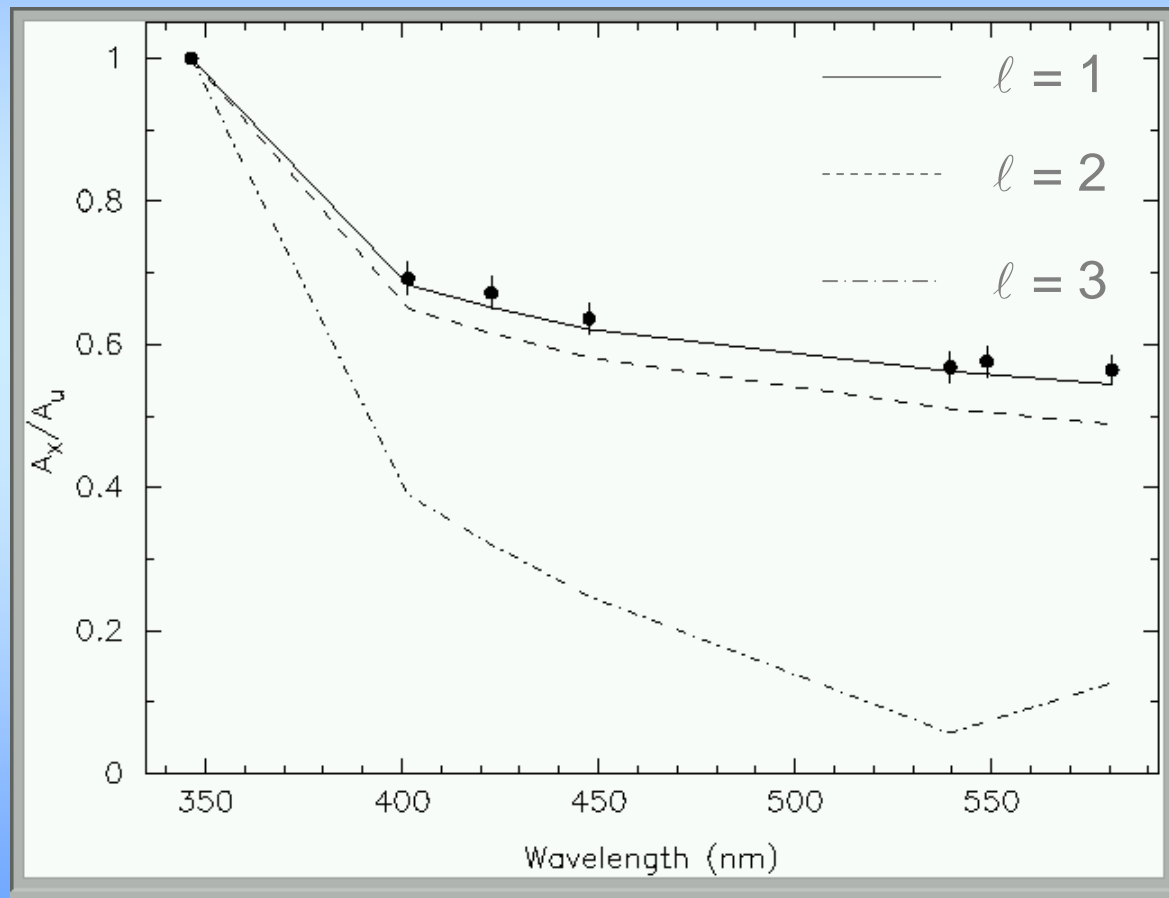
BAG meeting, asteroseismology of  $\gamma$  Dor stars  
Liège, 5th of May 2006



# Slowly Pulsating B stars

## Identification photométrique des modes

HD 74560



# $\delta$ Scuti

## Excitation mechanism

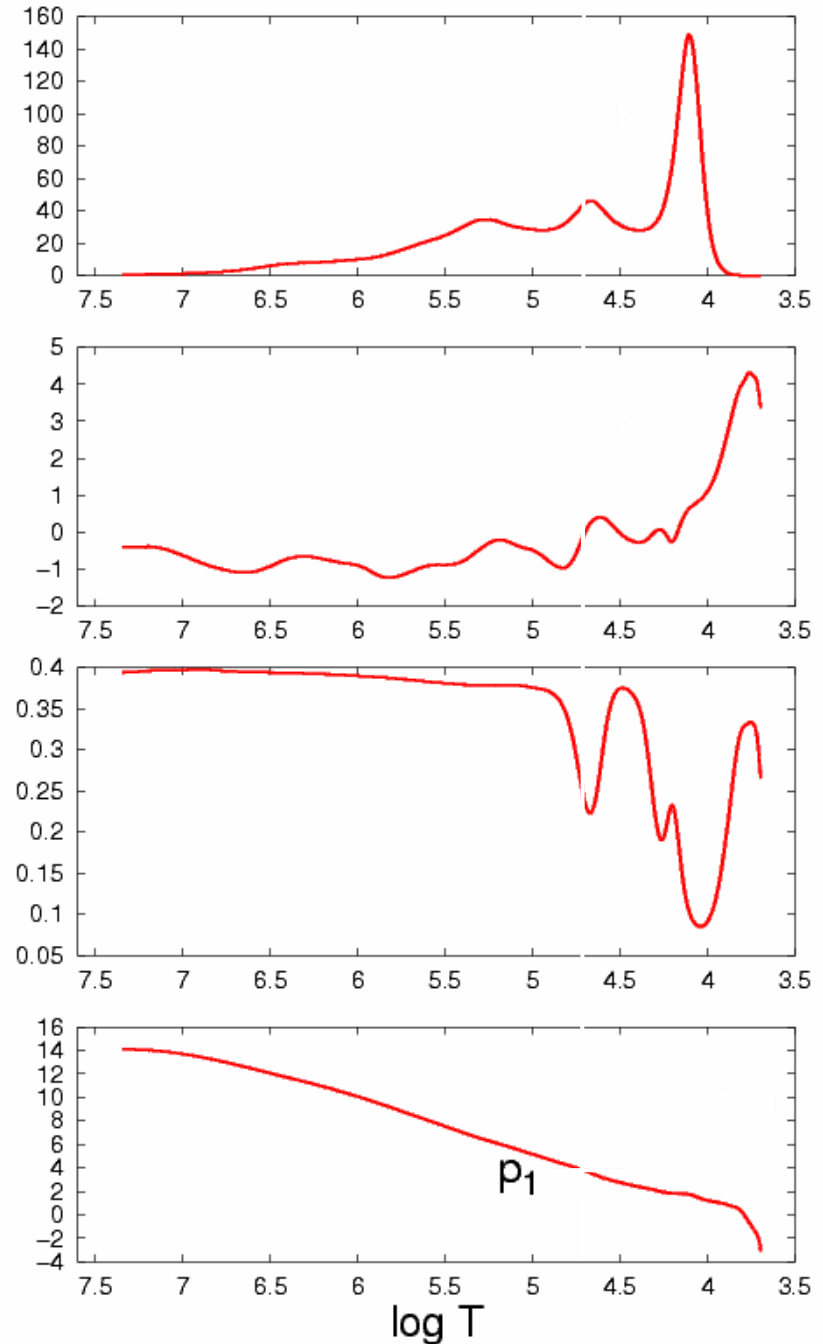
Opacity and thermal relaxation time from the center to the surface

Partial ionization zone of Helium II = Transition zone



$\kappa$ - $\gamma$  mechanism of excitation

BAG meeting, asteroseismology of  $\gamma$  Dor stars  
Liège, 5th of May 2006





# $\delta$ Scuti

## Excitation mechanism

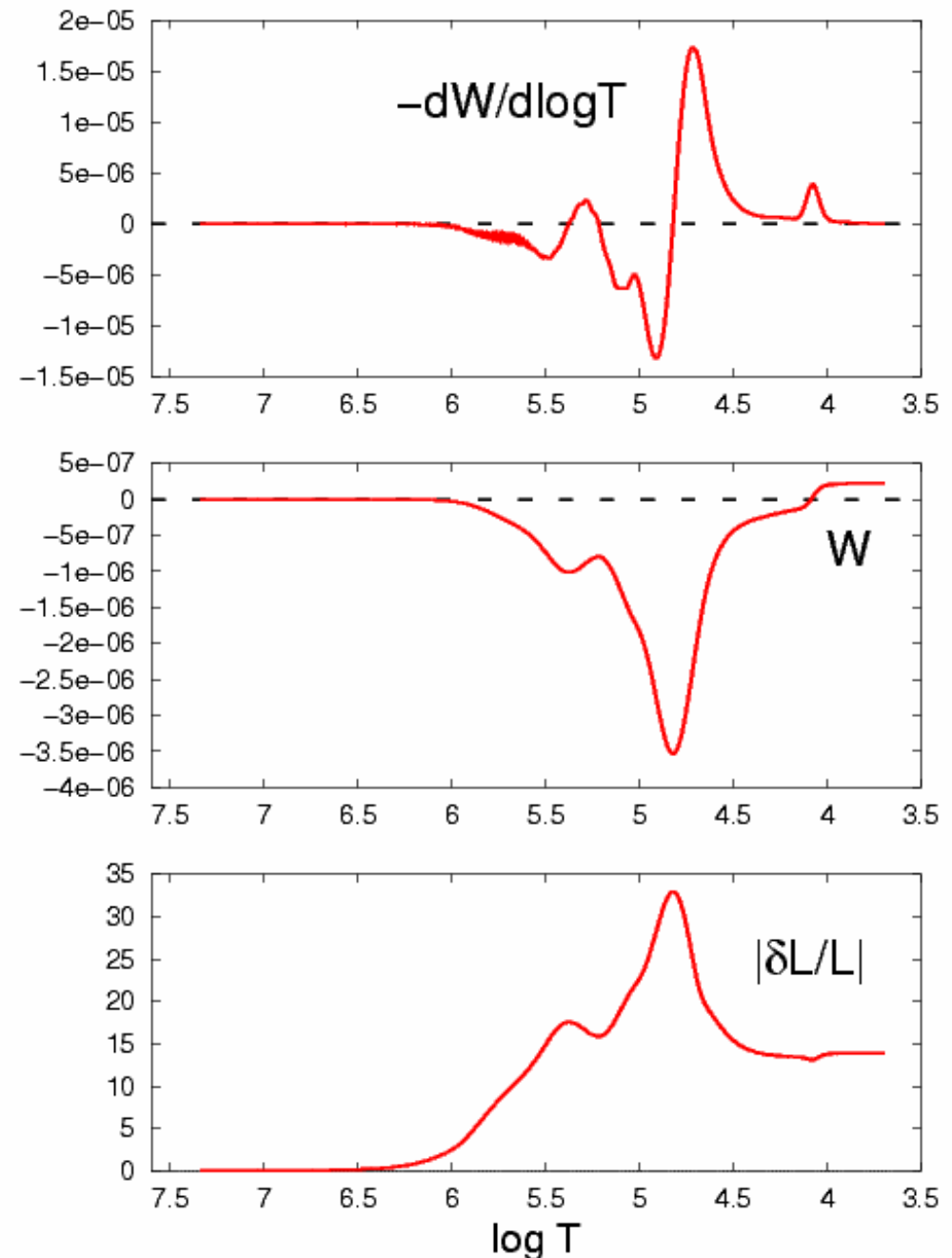
Work integral and luminosity variation from the center to the surface of the star

$$M = 1.8 M_0$$

$$T_{\text{eff}} = 7480 \text{ K}$$

$$Z = 0.02, \quad \alpha = 1$$

Radial  
fundamental mode

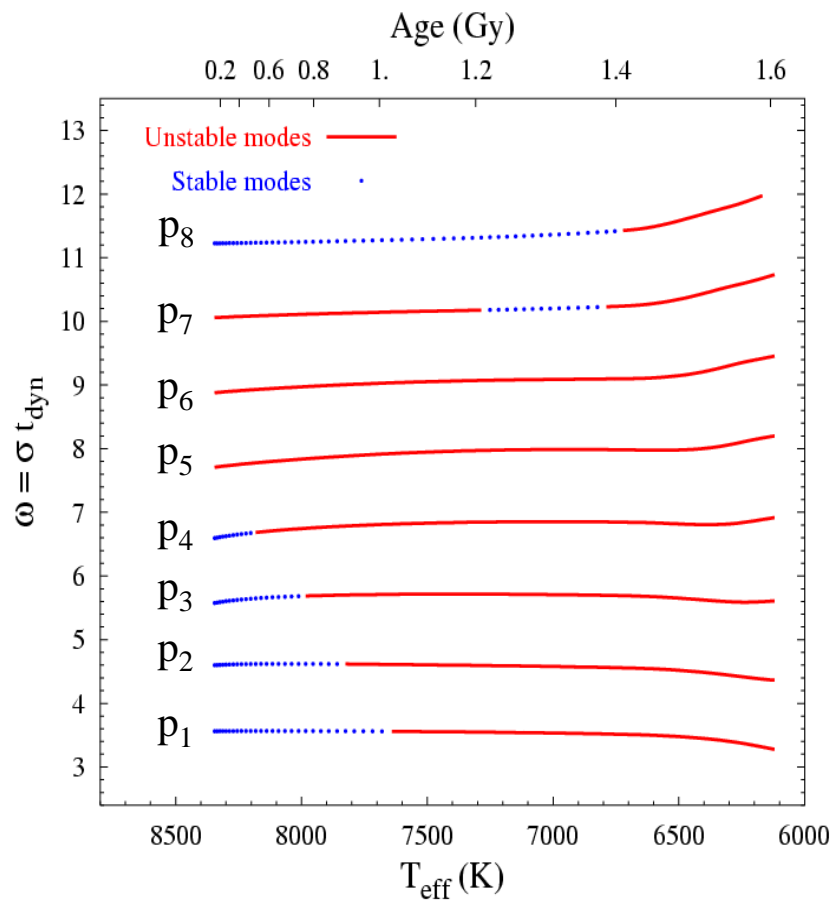


# $\delta$ Scuti

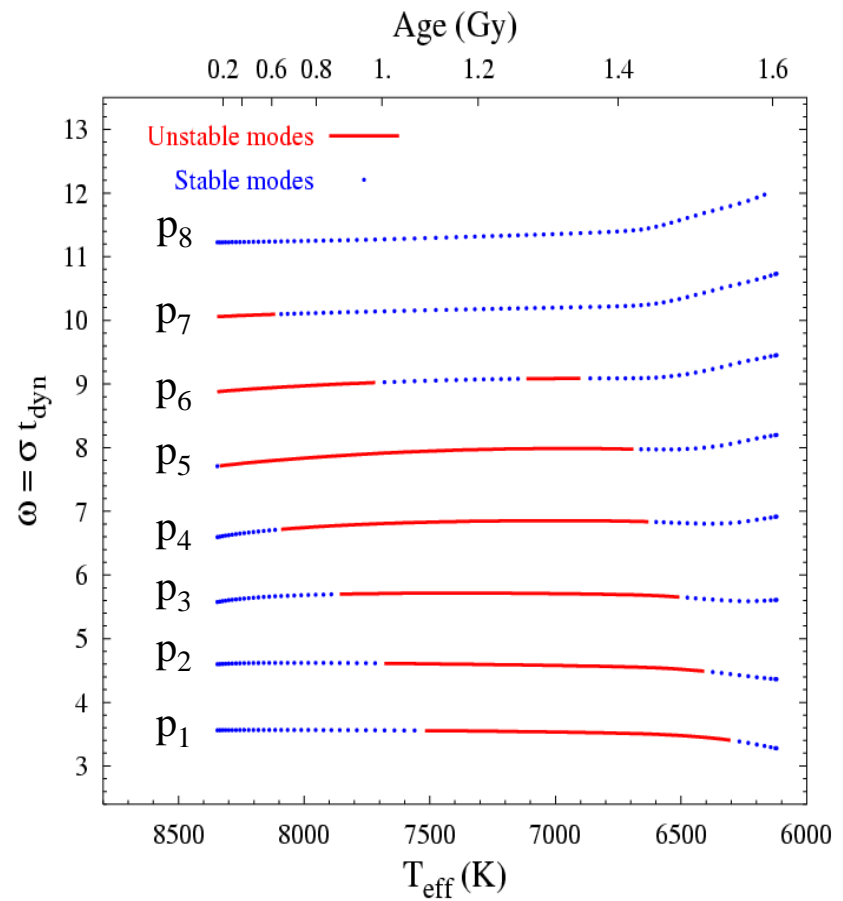
## Modes stables et instables

$$\ell = 0 - 1.8 M_0 - \alpha = 1.5$$

### Convection gelée



### Convection dépendant du temps



# $\gamma$ Doradus

Mécanisme d'excitation :



?? Bloquage convectif ?? (Guzik et al. 2000)

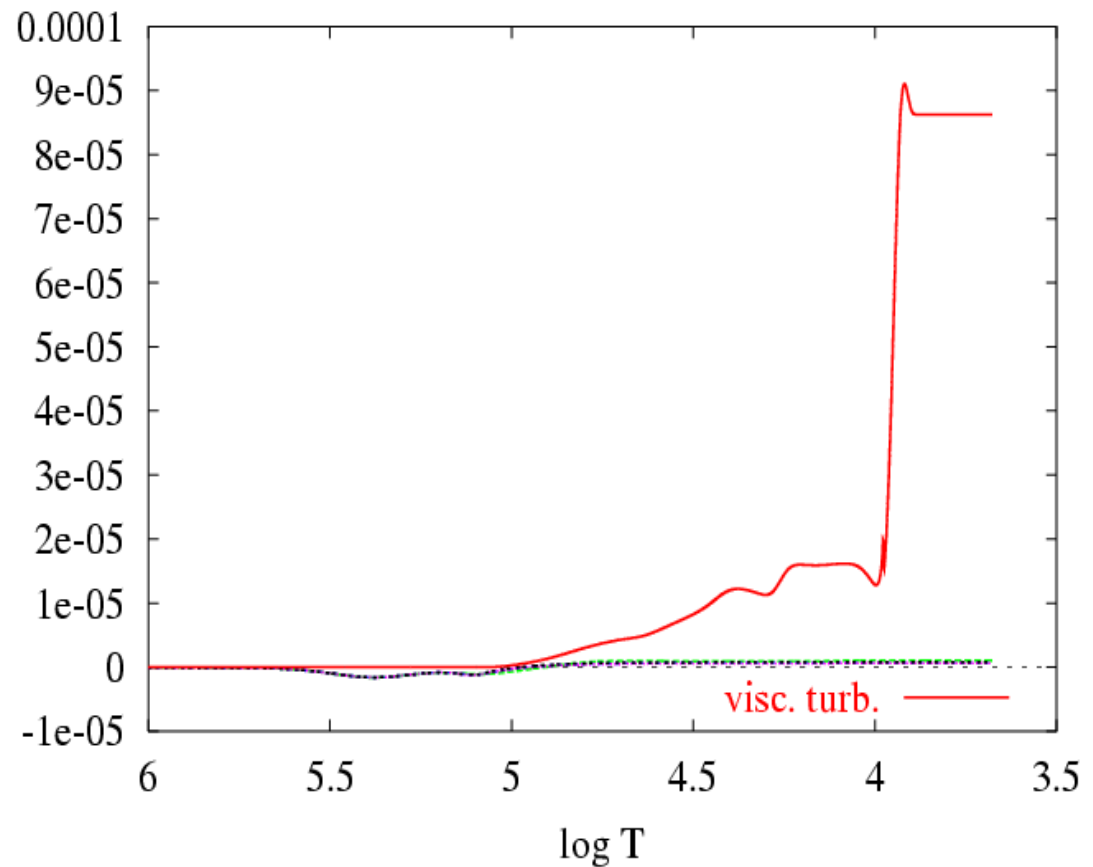
Travail intégré

$$M = 1.6 M_0$$

$$T_{\text{eff}} = 7000 \text{ K}$$

$$\alpha = 2$$

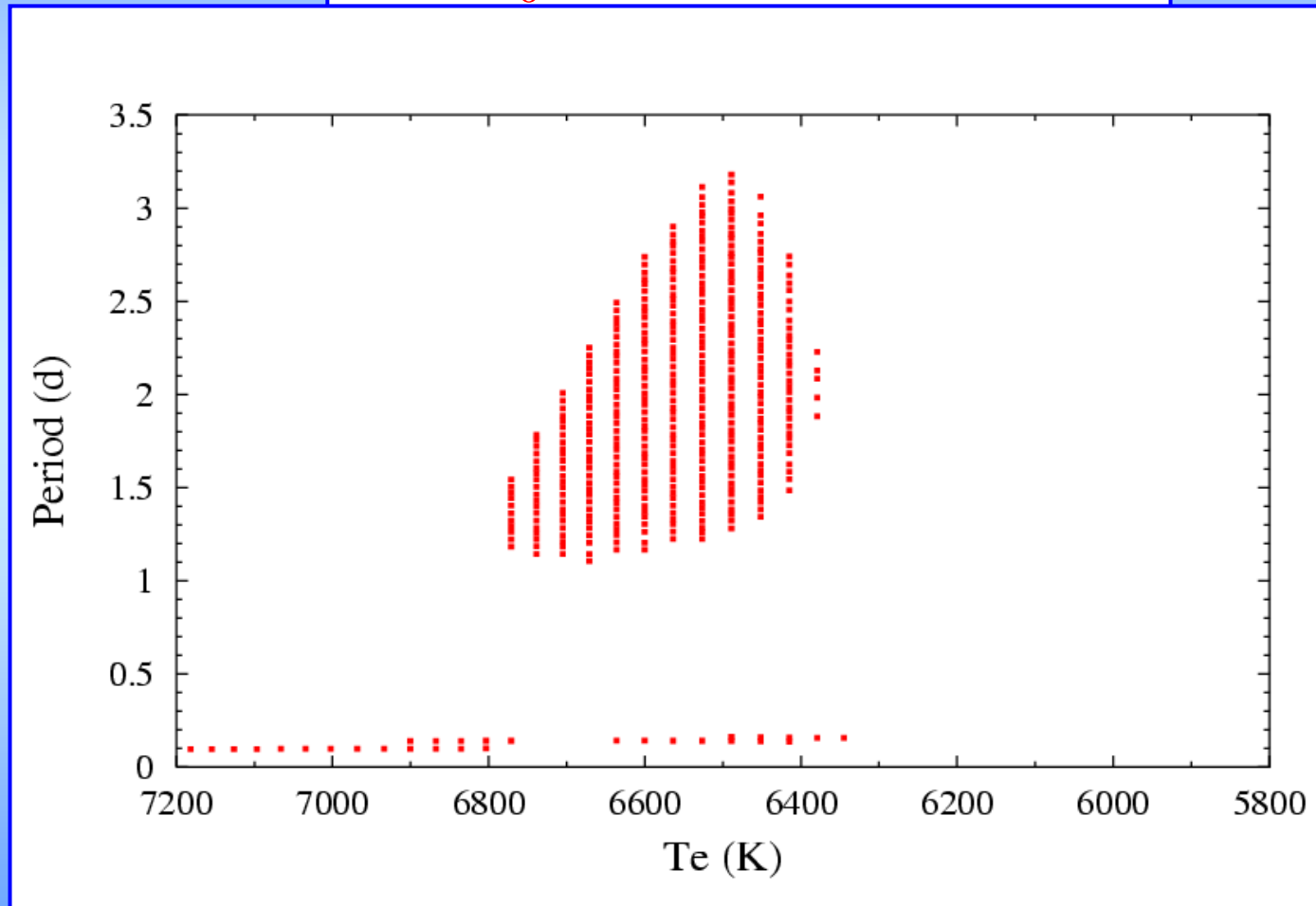
Mode  $\ell=1$ ,  $g_{47}$



$\gamma$  Doradus

Modes instables

$1.6 M_{\odot} - \ell = 1 - \alpha = 1.5$



# Convection – pulsation interaction: Gabriel's theory

## Hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial(\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = -\rho \nabla \Phi + \nabla \cdot P$$

$$\frac{\partial(\rho U)}{\partial t} + \nabla \cdot (\rho U \vec{v}) = \rho \varepsilon_N - \nabla \cdot \vec{F}_R - P \otimes \nabla \vec{v}$$

$$P = P_G + P_R$$

$$p = p_G + p_R \quad P: \text{Pressure tensor ; } p : \text{its diagonal component.}$$

$$P_X = p_X 1 - \beta_X$$

$\vec{F}_R$  Radiative Flux

# Convection – pulsation interaction: Gabriel's theory

## Mean equations

$$\frac{d\bar{\rho}}{dt} + \bar{\rho} \nabla \cdot (\bar{\mathbf{u}}) = 0$$

$$\bar{\rho} \frac{d\bar{\mathbf{u}}}{dt} = -\bar{\rho} \nabla \bar{\Phi} - \nabla (\bar{p}_G + \bar{p}_R + \bar{p}_T) + \nabla \cdot (\bar{\beta}_G + \bar{\beta}_R + \bar{\beta}_T)$$

$$\bar{\rho} \bar{T} \frac{d\bar{s}}{dt} = -\nabla \cdot (\bar{\mathbf{F}}_R + \bar{\mathbf{F}}_C) + \bar{\rho} \bar{\varepsilon}_N + \bar{\rho} \bar{\varepsilon}_2 + \overline{\bar{\mathbf{V}} \cdot \nabla (p_G + p_R)}$$

$$\bar{\rho} \frac{d}{dt} \left( \frac{1}{2} \frac{\overline{\rho V^2}}{\bar{\rho}} \right) = -\bar{\rho} \bar{\varepsilon}_2 - \overline{\bar{\mathbf{V}} \cdot \nabla (p_G + p_R)}$$

$$\overline{\rho \bar{\mathbf{V}} \bar{\mathbf{V}}} = \bar{p}_T \mathbf{1} - \bar{\beta}_T$$

Reynolds stress tensor

$$\bar{\rho} \bar{\varepsilon}_2 + \overline{\bar{\mathbf{V}} \cdot \nabla (p_G + p_R)}$$

Dissipation of turbulent kinetic energy into heat

$$\bar{p}_T = \overline{\rho V_r^2}$$

Turbulent pressure

# Convection – pulsation interaction: Gabriel's theory

## Convective fluctuations equations

$$\bar{\rho} \frac{d}{dt} \left( \frac{\Delta \rho}{\bar{\rho}} \right) + \nabla \cdot (\rho \vec{V}) = 0$$

$$\bar{\rho} \frac{d\vec{V}}{dt} = \frac{\Delta \rho}{\bar{\rho}} \nabla \bar{p} - \nabla \Delta p - \frac{8\rho \vec{V}}{3\tau_c} - \rho \vec{V} \cdot \nabla \vec{u}$$

$$\frac{\Delta(\rho T)}{\bar{\rho T}} \frac{d\bar{s}}{dt} + \frac{d\Delta s}{dt} + \vec{V} \cdot \nabla \bar{s} = -\frac{\Gamma^{-1} + 1}{\tau_c} \Delta s$$

### Approximations of Gabriel's Theory

$$\frac{\Delta \rho}{\bar{\rho}} \nabla \cdot (\bar{\beta}_G + \bar{\beta}_R + \bar{\beta}_T) - \nabla \cdot (\Delta \beta_G + \Delta \beta_R + \Delta \beta_T) = \frac{8\rho \vec{V}}{3\tau_c}$$

$$\rho \varepsilon_2 - \overline{\rho \varepsilon_2} + \rho T \nabla s \cdot \vec{V} - \overline{\rho T \nabla s \cdot \vec{V}} = \frac{(\nabla \cdot \vec{F}_R - \overline{\nabla \cdot \vec{F}_R})}{(\overline{\rho T})} - \frac{\Delta s}{\tau_c}$$

$$\frac{(\nabla \cdot \vec{F}_R - \overline{\nabla \cdot \vec{F}_R})}{(\overline{\rho T})} = -\omega_R \Delta s$$

$\omega_R$  Characteristic frequency of radiative energy lost by turbulent eddies

$\tau_c$  Life time of the convective elements

$\Gamma^{-1} = \omega_R \tau_c$  Convective efficiency

In the static case, assuming constant coefficients ( $\text{Hp} \gg 1$  !), we have solutions which are plane waves identical to the ML solutions.

# Convection – pulsation interaction: Gabriel's theory

Perturbation of the mean equations

Linear pulsation equations

## Equation of mass conservation

$$\frac{\delta\rho}{\rho} + \frac{1}{r^2} \frac{d(r^2 \delta r)}{dr} - l(l+1) \frac{\delta r_H}{r} = 0$$

## Radial component of the equation of momentum conservation

$$\sigma^2 \delta r = \frac{d\delta\Phi}{dr} - \frac{1}{\rho} \frac{d\delta p_g}{dr} - \frac{1}{\rho} \frac{d\delta p_{\text{turb}}}{dr} + g \frac{\delta\rho}{\rho} + \frac{2A-1}{A} \frac{\bar{p}_T}{r} \frac{\partial \delta r}{\partial r} - \delta(\nabla_j \bar{\beta}_T^{rj})$$

## Transversal component of the equation of momentum conservation

$$\sigma^2 \delta r_H = \frac{1}{r} \left( \delta\Phi + \frac{\delta p}{\rho} + \frac{r \text{Visc}H}{\bar{\rho}} + \frac{2A-1}{A} \frac{\bar{p}_T}{\bar{\rho}} \left( \frac{\delta r}{r} - \frac{\delta r_H}{r} \right) \right)$$



# Convection – pulsation interaction: Gabriel's theory

## Equation of Energy conservation

$$\begin{aligned}
 i\sigma T \delta s = & \delta \varepsilon_N + l(l+1) \frac{\delta r_H}{r} \frac{dL}{dm} - \frac{d\delta L_R}{dm} - \frac{d\delta L_C}{dm} \\
 & + \frac{l(l+1)}{4\pi r^3 \rho} \left( L_R \left( \frac{\delta T}{r(dT/dr)} - \frac{\delta r}{r} \right) - L_C \frac{\delta r_H}{r} \right) \\
 & + \frac{l(l+1)}{\bar{\rho} r} FCH + \underline{\underline{\delta \varepsilon_2}} + \underline{\underline{\delta \left( \vec{V} \cdot \frac{\nabla(p_G + p_R)}{\bar{\rho}} \right)}}
 \end{aligned}$$

*FCH* : Amplitude of the horizontal component of the convective flux

# Convection – pulsation interaction: Gabriel's theory

## Convective flux perturbation

**Convective Flux :**  $\vec{F}_C = \overline{\rho T \Delta s \vec{V}}$

**Perturbation :**  $\delta \vec{F}_C = \vec{F}_C \left( \frac{\delta \rho}{\bar{\rho}} + \frac{\delta T}{\bar{T}} \right) + \bar{\rho} \bar{T} \left( \delta(\Delta s) \vec{V} + \Delta s \delta \vec{V} \right)$

$$\frac{\delta F_{Cr}}{F_{Cr}} = a_1 \frac{\delta r}{r} + a_2 \frac{d\delta r}{dr} + a_3 \frac{\delta r_H}{r} + a_4 \frac{\delta \rho}{\rho} + a_5 \frac{\delta s}{c_v} + a_6 \frac{\delta l}{l} + a_7 \frac{d\delta s}{ds}$$

$$a_7 \cong \frac{\omega_R \tau_C + 1}{i \sigma \tau_C + \omega_R \tau_C + 1} \cong \frac{1}{i \sigma \tau_C} \quad \text{where convection is efficient}$$

# Convection – pulsation interaction: Gabriel's theory

## Turbulent pressure perturbation

**Turbulent pressure :**  $P_{\text{turb}} = \rho V_r^2$

**Perturbation :**  $\frac{\delta P_{\text{turb}}}{P_{\text{turb}}} = \frac{\delta \rho}{\rho} + 2 \frac{\delta V_r}{V_r}$

$$\frac{\delta V_r}{V_r} = b_1 \frac{\delta r}{r} + b_2 \frac{d\delta r}{dr} + b_3 \frac{\delta r_H}{r} + b_4 \frac{\delta \rho}{\rho} + b_5 \frac{\delta s}{c_v} + b_6 \frac{\delta l}{l} + b_7 \frac{d\delta s}{ds}$$

$$b_7 \cong \frac{a_7}{i \sigma \tau_c} \quad \text{near the surface}$$

## Perturbation of the mixing-length

$$\frac{\delta l}{l} = \frac{\delta H_p}{H_p}$$

$H_p$  Pressure scale height

# Convection – pulsation interaction: the Solar case

Difficulties:

2. Treatment of turbulent pressure perturbation

Local analysis

$$\delta X(r, t) = \delta X_0 e^{\lambda r + i\sigma t}$$

$$\lambda \delta p_g / p_g + \lambda^2 \varepsilon \delta s / c_p = \dots$$

Movement

$$\lambda \delta s / c_p + \lambda b \delta p_g / p_g = \dots$$

Transfer

...

Characteristic polynomial

$$\lambda^2 \varepsilon P_3(\lambda) + P_{40}(\lambda) = 0$$

New terms

$\varepsilon = 0$

New root:  $\lambda = 1/(b \varepsilon) \rightarrow \infty$  at the convective boundaries

The worse numerical case gives the best fits with observations

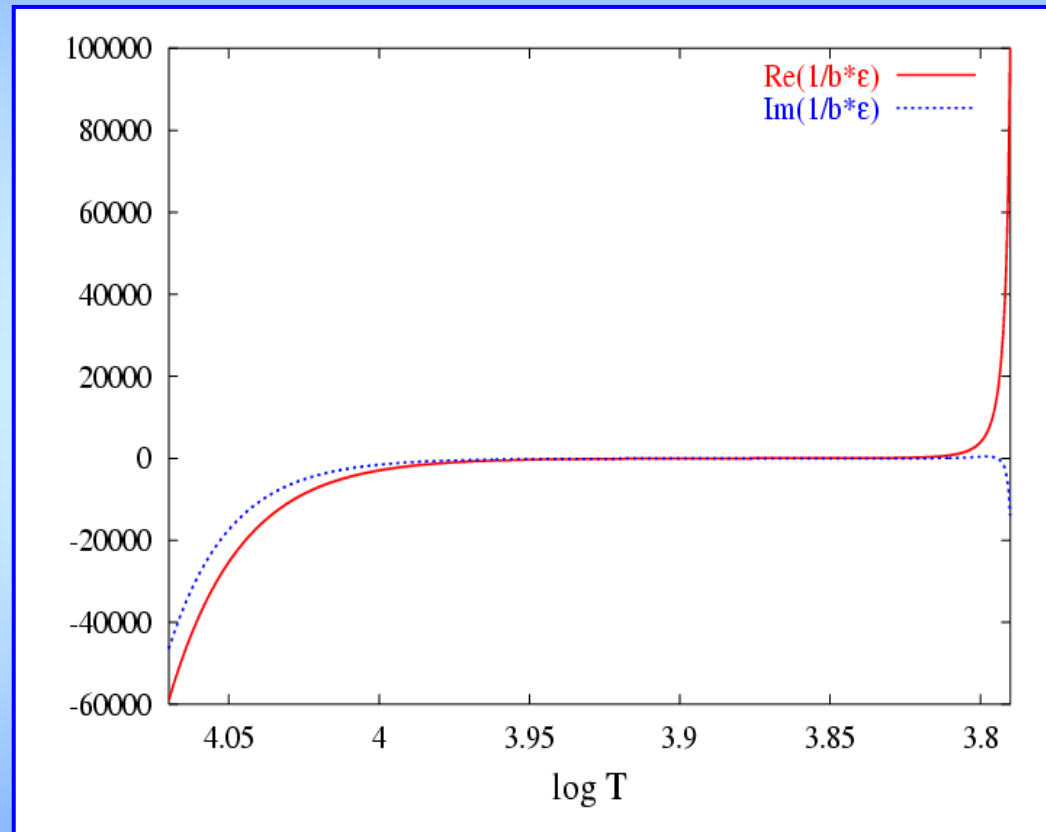
$\text{Re}(1/(b \varepsilon)) < 0$  at the left boundary

$\text{Re}(1/(b \varepsilon)) > 0$  at the right boundary

# Convection – pulsation interaction: the Solar case

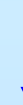
Difficulties:

2. Treatment of turbulent pressure perturbation



Example:

$$\frac{dz}{dx} = \frac{2x - i(1 + x^2)}{(1 - x^2)^2} z$$



$$z(x) = c \exp\left(\frac{1}{1 - x^2} - i \frac{x}{1 - x^2}\right)$$

The worse numerical case gives  
the best fits with observations

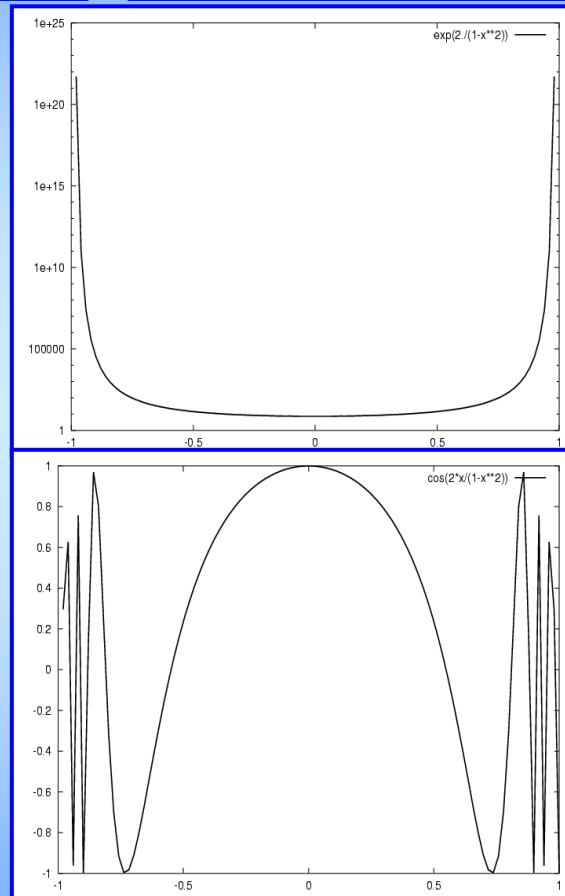
$\text{Re}(1/(b \epsilon)) < 0$  at the left boundary  
 $\text{Re}(1/(b \epsilon)) > 0$  at the right boundary

117

# Convection – pulsation interaction: the Solar case

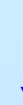
Difficulties:

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The worse numerical case gives the best fits with observations

$\text{Re}(1/(b \epsilon)) < 0$  at the left boundary  
 $\text{Re}(1/(b \epsilon)) > 0$  at the right boundary

# Plan of the presentation

1. Introduction

2. Convection-pulsation interaction

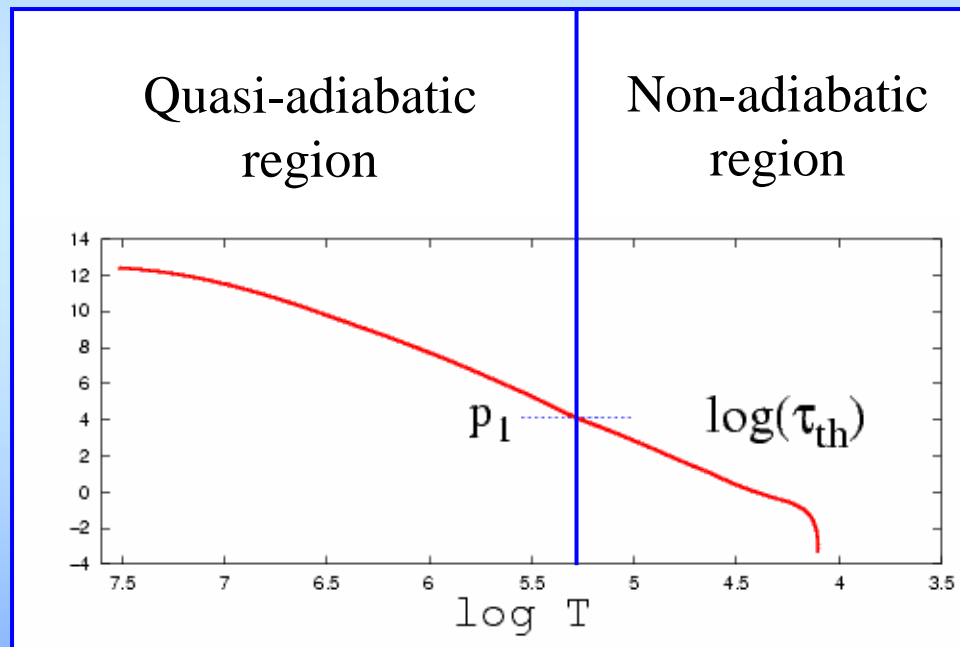
2.1. The MLT theory of Gabriel

2.2. The case of solar-like oscillations

3. Confrontation to observations

4. Conclusions

# Non-adiabatic stellar oscillations



$$\delta S \neq 0$$



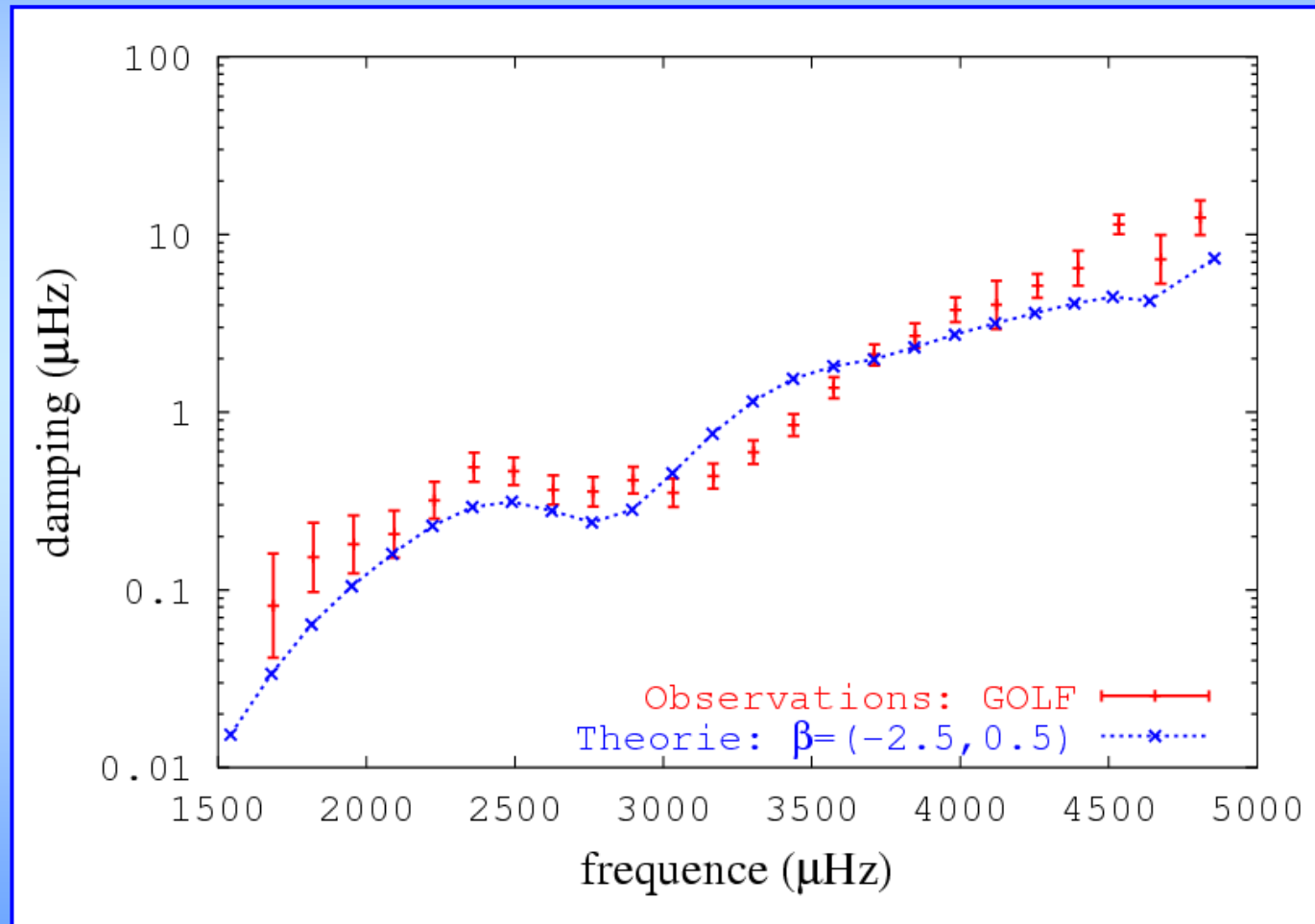
Coupling between

- the **dynamical** equations
- the **thermal** equations



## Confrontation to observations

Theoretical damping rates  $\leftrightarrow$  line-widths observed by GOLF



## Confrontation to observations

Theoretical damping rates  $\leftrightarrow$  line-widths observed by GOLF

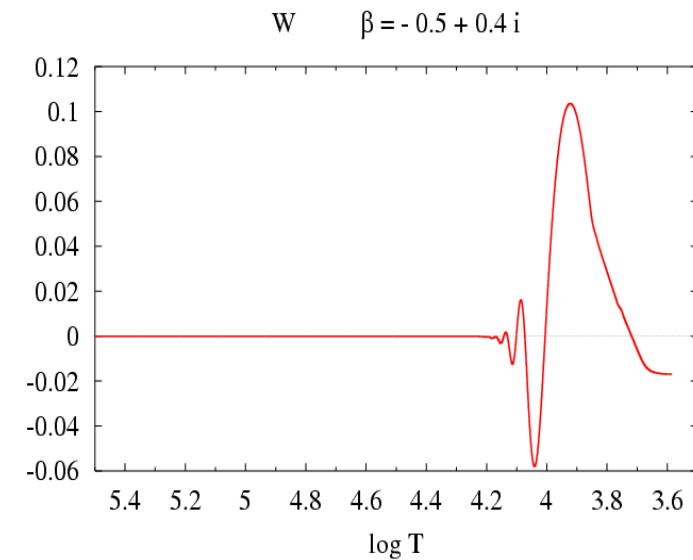
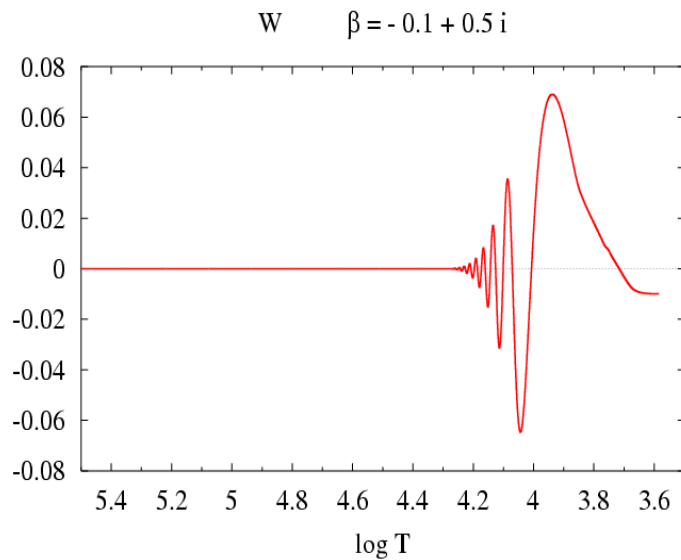
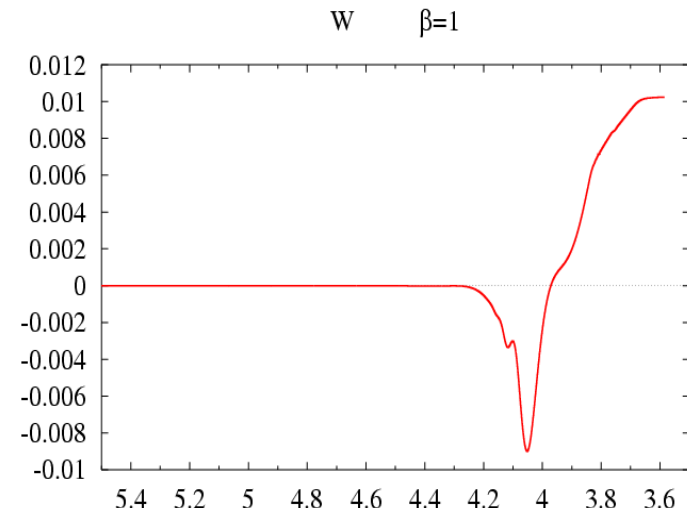
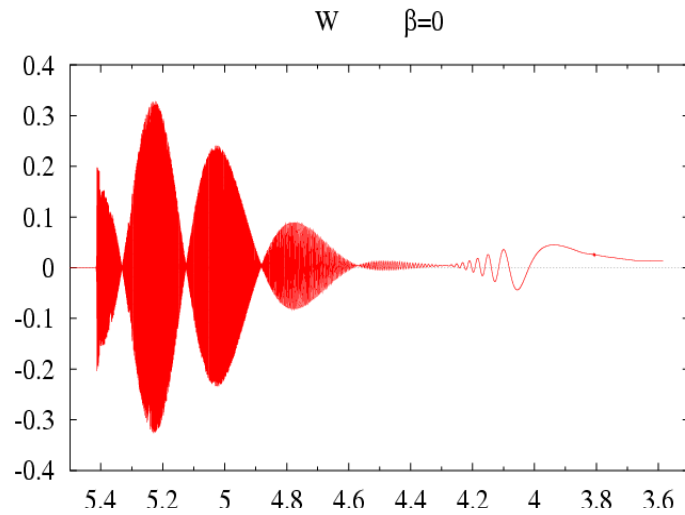
### Problem:

This solution fitting very well the observations is subject to numerical instabilities near the upper boundary of the convective envelope.

They come from the turbulent pressure perturbation term.

# Confrontation to observations

## Theoretical work integrals for different $\beta$ (mode $l=0, p_{22}$ )

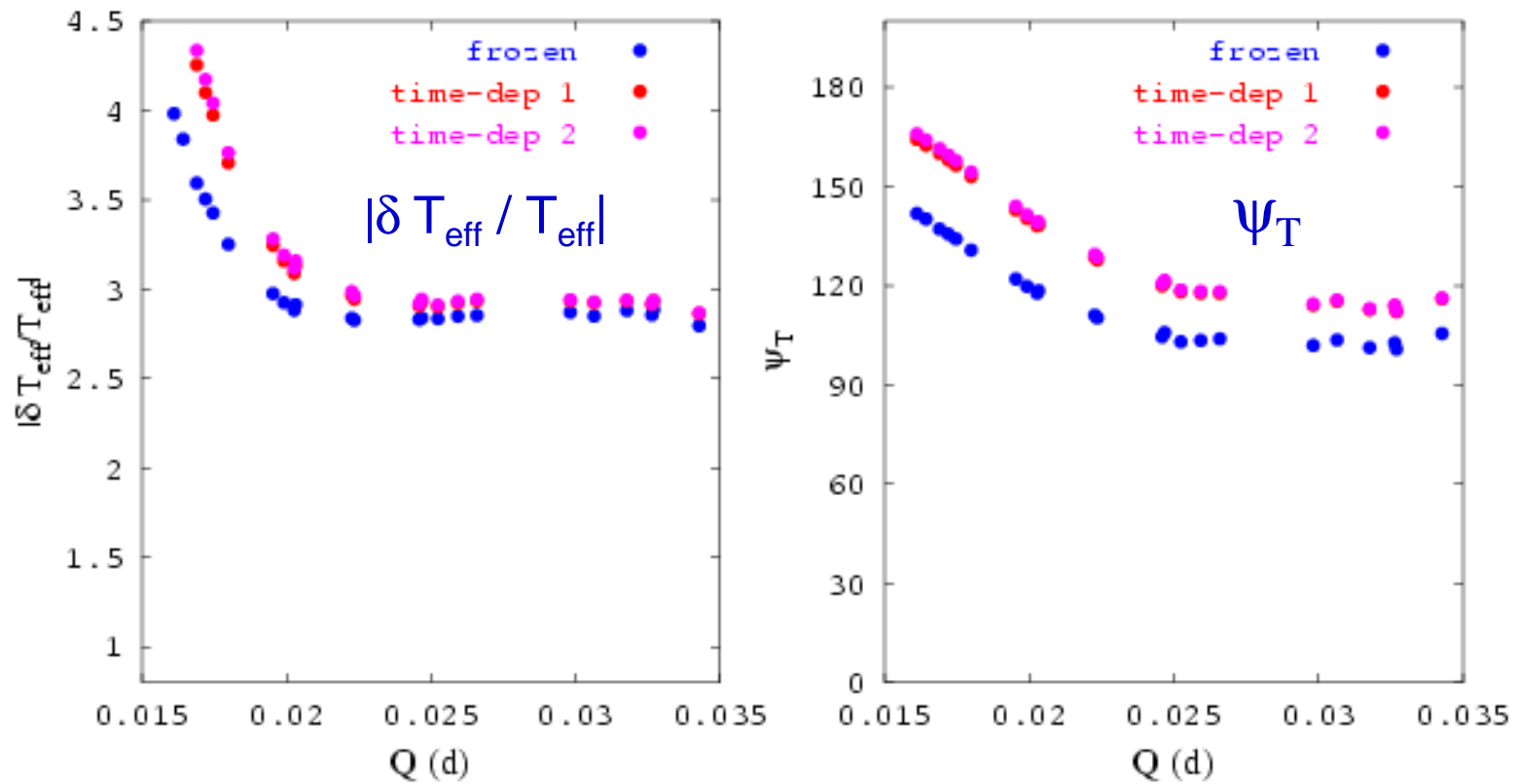


$\delta$  Scuti

# Amplitudes et phases photométriques

Convection dépendant du temps

$M = 1.8 M_0$  -  $T_{\text{eff}} = 7150 \text{ K}$  -  $\alpha = 0.5$

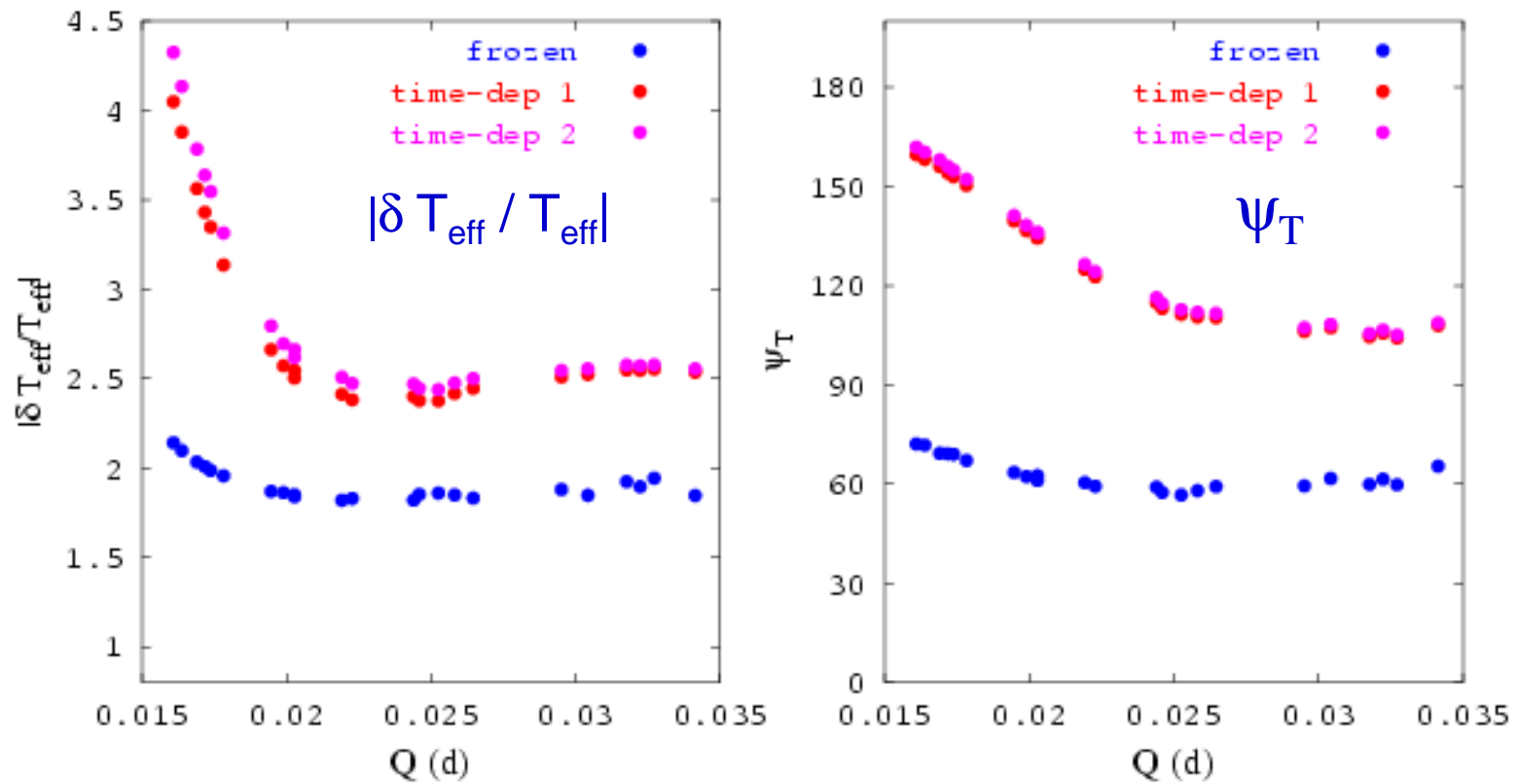


$\delta$  Scuti

# Amplitudes et phases photométriques

Convection dépendant du temps

$M = 1.8 M_0$  -  $T_{\text{eff}} = 7130$  K -  $\alpha = 1$

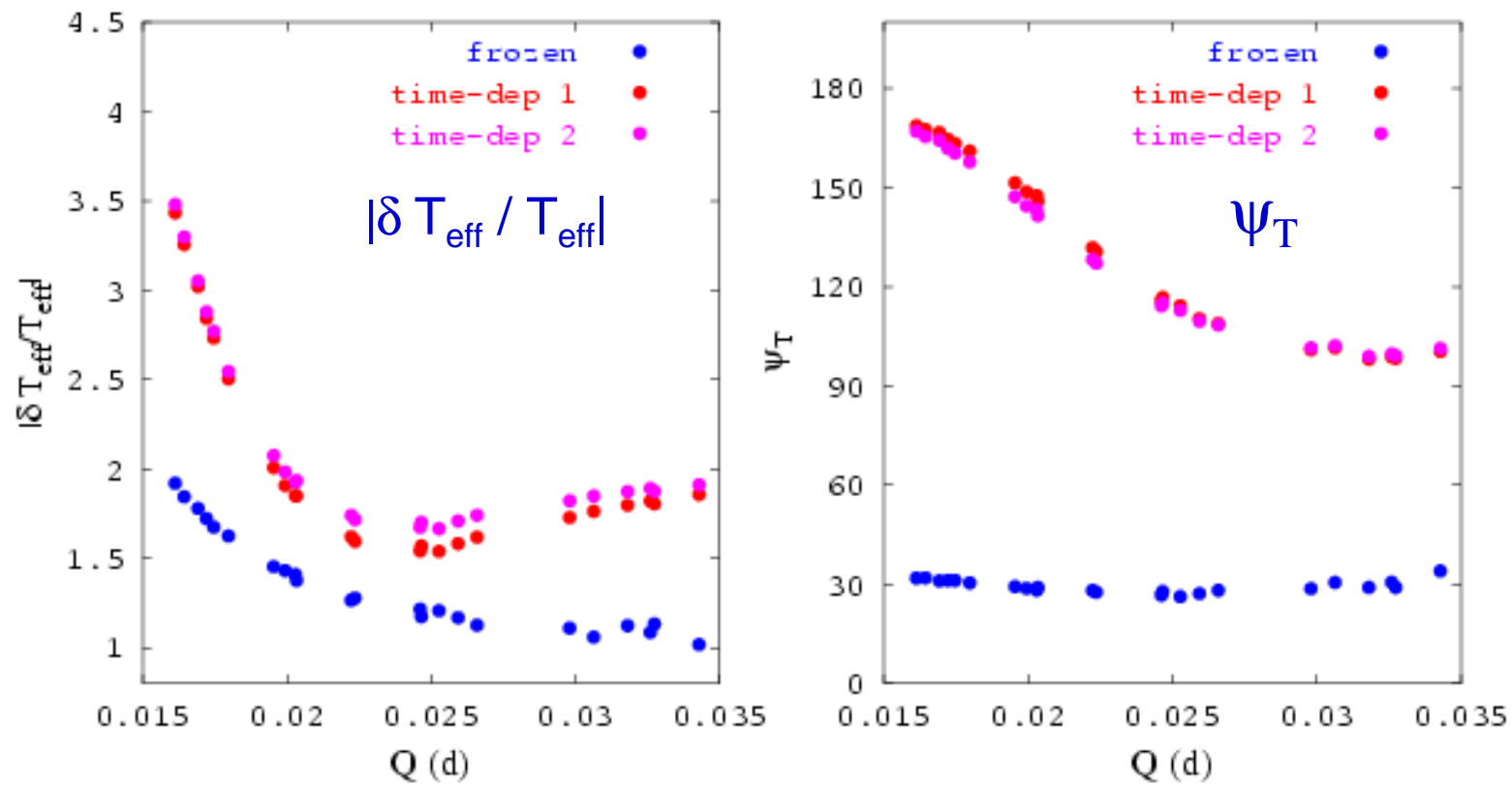


$\delta$  Scuti

# Amplitudes et phases photométriques

Convection dépendant du temps

$M = 1.8 M_0$  -  $T_{\text{eff}} = 7150 \text{ K}$  -  $\alpha = 1.5$



# Internal physics:

# Propagation cavities

$\xi_{50}$

